ASYMMETRIES AND TAILS IN STOCK INDEX RETURNS: ARE THEIR DISTRIBUTIONS REALLY ASYMMETRIC?

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Abstract: This paper examines the symmetry of the distribution of four major stock index returns: Standard and Poor's 500, Dow-Jones Industrial, Nikkei 225, and Financial Times 100, from the stock markets of New York, Tokyo and London. The symmetry of the whole distributions, of the different intervals, and of the tails, is analysed. Clear, strong asymmetries are not found. In particular, for different stock indexes and for different sample periods, the probabilities of occurrence of extreme downward and upward movements do not seem to be different.

Key words: Skewness, stock indexes, symmetry, tails.

Acknowledgement

I am very grateful to three anonymous referees for useful comments and suggestions. Financial support from the Ministerio de Ciencia y Tecnología (BEC2001-3126) is gratefully acknowledged.

1. Introduction

Though the unconditional distribution of stock returns has been studied in depth in the last decades, the possible asymmetry of their density function remains an open question. At present, no clear consensus has been reached on the existence of asymmetries, their characteristics and their importance. In fact, financial analysis has traditionally been restricted to the first and second order moments of the distribution of returns. As this practice, however, is appropriate only under the questionable assumptions of quadratic utility functions or normality of returns, several contributions have allowed for the third order moment or skewness.

Arditti and Levy (1975) and Kraus and Litzenberger (1976) have proposed financial models with three parameters that incorporate the effect of skewness on valuation. Simkowitz and Beedles (1978) and Conine and Tamarkin (1981) have explained the low diversification of many investors' portfolios by the preference for positive skewness, and Prakash *et al* (1996), Golec and Tamarkin (1998) and Garrett and Sobel (1999) have argued that this preference would explain why rational people take unfair gambles. Lai (1991) and Chunhachinda *et al* (1997) have analysed the problem of portfolio selection taking into account the skewness of returns. Corrado and Su (1996 and 1997) attribute the anomaly known as 'volatility skew' in option pricing to the skewness and kurtosis of the returns' distribution. Harvey and Siddique (1999) extend autoregressive conditional heteroskedasticity (ARCH) models by incorporating conditional skewness, and Harvey and Siddique (2000) present an asset pricing model that includes conditional skewness.

Though all these contributions, among many others, are based on the asymmetric distribution of returns, it is surprising that the number of studies that specifically address the question of the existence and types of asymmetries is so sparse. Often, the existence of strong asymmetries that justify all the different financial models that take these supposed asymmetries into account is implicitly assumed. Four examples will illustrate this presumption. They have been obtained from four excellent articles published recently, two of which in this same journal. Harvey and Siddique (1999) begin their paper on autoregressive conditional skewness with the following words: "Skewness, asymmetry in distribution, is found in many important economic variables such as stock index returns.” Chen *et al* (2001) also begin their paper as follows: “Aggregate stock market returns are asymmetrically distributed (...) the very largest movements in the market are usually decreases, rather than increases (...) of the ten biggest one-day movements in the S&P500 since 1947, nine were declines.” In a survey on stylised facts of asset returns, Cont (2001) enumerates 11 stylised facts “obtained by taking a common denominator among the properties observed in studies of different markets and instruments.” Cont (2001) states as the third of these 11 stylised facts: “Gain/loss asymmetry: one observes large drawdowns in stock prices and stock index values but not equally large upward movements.” In another survey on volatility, Engle and Patton (2001) use daily close data on the Dow Jones Industrial Index over the period from 23 August 1988 to 22 August 2000, and report that: “The skewness coefficient indicates that the returns distribution is *substantially* negatively skewed; a common feature of equity returns.” (Engle and Patton 2001, p. 241, italics added). I think that these assertions are disputable, and that it is not obvious that asymmetry should be considered as a stylised fact, or that returns distribution is substantially negatively skewed.

This paper aims to question or, at least, to qualify these statements, by studying the existence of asymmetries in the whole distribution of stock index returns, and, specially, in the tails. In order to cast some light on these issues, section 2 presents the data used, daily returns from four stock indexes. In section 3 symmetry is examined from three perspectives, which, listed in order of generality, are the following: analysis of the whole distribution of returns, analysis of the different intervals of the distribution, and analysis of the tails of the distribution. Finally, section 4 concludes.

2. Data

In what follows I have used the daily returns of four stock indexes. I collected the Standard & Poor’s 500 Composite (SP), Dow-Jones Industrial (DJ), Nikkei 225 (NI) and Financial Times 100 (FT) of the stock exchanges of New York (the first two), Tokyo and London. Once Saturdays and Sundays were excluded from the samples, the series still had some missing values due to non-trading days. Daily returns were obtained by logarithmic differences; that is by $R_t = \ln(I_t / I_{t-1})$, where R_t is the return for day t and I_t is the daily index for the same day. The returns whose calculation involved missing values have been considered missing values. So, all the observations are one-day returns, excluding the Monday ones, which are three-day returns. For SP, DJ and NI, the returns are from 3 January 1980 to 14 August 2002, and they provide 5535 observations for SP and DJ, and 5326 observations for NI. FT returns extend from 3 January 1984 to 14 August 2002, and they imply 4599 observations.

With regard to these data, two important features must be stressed. First, the samples cannot be considered completely independent of each other, due to the integration of capital markets, as is clearly indicated by the common movements in these markets. Especially in recent years, the correlations between the returns of different markets have been considerable, thus proving a certain degree of linear relationship. Secondly, all the returns used are index returns. Therefore, the results really refer to portfolio returns, of equal composition and weights to those of the index, and not to individual stocks.

For each market, some basic statistics on the return distribution are shown in Table 1. Under normality, the coefficient of skewness follows asymptotically a N(0, 6/T) distribution, where T is the sample size. In all cases the coefficients of skewness are significantly different from zero. When comparing panels A and B in Table 1, it is interesting to observe the dramatic changes that the coefficients of skewness, as well as the other statistics that involve high order moments, experiment when excluding a few extreme observations. Thus, panel B in Table 1 shows the same statistics as panel A once the observation corresponding to the crash of October 1987, the ten preceding and the ten subsequent returns are excluded. In any case, these departures of the coefficient of skewness from zero must be understood as rejections of normality, rather than rejections of symmetry. In fact, the evidence of non-normality is overwhelming for daily returns. This is confirmed in Table 1 by the kurtosis, Jarque-Bera and Kolmogorov-Smirnov tests, which clearly reject normality.

3. Symmetry or asymmetry of returns

3.1. Asymmetries in the whole distribution

Given these rejections of normality, in order to test for symmetry distribution-free procedures should be used. If the distribution of returns is symmetric, then the median must necessarily be the axis of symmetry and coincides with the mean, if it exists. When the whole distribution is shifted by subtracting the mean from the returns, the new axis of symmetry is the origin. The symmetry of the distribution of returns will imply the symmetry of those new returns in excess of the mean about zero. Following Peiró (1999 and 2002), the symmetry of returns may then be tested by comparing the distribution of the negative excess returns,

$$R_t^- : \left\{ R_t - \overline{R} \,\middle|\, R_t < \overline{R} \right\},$$

taken in absolute values, with the distribution of the positive excess returns,

$$R_t^+ : \left\{ R_t - \overline{R} \,\middle|\, R_t > \overline{R} \right\}.$$

The comparison may be carried out with conventional tests, such as the t-test for the equality of means or, given the clear non-normality, with distribution-free tests, such as the Wilcoxon (W), Siegel-Tukey (ST) or Kolmogorov-Smirnov (KS) two-sample tests. In all cases, the null hypothesis states the equality of the distributions of both types of excess returns, but some of these tests are more sensitive to certain differences in the distributions than others (Gibbons and Chakraborti 1992). Table 2 shows the results of these tests. The null hypothesis of equal distributions can never be rejected at the 5% significance level for SP and DJ. Therefore, the distributions of these excess returns seem to be symmetric about zero, and, consequently, observed returns seem to be symmetric about their mean. For NI and FT returns, neither the t-, W- or the KS-tests allow the rejection of the null of symmetry at the 5% significance level. Nevertheless, the ST-tests clearly reject the null of symmetry. As this test is specially sensitive to differences in dispersion, these results suggest a different dispersion of negative and positive excess returns. In fact, for NI returns, the sample standard deviation of negative excess returns is clearly higher than that of positive excess returns (0.0097 and 0.0092, respectively); the difference is even larger for FT returns (0.0078 and 0.0064, respectively). Given the nature of the *ST*-test, these rejections are not due to a few extreme returns; on the contrary, they are probably due to a substantial proportion of returns. This is the only sign of asymmetry that can be observed in these indexes, and is consistent with Peiró (1999 and 2002), which analyse shorter sample periods, and other stock indexes or individual stocks.

These tests, however, may have little power, a common characteristic of many distribution-free tests. It is the price of not relying on a hypothetical or arbitrary distribution, given the absence of normality. Therefore, it would be interesting to examine the power of these tests under different distributions. In these circumstances, there arises the problem of the selection of the distributions. One can always select a strongly asymmetric distribution, and the tests will possibly have high power. On the other hand, one can use a distribution whose asymmetry is extremely weak and, then, the power will probably be low. To avoid an arbitrary choice, nine distributions, which have been widely used to examine the properties of different symmetry tests (Randles *et al* 1980, McWilliams 1990), will be considered. They are obtained from the generalised lambda family with four parameters, l_1, l_2, l_3, and l_4, which include the original symmetric lambda distribution, when $l_3 = l_4$, and unimodal asymmetric distributions, when $l_3 \neq l_4$ (see Ramberg and Schmeiser, 1974). 2000 samples of size N, N = 100, 1000, and 5000, were generated for each of the generalized lambda distributions, with parameter values equal to those shown in Table 3. The shapes of the density functions can be seen in McWilliams (1990). The samples were generated by using the inverse cumulative distribution function,

$$F^{-1}(u) = l_1 + \frac{u^{l_3} - (1-u)^{l_4}}{l_2},$$

where u is a uniform (0, 1) random variable.

Table 3 shows the performance of these tests, under one symmetric distribution (D1) and under eight asymmetric distributions (D2-D9). The *ST* test holds the α level equal to 0.05 very well. The *W* statistic increases the level somewhat, and does not seem to approach 0.05 as the sample size increases. More interestingly, the power of the three tests is relatively high for most asymmetric distributions. This is unambiguous for the distributions that are strongly asymmetric (D2, D3, D8 and D9), but not for the distributions slightly asymmetric (D4, D5, D6 and D7). In particular, the power for D6 (a distribution very weakly asymmetric) and sample size equal to 100 is extremely low. However, it tends clearly to unity as sample size increases, and for sample sizes close to those of the stock indexes, such as $N = 5000$, the power is virtually equal to unity. Therefore, the simulations indicate that these distribution-free tests may be useful in the detection of asymmetry, specially when large sample sizes are used as is generally the case in the analysis of daily financial returns. Of course, these results do not preclude the possibility that stock index returns present a distribution so slightly asymmetric that symmetry cannot be rejected in these (or any other) tests. However, in that case, one could not maintain that stock index returns are *clearly or strongly* asymmetric.

3.2. Asymmetries in the different intervals

As the preceding tests examine the whole density function of returns, they may have little power against specific asymmetries in certain ranges of the distribution. In other words, particular asymmetries (for example, in a certain interval) could exist that cannot be detected with those tests that consider the whole distribution. These possible asymmetries may only be detected by examining the different intervals of the distribution more closely. Figure 1 shows the histogram of SP excess returns. Though the symmetry of the whole distribution cannot be rejected, one could think that the frequency of excess returns comprised, for example, between -0.5% and 0% (1382) is substantially different from the frequency of excess returns comprised between 0% and $+0.5\%$ (1311). A formal comparison can be made by using the binomial distribution. Given the number of excess returns comprised in two certain intervals symmetric with respect to the origin, *n*, if the distribution is symmetric, then both the number of negative and positive excess returns will follow binomial distributions with parameters *n* and *p*, with $p = 0.5$. Therefore, the following test:

$$\mathrm{H}_0 : p = 0.5$$
$$\mathrm{H}_1 : p \neq 0.5,$$

can be carried out, where the null states that both types of excess returns have the same probability (0.5). If the null is rejected, then the probabilities of returns being comprised in both intervals will be different, and asymmetry will follow. Binomial tables provide the different probabilities for $p = 0.5$, but are usually available for $n \leq 20$. The binomial distribution with parameters n and p may be approximated by the normal distribution with parameters np and $np(1-p)$. The approximation is especially good when $p = 0.5$, as is the case. As a discrete distribution is approximated by a continuous distribution, a continuity correction of 0.5 must be included. Then, the *P*-values of the test will be computed as:

$$2\left(1-\Phi\left(\frac{Max\left(n^-,\, n^+\right)+0.5-np}{\sqrt{np(1-p)}}\right)\right) = 2\left(1-\Phi\left(\frac{Max\left(n^-,\, n^+\right)+0.5-n/2}{\sqrt{n/4}}\right)\right),$$

where n^- (n^+) is the number of negative (positive) excess returns, with $n^- + n^+ = n$, and Φ is the cumulative standard normal distribution function.

The result of this test in Table 4 shows that the number of SP excess returns comprised between -0.5% and 0% is not significantly different from the number comprised between 0% and $+0.5\%$, as the null hypothesis of equal probabilities cannot be rejected (*P*-value equal to 0.17). The same occurs for most intervals in the different indexes. Only significant differences are observed for negative and positive excess returns whose absolute values are comprised between 1.0% and 1.5% in SP, and between 0.5% and 1.0% in NI and FT. Thus, Table 4 shows that DJ does not present any difference in the intervals, SP and FT present only one difference at the 5% significance level, and NI presents only one, which is significant at the 1% level (*P*-value equal to 0.0001). Other interval divisions (both shorter and longer intervals) yielded similar results. Therefore, it is reasonable to conclude that asymmetry is not a strong characteristic of stock index returns, though some asymmetries are observed in excess returns whose absolute values are comprised between 0.5% and 1.5%.

3.3. Asymmetries in the tails

Let us now have a closer look at the tails of the distribution. The tails are specially important from a financial point of view, for example in value at risk analysis. The definition of the tails of the distribution (or, more exactly, the points where the left tail finishes and the right tail begins) is arbitrary. If we define the tails as those areas below -3% and beyond $+3\%$, SP tails include 70 excess returns. 37 of these 70 excess returns are lower than -3%, and 33 are higher than $+3\%$ (see Table 4). Analogously, for the other indexes there are more negative excess returns than positive ones (in line with the statements of Chen *et al* 2001, Cont 2001 and Engle and Patton 2001), but the differences are not significant in any case.

The preceding definition of the tails implies a number of returns comprised between 70 (for SP) and 189 (for NI); that is, between 1.3% (for SP) and 3.5% (for NI) of the total number of returns. Similar results are found when one considers a wider definition of the tails (for example, excess returns below -2% and over $+2\%$). Conversely, if one wishes to focus only on the most extreme movements, one could wonder whether these are negative or positive. That is, one could wonder, with the words of Cont (2001), whether the drawdowns are larger than the upward movements in stock index values.

Table 5 shows the 20 most extreme excess movements in these stock indexes and their corresponding dates. As these movements are very large compared with the mean of all returns, very similar results are obtained with excess returns or with observed returns. For SP, DJ and FT, the number of negative returns is higher than the number of positive returns. The contrary occurs for NI. However, usually these differences are not significant statistically. This can be tested with the binomial distribution, analogously to the previous sub-section. For different characterisations of the tails (that is, for different sets of the most extreme returns), Table 6 shows that the probabilities of the most extreme returns' being negative or positive are not different. This questions the statistical validity of the statements of Chen *et al* (2001) and Cont (2001), which were reproduced in section 1. Only when one considers the ten most extreme excess returns, are the differences significant for SP and NI at the 5% significance level, although not at the 1% level. The conclusion, then, is clear; although negative excess returns seem to be slightly more frequent among the most extreme in three series, the null hypothesis of equal probability cannot be rejected in most cases.

Of course, one could think that this conclusion is due to the stock indexes and sample periods taken into account. With regard to the first objection, it is important to note that these indexes are undoubtedly among the main stock indexes in the world. With regard to the sample periods, these begin in 1980 (1984 for FT), which implies more than twenty years of daily returns. This is a rather long sample period. Nevertheless, it is also opportune to report two additional pieces of evidence that cover longer periods.

Schwert (1989) examines the largest increases and decreases in daily returns from 1885 to 1987. This sample period covers more than a century, and includes almost 30,000 daily returns. The series includes DJ returns from 1885 through 1927, and SP returns from 1928 to 1987. In Schwert (1989, Table 1), the 50 largest increases and decreases are listed by order of magnitude.[1] Table 7 shows the number of negative and positive returns among the 10, 20, 30, 40 and 50 most extreme returns listed in Schwert (1989). Table 7 also shows that, in clear opposition to Chen *et al* (2001) and Cont (2001), the number of positive returns is always higher than the number of negative returns, and presents the results of the tests for these differences following the same procedure outlined above. The null hypothesis of equal probabilities cannot be rejected in any case.

With regard to the Nikkei index, on the Nihon Keizai Shimbun (Nikkei) website (http://www.nni.nikkei.co.jp/FR/SERV/nikkei_indexes/nifaq225.html) one can find the top 10 percentage gains and the top 10 percentage falls in Nikkei Stock Average since 1949. Table 8 reproduces the seventeen (the maximum number of returns listed in the website that can be ordered by absolute value) most extreme returns. Though the most extreme observed returns are virtually identical to excess returns (as the mean is very low in comparison with them), the returns in the period 1980-2002 in this Table 8 do not coincide exactly with those in Table 5 for two reasons. First, the returns in Table 8 have been calculated as $R_t^* = \left(I_t - I_{t-1}\right)/I_{t-1}$, and not as logarithmic differences. When

[1] Schwert (1990) lists only the 25 largest increases and decreases.

one transforms the returns in Table 8 in continuously compounded returns, $R_t = \ln\left(1 + R_t^*\right)$, they coincide exactly with those in Table 5. Second, according to the criteria exposed in section 2, the return from 21 March 2001 has not been included in Table 5 as the preceding day, 20 March 2001, Tuesday, was not a trading day in Tokyo's exchange. Among the ten most extreme returns in Table 8, four are negative and six positive (P-value equal to 0.34), and among the seventeen most extreme returns, ten are negative and seven positive (P-value equal to 0.33). Therefore, the evidence for these two very long daily series does not allow rejecting the hypothesis of equal probability of extreme rises and falls in stock markets.

Before concluding, some reflections on the extensions and implication of the results contained herein are interesting. First, though the two most extreme SP excess returns occurred on Monday, and though Monday returns present a higher dispersion than those of the other days (probably due to the longer generating interval), asymmetries linked to the day of the week are not observed. The results concerning the symmetry of the distributions hold after excluding these Monday returns. Second, weekly and monthly returns, obtained by aggregation of daily returns, do not present clear asymmetries when analysed with the same methods outlined above either. This is not surprising, as one may invoke a central limit theorem, which, under relatively general conditions, implies convergence to normality (and, therefore, to symmetry), even if daily returns are not symmetric (which does not seem to be the case). Third, for the same reason, individual stocks could well present stronger asymmetries than stock indexes, as these are obtained as averages of the first, and convergence to normality could take place. Further research should examine the behaviour of individual stocks. Fourth, as commented on in section 1, the implications of the hypothesis of symmetry are numerous. They question different financial models that incorporate skewness as a crucial assumption. In particular, models such as those proposed in Corrado and Su (1996 and 1997) or Harvey and Siddique (1999 and 2000) are clearly questioned. Finally, a caveat is in order. In section 2, the interdependence of the movements in the different indexes and markets has been stressed. With respect to the largest movements, the interdependence seems to be stronger. This is evident for SP and DJ, but the interdependences are also very tight between the different markets. Thus, for example, according to Table 5, in the period 1980-2002, the largest drop in SP and DJ occurred on the same day, 19 October 1987, and on the following day in NI and FT. The four largest movements in SP occurred on the same day as in DJ. Two of the four largest movements in NI, and the four largest movements in FT also occurred on the same day (or on a contiguous day) as the four largest movements in SP. Therefore, the evidence reported for each of these indexes should not be considered completely independent of each other.

4. Conclusions

The presumption of skewness or asymmetry in financial returns is implicitly or explicitly assumed in many financial models. In fact, several recent prominent contributions declare or are based on this property. Nevertheless, in spite of its crucial role, empirical research has hardly ever examined the existence and types of asymmetries in financial series. In the preceding pages, the possible existence of asymmetries has been examined in the whole distribution, in the different intervals and in the tails of four major stock index returns: Standard & Poor's 500 Composite, Dow-Jones Industrial, Nikkei 225 and Financial Times 100.

Distribution-free methods have been used in the analysis of (a)symmetry. Simulation results suggest that these tests have good properties for large sample sizes. Asymmetries are not found when examining the whole distribution of SP and DJ returns, but one test (Siegel-Tukey) detects an asymmetric distribution in NI and FT returns, possibly due to a different dispersion in negative and positive excess returns. Other tests, such as the *t*-test and the Wilcoxon test, specially sensitive to differences in location, do not detect asymmetries in any series. Nor does the Kolmogorov-Smirnov test detect any clear asymmetry in any series. When looking with detail at the different intervals of the distributions, a few asymmetries are observed which are relatively limited. Similar results are obtained in the analysis of the tails. Though SP and FT present a significantly high proportion of negative returns among the ten most extreme returns in the period 1980-2002, this feature disappears when considering more extreme returns or when considering longer sample periods. All these results indicate that, though there could exist some specific asymmetries that are relatively weak, asymmetry or skewness is not a stylized fact of stock index returns.

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Table 1. Return statistics.

PANEL A				
	SP	DJ	NI	FT
Observations	5535	5535	5326	4599
Mean	0.0398%	0.0410%	0.0048%	0.0321%
Std. Dev.	0.0105	0.0107	0.0129	0.0104
Skewness	-2.019*	-2.640*	-0.163*	-0.886*
Kurtosis	47.19*	67.60*	12.90*	14.57*
Jarque-Bera	454,027*	968,947*	21,757*	26,253*
KS	0.071*	0.069*	0.082*	0.047*
PANEL B				
	SP	DJ	NI	FT
Observations	5514	5514	5305	4578
Mean	0.0445%	0.0461%	0.0072%	0.0403%
Std. Dev.	0.0098	0.0098	0.0126	0.0098
Skewness	-0.232*	-0.254*	0.169*	-0.182*
Kurtosis	6.91*	7.16*	8.48*	5.45*
Jarque-Bera	3,571*	4,043*	6,671*	1,174*
KS	0.057*	0.051*	0.078*	0.034*

Skewness: m_3 / s^3, *Kurtosis*: m_4 / s^4, *Jarque-Bera*: $T(Skewness^2/6 + (Kurtosis - 3)^2/24)$, where m_k is the central moment of order k, s^2 is the sample variance, $\overline{R}$ is the sample mean of returns and T is the number of observations. *KS* is the Kolmogorov-Smirnov statistic for testing normality. * denotes significant at the 1% level. Panel A uses all the returns in the sample, while in Panel B the observation corresponding to the crash of October 87 has been excluded, as well as the ten preceding and the ten subsequent daily returns.

Table 2. Symmetry tests.

	SP	DJ	NI	FT
t	0.19 (0.85)	0.26 (0.79)	1.14 (0.25)	1.78 (0.08)
W	1.66 (0.10)	0.67 (0.50)	0.60 (0.55)	0.22 (0.83)
ST	1.08 (0.28)	0.28 (0.78)	4.85 (0.00)**	2.66 (0.01)**
KS	0.03 (0.08)	0.02 (0.51)	0.04 (0.05)	0.03 (0.18)

t is the usual test statistic for equality of means. *W*, *ST* and *KS* are respectively the standardized Wilcoxon, the standardized Siegel-Tukey and the Kolmogorov-Smirnov two-sample test statistics for equality of distributions. *P*-values are in parenthesis and ** denotes statistics significant at the 1% significance level. In all cases, the first sample is formed by negative excess returns, and the second sample is formed by positive excess returns.

Table 3. Empirical rejection probabilities (%) for $\alpha = 0.05$

Distribution	Test	$N = 100$	$N = 1000$	$N = 5000$
Symmetric				
D1 (normal-like)	*KS*	8.2	7.6	6.4
	W	8.4	11.2	11.2
	ST	5.8	5.1	5.2
Asymmetric				
D2	*KS*	86.1	100	100
	W	59.3	99.9	100
	ST	81.9	100	100
D3	*KS*	99.7	100	100
	W	83.5	100	100
	ST	97.9	100	100
D4	*KS*	50.8	100	100
	W	38.2	99.8	100
	ST	49.0	100	100
D5	*KS*	70.2	100	100
	W	50.4	100	100
	ST	64.5	100	100
D6	*KS*	9.5	30.7	96.3
	W	7.8	30.3	85.0
	ST	6.7	34.7	94.0
D7	*KS*	27.5	99.6	100
	W	22.2	95.7	100
	ST	25.0	99.0	100
D8	*KS*	100	100	100
	W	91.9	100	100
	ST	99.2	100	100
D9	*KS*	100	100	100
	W	91.4	100	100
	ST	99.7	100	100

D1: $l_1 = 0$, $l_2 = 0.197454$, $l_3 = 0.134915$, $l_4 = 0.134915$
D2: $l_1 = 0$, $l_2 = 1$, $l_3 = 1.4$, $l_4 = 0.25$
D3: $l_1 = 0$, $l_2 = 1$, $l_3 = 0.00007$, $l_4 = 0.1$
D4: $l_1 = 3.586508$, $l_2 = 0.04306$, $l_3 = 0.025213$, $l_4 = 0.094029$
D5: $l_1 = 0$, $l_2 = -1$, $l_3 = -0.0075$, $l_4 = -0.03$
D6: $l_1 = -0.116734$, $l_2 = -0.351663$, $l_3 = -0.13$, $l_4 = -0.16$
D7: $l_1 = 0$, $l_2 = -1$, $l_3 = -0.1$, $l_4 = -0.18$
D8: $l_1 = 0$, $l_2 = -1$, $l_3 = -0.001$, $l_4 = -0.13$
D9: $l_1 = 0$, $l_2 = -1$, $l_3 = -0.0001$, $l_4 = -0.17$

Table 4. Tests of equal probability of negative and positive excess returns.

Index →	SP			DJ			NI			FT		
Intervals ↓	*Neg.*	*Pos.*	*P*	*Neg.*	*Pos.*	*P*	*Neg.*	*Pos.*	*P*	*Neg.*	*Pos.*	*P*
0.0%, 0.5%	1382	1311	0.17	1318	1314	0.92	1171	1153	0.69	976	1022	0.29
0.5%, 1.0%	727	784	0.14	784	778	0.86	632	783	0.00**	669	755	0.02*
1.0%, 1.5%	306	370	0.01*	339	378	0.14	335	342	0.76	338	360	0.38
1.5%, 2.0%	201	169	0.09	172	173	0.91	179	184	0.75	133	115	0.23
2.0%, 2.5%	72	78	0.57	78	73	0.63	131	105	0.08	66	61	0.59
2.5%, 3.0%	35	30	0.46	26	31	0.43	69	53	0.12	23	20	0.54
> 3.0%	37	33	0.55	40	31	0.24	101	88	0.31	37	24	0.07
All	2760	2775	0.83	2757	2778	0.77	2618	2708	0.21	2242	2357	0.09

For each index and for each interval, the column *Neg.* (*Pos.*) indicates the number of negative (positive) excess returns whose absolute values are comprised in the interval, and the column *P* shows the *P*-values corresponding to the test of equal probability of negative and positive excess returns. * (**) denotes significant at the 5% (1%) significance level.

Table 5. Most extreme excess returns (%), 1980-2002.

Ranking	SP		DJ		NI		FT	
1	–22.87	(191087)	–25.67	(191087)	–16.14	(201087)	–13.06	(201087)
2	–8.68	(261087)	9.63	(211087)	12.43	(021090)	–11.51	(191087)
3	8.67	(211087)	–8.42	(261087)	8.89	(211087)	7.57	(211087)
4	–7.15	(271097)	–7.50	(271097)	7.66	(171197)	–6.42	(261087)
5	–7.08	(310898)	–7.20	(131089)	7.55	(310194)	–5.92	(110901)
6	–7.05	(080188)	–7.14	(080188)	7.27	(100492)	–5.89	(221087)
7	–6.36	(131089)	–6.62	(310898)	–7.24	(170400)	–5.62	(150702)
8	–6.04	(140400)	6.11	(240702)	–6.87	(120901)	5.41	(100492)
9	5.53	(240702)	–5.86	(140400)	–6.83	(020490)	–5.11	(220702)
10	–5.34	(161087)	5.68	(201087)	6.37	(021181)	–4.90	(010802)
11	5.23	(290702)	5.23	(290702)	–6.14	(190891)	4.85	(250702)
12	5.09	(201087)	–4.97	(140488)	6.07	(070795)	–4.77	(190702)
13	–4.97	(110986)	4.80	(291087)	6.03	(210892)	4.50	(290702)
14	4.95	(281097)	–4.79	(190702)	–6.02	(230890)	–4.47	(301187)
15	4.85	(030101)	4.77	(160300)	6.01	(081082)	–4.43	(110702)
16	4.77	(291087)	–4.76	(110986)	5.99	(071098)	4.31	(170992)
17	4.61	(160300)	–4.75	(161087)	–5.96	(081098)	4.31	(121098)
18	4.61	(170882)	4.74	(170882)	5.95	(270892)	4.29	(061098)
19	–4.49	(140488)	4.56	(281097)	–5.77	(230195)	–4.19	(220301)
20	–4.45	(120301)	–4.51	(200901)	5.73	(040302)	–4.17	(051092)

Most extreme excess returns from 3 January 1980 to 14 August 2002, for SP, DJ and NI, and from 3 January 1984 to 14 August 2002, for FT. The corresponding dates are in parenthesis in the format *ddmmyy*.

Table 6. Tests of equal number of positive and negative most extreme excess returns, 1980-2002.

	SP			DJ			NI			FT		
	Neg.	*Pos.*	*P*	*Neg.*	*Pos.*	*P*	*Neg.*	*Pos.*	*P*	*Neg.*	*Pos.*	*P*
10	8	2	0.03*	7	3	0.11	4	6	0.34	8	2	0.03*
20	11	9	0.50	12	8	0.26	8	12	0.26	13	7	0.12
30	17	13	0.36	17	13	0.36	14	16	0.58	17	13	0.36
40	21	19	0.64	23	17	0.27	18	22	0.43	23	17	0.27
50	24	26	0.67	29	21	0.20	22	28	0.32	28	22	0.32

For each index and for the different numbers of most extreme excess returns indicated in the first column, the column labelled *Neg.* (*Pos.*) indicates the number of negative (positive) excess returns, and the column labelled *P* shows the *P*-values corresponding to the test of equal probability of negative and positive excess returns. * denotes significant at the 5% significance level. The sample period covers from 3 January 1980 to 14 August 2002, for SP, DJ and NI, and from 3 January 1984 to 14 August 2002, for FT.

Table 7. Tests of equal number of positive and negative most extreme daily returns, 1885-1987.

	Negative	Positive	*P*-value
10	4	6	0.34
20	8	12	0.26
30	14	16	0.58
40	18	22	0.43
50	23	27	0.48

Number of negative and positive returns and *P*-values corresponding to the test of equal probabilities of negative and positive returns for the different numbers of most extreme returns indicated in the first column. The series includes DJ returns from 1885 through 1927, and SP returns from 1928 to 1987. This Table has been elaborated from Schwert (1989, Table 1).

Table 8. Most extreme Nikkei daily returns (%), 1949-2002.

Ranking	Date	Return
1	October 20, 1987	–14.90
2	October 2, 1990	13.24
3	December 15, 1949	11.29
4	March 5, 1953	–10.00
5	October 21, 1987	9.30
6	April 30, 1970	–8.69
7	November 17, 1997	7.96
8	January 31, 1994	7.84
9	August 16, 1971	–7.68
10	April 10, 1992	7.55
11	March 21, 2001	7.49
12	April 17, 2000	–6.98
13	December 14, 1949	–6.97
14	March 30, 1953	–6.73
15	September 12, 2001	–6.63
16	June 24, 1972	–6.61
17	April 2, 1990	–6.60

Most extreme Nikkei daily returns from 1949 to 2002. Source: Nihon Keizai Shimbun. (http://www.nni.nikkei.co.jp/FR/SERV/nikkei_indexes/nifaq225.html)

Figure 1. Histogram of SP daily excess returns.