PUBLIC FUNDING OF POLITICAL PARTIES*

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A B S T R A C T

This paper concerns public funding of parties. Parties receive public funds depending on their vote share. Funds finance electoral campaigns. Two cases are investigated. In the first some voters are policy motivated and some are “impressionable” - their vote depends directly on campaign expenditures. In the second campaigning is informative and all voters are policy motivated.

Public funds increase policy convergence in both cases. The effect is larger, the more funding depends on vote shares. When campaigns are informative, there may be multiple equilibria. Intuitively, a large party can stay large since it receives large funds.

Keywords: Public Funding, Political Competition, Information.
1 Introduction

The funding of political parties is a fundamental aspect of democracy, but although most countries have some sort of public-funding policy for parties, the way it is provided varies considerably from country to country. In the U.S., for instance, public funding takes the form of matching funds, with certain limitations being imposed on contributions and expenditures. In many other countries, however, parties receive government funding in relation to the share of the electoral vote they achieved in the last election. Le Duc et al. (1996), investigate 27 democracies and report that in 17 cases, public subsidy depends on vote (or seat) share. In Denmark, for example, one vote is equivalent to approximately 20 Danish kroner per year (approximately $3 U.S.). See Bille (1997). At first glance, such a system would certainly seem to favour the larger parties. Mair (1994) reports that in many European countries public finance is at least as important for political parties as private finance is and adds that in some cases it may be even more important.

In this paper, we provide a simple theoretical model that casts some light on the issue of public funding of political parties. While the debate about public funding has often focused on fairness and on reducing the power of wealthy private lobbies, we show that an important consequence of the system, which prevails in many European countries, is policy convergence. There is substantial empirical evidence to support the hypothesis that election campaigns directly affect how the electorate votes, (see e.g. Holbrook

\footnote{In Denmark, direct public support of political parties was enacted in 1986. In 1995 the level was four-doubled. Billie (p201) concludes that in comparison to 1985, the proportion of the parties' revenues that is provided by state funding has increased dramatically. He investigates all of the major Danish parties and finds that state funding accounts for between 48% and 98% of a party's income.}
1996), which, of course, is precisely why such campaigns are carried out.\(^2\) What one could discuss, however, is just why this is so. Baron (1994) (and McKelvey and Ordeshook (1987)) suggest that there are two types of voters: “informed” and “uninformed” or “impressionable” voters. The informed electorate votes according to the policies proposed by the different political parties (or candidates). Impressionable voters are, however, poorly informed about the policies of the different parties and their vote is directly influenced by campaign spending\(^3\). Alternatively, one may assume that all voters are policy motivated but not necessarily well informed about the parties’ policies. Campbell et al. (1960) provide evidence that most voters cannot correctly identify the position of parties or candidates on the main political issues, while Popkin et al. (1976) try to explain why voters have such a poor level of information, by arguing that the gathering of information is a costly investment. In such a case, campaign spending could spread information about a party’s policies, and as such, political campaigns are rather like informative advertising campaigns, as studied by Butters (1977) or Grossman and Shapiro (1984) among others. Political campaigns then affect the outcome of an election since voters will cast their vote depending on their information.\(^4\)

Since both of these views seem reasonable, we shall investigate them both within the framework of the same basic model, with two policy motivated parties and a uni-dimensional policy space. When some voters are impressionable, a party directly receives a larger vote-share if it spends more on its campaigns. We show that public funding that depends on vote-share makes

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\(^2\)See also Lazarsfeld et al. (1944), Campbell et al. (1960) and Finkel (1993).

\(^3\)This type of campaign is similar to the persuasive advertising analyzed in the economics literature, see for example Shy (1995).

\(^4\)See Morton and Cameron (1992) and Austen-Smith (1997) for a critique to the informative role of the campaign advertising.
the parties’ policies converge. The reason for this is intuitive. The likelihood that a party’s policy is implemented is assumed to depend positively on its share of the total vote. For a politically motivated party this creates a trade-off, a more moderate policy is less good but more likely to be implemented. This trade-off is changed when public funds depend vote shares. In this case the vote gaining effect of policy moderation is enhanced (as the public funds gained buys even more votes) and in equilibrium, therefore, policies are more moderate. The more responsive votes are to money, and the more responsive public funding is to votes, the more the parties’ policies converge.

In countries where public funds depend on the vote shares, they are distributed after the election where the vote shares are known. If the funds that are earned in one election are used to finance the following campaign, therefore, a dynamic process arises. If, however, a party is able to borrow funds on a credit market, it may spend the money before the election and repay the debt using the public funds after the election. In order to keep our model simple and avoid complicated dynamics, we shall assume that this is the case. In the model public funding depends on the expected vote shares. We study a rational expectations solution, where expectations are correct.

For the sake of simplicity, we assume that parties and voters are risk-neutral. This, however, is not essential for our qualitative results. If agents are risk-averse, the European system for public funding of parties is welfare-enhancing, since it reduces policy polarization and, therefore, the general risk for everyone. One could argue, therefore, that such a system is most called for in countries where other political institutions do not make for policy convergence.

We briefly consider the case where parties have some private (lump sum) funding. This mitigates the effects of public funding. Since they become
relatively less important, parties are less eager to moderate their policies to obtain such funds. We also briefly study an extension to a case with more parties. With a uni-modal distribution of the voters’ preferred policies, the results previously obtained are easily transferable to the examples we have studied. Public funding increases the incentive to moderate policies for all parties, so that, in equilibrium, the policies will be compressed more around the middle.

In the second case, with informative campaigns, all voters are policy motivated but only some are informed about a party’s policy. An uninformed voter, however, has an expectation regarding the policy. For the usual reasons - our results should not hinge on arbitrary assumptions about expectations - we study a rational expectations solution. Campaign spending influences the voters’ information in different ways. Voters may see television spots and newspaper advertisements. However, the effect may also be more indirect. A party can get more exposure in the mass media by spending more money on staging events, or having a campaign staff that caters to the journalists and provides regular press releases, etc. Whatever the reason may be, however, we assume that the more a party spends on its campaign, the more voters learn its policy.

Since only informed voters learn about a policy change, the gain in vote-share the party makes from having moderated its policy is larger, the larger is the fraction of informed voters. The more effective funds are in spreading information and the greater the funds available for campaigns are, the more policies will therefore converge. In this setting, however, it is not important whether funds depend on vote-share or not. Our model also shows that a party who has access to more efficient campaign technology will receive a larger share of the vote. Thus, it also highlights the importance for parties
to have good relations with the mass media.

Furthermore, multiple equilibria might well exist. In some cases, there are at least three equilibria: one which is symmetric and two asymmetric ones. In an asymmetric equilibrium, a large party receives large funds, which implies that many voters are informed about its policy and that its gain in votes from moderating its policy will also be large, so the incentive to do so is also greater. The smaller party, on the other hand, finds itself trapped in a situation in which it has few votes and a minimum of public funding. Hence, very few voters are informed about its policy and few people will notice if it is altered, so that the gain in votes from doing so will also be minimal. In equilibrium, therefore, such a party proposes a rather extreme policy and receives very few votes.

We have not found many theoretical papers on public funding of political parties. Baron (1994) considers lobbying in a model with informed and uninformed voters, in which parties seek to maximize their probability of winning. If all of the voters are informed, each party chooses the median voter's most preferred policy. As a result of the uninformed voter's behavior however, parties also have an incentive to raise funds. Courting lobby groups, who are generally considered to be extreme, can do this. In their desire to please the lobbies, therefore, political parties tend to propose more extreme policies. The introduction of public funding (as a lump-sum and independent of vote-share), mitigates the power of interest groups and their contributions become relatively less important. As a result, the parties' policies are less polarized.

Our argument here is different. It does not depend on the mitigating of the lobby groups' power. We consider partisan parties which, by themselves, would choose polarized policies even in the absence of lobbies. When public
funding depends on vote-share, parties get an extra incentive to moderate their policies. When public funding does not depend on the vote share, (as in the U.S. system which Baron analyses), there is no policy convergence in our model with impressionable voters. Hence, our analysis shows that the precise sort of public funding system employed is significant to the effects on policy convergence. Furthermore, we extend the analysis to informative advertising campaigns and analyse the effects of public funding in this framework. The amount of funds made available to the parties and the information technology they employ play a major role, but again, the results are not related to mitigating the power of lobby groups.

Mueller and Stratman (1994) consider both informative and persuasive campaigns where money directly moves votes. They have, however, no formal model for informative campaigning and focus on private contributions to political parties.

The organization of the paper is as follows: Section 2 presents the basic model with impressionable voters and uninformative campaigning. Section 3 derives the equilibrium created by public funding. Section 4 considers, briefly, the case in which there are more parties; Section 5 analyses the case of both public and private funding. Informative campaigning is the subject of the rest of the paper: Section 6 presents the basic model and discusses equilibria. Section 7 introduces public funding and contains examples of different information technologies that lead to different kinds of equilibria, and Section 8 concludes. A few proofs are relegated to an Appendix.
2 The Model With Impressionable Voters

There are two parties, L and R; and a continuum of voters of measure one. Politics is uni-dimensional. The parties each propose a policy, \( l \) and \( r \) respectively, receive campaign money, \( c_L \) and \( c_R \); which is spent, and then an election is held. The implemented policy will be either \( l \) or \( r \). There are two kinds of voters, informed and impressionable. The fraction of informed voters is \( (1 - \delta) \); where \( 0 < \delta < 1 \):

If the policy is \( \frac{1}{4} \) an informed voter with bliss point \( x \) gets utility

\[
 u_{\frac{1}{4}}(x) = \frac{1}{4} x - \frac{1}{4} x^2
\]

The bliss points are distributed on the interval [0,1] according to the cdf \( F(x) \); the corresponding (strictly positive, differentiable) density is \( f(x) \): Let \( m \) be the median bliss point, \( F(m) = \frac{1}{2} \): The parties are committed to their policy proposals, \( l \) and \( r \): As \( l \rightarrow r \) an informed voter with bliss point \( x \) prefers party L's policy if \( x \cdot \frac{l+r}{2} \): Since voting is sincere, \( F(\frac{l+r}{2}) \) of the informed voters vote for party L. Parties are policy motivated and have the same type of utility function as voters (see Wittman 1990). The bliss point of party L is \( x_L \); the bliss point of party R is \( x_R \); and \( x_L < x_R \):

The difference in campaign spending by the parties is \( \epsilon = c_L - c_R \): A fraction \( H(\epsilon) \) of the impressionable voters votes for party L: We will assume that \( H(0) > 0 \); \( H(0) = \frac{1}{2} \) and \( H^0(\epsilon) \cdot 0 \) for all \( \epsilon \); \( 0 \): The party spending most is most popular. We further assume that the \( H \) function is symmetric: \( H(\epsilon) = 1 - H(-\epsilon) \); so \( H^0(\epsilon) \cdot 0 \) for all \( \epsilon \); \( 0 \) and \( H^0(0) = 0 \). \footnote{In principle, the parties can of course propose policies \( l; r \) where \( r < l \): This will never occur in equilibrium, however, so we will just disregard this case.}

\footnote{The way electoral campaigning works here is similar to the “predatory advertising” analyzed in oligopoly theory, see Friedman (1983). It is the formulation chosen by}
Party L's total vote share is therefore

\[ v = \mathcal{H}(z) + (1 - \mathcal{H})F \left( \frac{A}{2} \right)^{1} \]

As Grossman and Helpman (1996) do, we assume that the chance of a party’s policy being implemented is increasing in its vote share. This is slightly ad hoc, although it may not be unreasonable at all, and may be rationalized in several ways. It may be due to the fact that the larger a party is, the larger the influence it has in parliament. One might also assume that parties are uncertain about the outcome of the election, either because of polling errors or uncertainty about who will vote and who will abstain. The larger the (ex-ante) vote share a party has, the larger its probability of winning the election and implementing its policy, (see Roemer 1994 for micro-foundations). Alternatively, one can assume, as in Alesina and Rosenthal (1995, 1996), Ortuno-Ortín (1997) and Gerber and Ortuno-Ortín (1998), that the policy ultimately implemented is a convex combination of the two policies originally proposed, and that the weights are given by their different shares of the electoral vote. Since our agents are risk-neutral, such an approach is equivalent to the one we employ here. For whatever the reason might be, we assume that party L's policy is implemented with a probability of \( p(v) \); where, for the sake of simplicity, \( p(v) = v \). The linearity simplifies formulas but has no qualitative importance. Intuitively, the steeper the \( p \) function at \( 1/2 \) is, the greater the incentive that the parties have to moderate their policies.\(^7\)

Parties receive public funds for campaigning. Such funds typically depend

\[ \text{Helpman and Grossmann (1996): Baron (1994) assumes that } H \text{ depends on } c_L = c_R; \text{ In our setting, this would complicate a few formulas, but the qualitative results would not change.} \]

\( ^7 \text{We see that our linear formulation of the } p \text{ function makes for less policy convergence than if we assumed that } p \text{ was very steep at } 1/2. \text{ On the other hand the linear formulation} \]
on the actual vote-share received in an election. If a party wants to spend money before an election, therefore, it must either take a loan or spend the money it received in the last election. Furthermore, if the party gets a loan, the issue of credit-worthiness arises. On the other hand, if the funds it earned in the last election are used, the problem becomes dynamic. Although these aspects may be important in real life, we shall disregard credit-worthiness and dynamic issues here, and study a rational expectations solution instead.

The timing is as follows: First, the different parties propose certain policies. They then receive public funding that depends on their expected vote-share. In equilibrium, the expectation is correct. This is equivalent to the case in which parties borrow on a perfect credit market with rational expectations before an election, and use the public funding they later receive for their actual vote-share to repay the loans afterwards. Notice that if parties have access to a perfect credit market, public funding will not prevent the entry of new parties to the political arena. For the time being, however, we shall disregard the issue of entry.

We normalize the size of the public funds available to parties to one. We shall assume that public funding treats all parties equally. If party $L$ is expected to receive a share $v^e$ of votes, it therefore receives $c_L = \bar{A}(v^e)$ from public funding, and party $R$ then receives $c_R = \bar{A}(1 - v^e)$. Accordingly, $\bar{A}(\frac{1}{2}) = \frac{1}{2}$: We assume that the funding system fulfills $\bar{A}^0 = 0$ for all $v^e$ and $\bar{A}^{00}(v^e) = 0$ for all $v^e > \frac{1}{2}$; and $\bar{A}^{00}(v^e) > 0$ for all $v^e < \frac{1}{2}$: A party that has already obtained more than 50% of the electoral vote (weakly) increases its funds by receiving more votes, but at a non-increasing rate. The difference of the parties’ utility functions makes for more convergence than if parties utility functions were strictly concave. In the latter case the marginal disutility of moderating the policy would be increasing.
in funds received, therefore, is $\xi(v^e)$ \(\tilde{A}(v^e) \cdot (1 \cdot \tilde{A}(v^e)) = 2\tilde{A}(v^e) \cdot 1; \xi(q(v^e)) \cdot 0\) for all $v^e$; and $\xi(q(v^e)) \cdot 0$ for all $v^e \geq 1/2$ and $\xi(q(v^e)) \cdot 0$ for all $v^e \geq 1/2$.

In the Danish system described in the introduction, in which a party receives approximately 20 Danish kroner. (US$ 3) per year per vote, $\tilde{A}^0 > 0$ and $\tilde{A}^0 = 0$:

Given an expected vote share of $v^e$ and policies $l; r,$ the actual vote-share of party $L$ is

$$v = \tilde{A}H(\xi(v^e)) \cdot \frac{1 \cdot r}{2}$$

Under rational expectations, $v^e = v$; hence $v$ solves equation (3) with $v$ inserted for $v^e$: If

$$H(\xi(v^e)) \cdot q(v^e) < 1; \text{ for all } v \in [0; 1]$$

the solution is unique. We shall assume this is the case, and denote the solution by $v(l; r)$. This means that the responsiveness of public funding and of the impressionable voters are sufficiently limited to make the problem well-behaved. Note that we are not assuming that the final equilibrium is unique, but just that there is a unique vote-share for each party for a given pair of policies. The implicit function theorem yields:

$$\frac{\partial v(l; r)}{\partial l} = \frac{\frac{3}{2} \cdot \frac{r}{2}}{1 \cdot H(\xi(q(v^e)) \cdot q(v))} > 0$$

When party $L$ changes its policy, some informed voters change their votes. This, in turn, changes the size of the public funds allocated to the party and, therefore, the share of impressionable votes it gains as well. But this, once again, changes the size of the funds received, etc., etc. All of these repercussions are taken into account in the rational expectations solution.
The assumption \( Q(v)H \{ Q(v) \} < 1 \) ensures that the comparative statics make sense, i.e., if party L increases its policy (towards the middle), it will also increase its vote-share.

3 Political Equilibrium

The parties are policy motivated and seek to maximize expected utility. They take the other party’s policy as given and recognize how the choice of policy might influence the distribution of funds and the relative campaign spending. Given the pair of policies \((l; r)\), party’s L expected utility is

\[
\begin{align*}
v(l; r) & (l \leq x_L) + (1 - v(l; r)) (r \leq x_R) \end{align*}
\]

Similarly, party’s R expected utility is given by

\[
\begin{align*}
v(l; r) & (l \leq x_R) + (1 - v(l; r)) (r \leq x_L) \end{align*}
\]

A Political Equilibrium with uninformative campaigning is a pair of policies \((l^*; r^*)\); such that \(l^*\) maximizes (6) given \(r^*\) and \(r^*\) maximizes (7) given \(l^*\).

In principle, a party’s optimal policy may be equal to the party’s bliss point where the utility function is non-differentiable. We shall, however, only consider cases where solutions are interior, i.e., where \(x_L \leq l < r < x_R\): We have

Proposition 1 In an interior equilibrium, each party has a winning probability of 1/2. The expected policy is equal to the median informed voter’s preferred policy

\[
\frac{l^* + r^*}{2} = m
\]
The policies are symmetric around \( m \) and the policy polarization is given by

\[
\frac{1}{\log(1 + \frac{1}{2})} \frac{1}{f(m)}
\]

(9)

Proof: see the Appendix.

The policies are symmetric around the median informed voter’s bliss point. The difference in the policies depends on the density of voters at the median, \( f(m) \): If the density is high, the policies are close, as many votes can be gained by moving the policy towards the middle. If the density is small, parties choose policies that are closer to their own preferred policy.

The numerator in (9) reflects the fraction of impressionable voters, \( \phi \); how responsive the impressionable voters are, \( H(0) \); and how responsive the public funding system is, \( \frac{1}{2} \): The larger the responsiveness, the closer the policies.

No public funding corresponds to the special case of the model where \( \phi(0) = 0 \); then

\[
\frac{1}{\log(1 + \frac{1}{2})} \frac{1}{f(m)}
\]

Comparing with (9) we see that public funding makes the parties’ policies converge. The intuition is simple. Public funding gives an extra incentive to moderate policies, since the extra votes they gain makes more public funds available, which, in turn can be used to gain further votes.

We can also compare this with the case of no impressionable voters, \( \phi = 0 \): In this case the difference in policies would be

\[
\frac{1}{f(m)}
\]

> From (9) it directly follows that if \( H(0) \phi(\frac{1}{2}) > 1 \); then policies are more convergent than in the benchmark case with no impressionable voters, and if \( H(0) \phi(\frac{1}{2}) < 1 \); policies are more divergent. In the knife-edge case,
where \( H(0) \cdot q(\frac{1}{2}) = 1 \); the vote-share of a party responds in the same way to changes in policy whether there are impressionable voters or not. The impressionable voters distribute their votes just as the informed ones do and the policies are the same as when there are no impressionable voters: Hence, in such a case, the publicly funded electoral system works as if there were only informed voters. Public funding, therefore, offsets the presence of poorly informed voters.

In the Appendix, we show that the second order condition for maximum is fulfilled if

\[
\frac{f^0}{f} < 4; \quad \text{and} \quad \frac{\partial^2 \xi}{\partial l^2} \cdot \frac{d \xi}{d l} + \frac{H^{(0)}}{H^0} < 2; \tag{10}
\]

The latter part is a joint condition on \( \xi \) and \( H \); and it is diﬃcult to provide general conditions that ensure that the inequalities in (10) are fulfilled. We show in the Appendix, however, that if \( H^{(0)} = 0 \) and \( \tilde{A}^{(0)} = 0 \); so \( \xi^{(0)} = 0 \); it is then fulﬁlled. As is clear from the derivation, it is not necessary that \( H^{(0)} = 0 \) and \( \xi^{(0)} = 0 \), but rather that they should simply not be numerically too large.

We have assumed that the equilibrium is interior, \( x_L < l < r < x_R \): This obviously implies that \( r - l < x_R - x_L \): As can be seen from equation (9) this requires \( f(m) \) to be sufficiently large and/or \( 1 \cdot 1 \cdot \cdot \cdot H(0) \cdot q(\frac{1}{2}) )/(1 \cdot \tilde{A}) \) to be sufficiently small. An example that fulﬁls this condition is, \( x_L = 0; x_R = 1; m = 1=2; H(0) \cdot q(\frac{1}{2}) = 1 \) and \( f(m) > 1 \): From the derivations in the appendix, it is clear that an interior equilibrium then exists, provided that the second-order condition is fulﬁlled, which is the case if, for instance, \( H^{(0)} = \xi^{(0)} = 0 \) and \( f^0f < 4 \):
In this section we sketch the case with \( N \) parties. Party \( n \)'s policy is \( x_n \) and its bliss point is \( b_n \); and \( b_1 < b_2 < \cdots < b_N \): If the parties propose policies \( x_1 < x_2 < \cdots < x_N \); the fraction of the informed electorate who vote for party \( n \) is

\[
F \left( \frac{\mu x_{n+1} + x_n}{2} \right) i \ F \left( \frac{\mu x_n + x_{ni_1}}{2} \right).
\]

Let \( v^n \) be the expected vote-share for party \( n \): The public funds allocated to party \( n \) depends only on \( v^n \); and not on the vote-shares of the other parties. This means, in effect, that \( \mu(v^n) \) is proportional to the vote-share, which is the case in most real-life public-funding systems. Similarly, a party's impressionable votes depend only on its own campaign spending and not on what the other parties spend. This implies that the fraction of impressionable votes that a party wins is directly proportional the fraction of public funds it receives. The total number of votes for party \( n \) is therefore:

\[
v^n = \circ(v^n) + (1 \circ) \ F \left( \frac{\mu x_{n+1} + x_n}{2} \right) i \ F \left( \frac{\mu x_n + x_{ni_1}}{2} \right).
\]

Applying rational expectations \( v^n = v^n \); we get

\[
v^n = \ F \left( \frac{\mu x_{n+1} + x_n}{2} \right) i \ F \left( \frac{\mu x_n + x_{ni_1}}{2} \right).
\]

With proportionality in the \( \mu \) and \( H \) functions, the public funding system makes the model work as if there were only policy-motivated voters. Without such public funding, party \( n \)'s vote-share would be

\[
v^n = \circ(1 \circ) + (1 \circ) \ F \left( \frac{\mu x_{n+1} + x_n}{2} \right) i \ F \left( \frac{\mu x_n + x_{ni_1}}{2} \right).
\]

As \( \circ > 0 \); public funding enhances the effects of changes in policy on vote-shares, just as in the case of two parties.
Party n’s problem is \( \max_{x_n} \sum_{i=1}^{N} v^i x_i \mid b_i j \): Clearly, \( x_n < x_{n+1} \) or \( x_n > x_{n+1} \) is not optimal. Assuming \( x_n > b_n \); the first-order condition is
\[
i \frac{\partial^{n+1}}{\partial x_n} x_n \mid b_i j \frac{\partial^n}{\partial x_n} x_n \mid b_i j = 0
\]
(if \( x_n < b_n \); \( v^n \) enters with a plus). Using \( \frac{\partial^{n+1}}{\partial x_n} + \frac{\partial^n}{\partial x_n} + \frac{\partial^{n+1}}{\partial x_n} = 0 \); we get
\[
i \frac{\partial^n}{\partial x_n} (j x_n + b_i j) \frac{\partial^{n+1}}{\partial x_n} (j x_n + b_i j) = 0
\]
(11)

An increase in \( x_n \) has three effects, each of which correspond one of the three terms. First, when \( x_n > b_n \); it lowers the utility from the party’s own policy, which is implemented with a probability of \( v^n \): Secondly, the probability that \( n \mid 1 \); rather than \( n \); wins increases, which gives an expected utility loss for party \( n \): Thirdly, the probability that party \( n \); rather than \( n + 1 \); wins increases, which gives an expected utility gain for party \( n \).

In general, therefore, whether public-funding promotes policy convergence or not depends on the distribution of voters. However, look at the extreme parties. They only face competition from one side. The problem they face is just as if there only were two parties: i.e., moving towards the middle increases the chances of their policies being implemented, but their policies become less attractive. As we have already seen, public funding increases the vote-gaining effect and, ceteris paribus, this makes the parties move closer to the middle. Let us suppose that the distribution of the voters’ bliss points is unimodal with the peak in the middle. A party facing competition on both sides will have a net gain in votes by moving closer to middle, as the density of voters there is higher. The introduction of public funds only reinforces this effect. Furthermore, since government funding makes the extreme parties move closer to the middle, the neighboring party finds that the policy of the extreme party is not as bad as it seemed to be before. The neighbor-party,
therefore, has an increased incentive to moderate its own policy, just slightly more than it did before. This has repercussions on the following neighbor, etc., etc. There is therefore the strong intuition that if the distribution of voters is uni-modal with its peak in the middle, public funding will induce parties to have more moderate policies.

Analytical solutions are hard to obtain (as the problem is one of \( n \) non-linear equations in \( n \) unknowns), but we have investigated a large number of examples with Mathematica that all yield the result described above, whenever the distribution is uni-modal.\(^8\) Public funding compresses the policies around the peak of the distribution. We present just one example here, with 4 parties, in which both the parties and the voters have quadratic preferences (to avoid non-differentiabilities). Party \( i \)'s utility function is 
\[
u(x; b_i) = (x - b_i)^2;
\]
We let, \( b_1 = 0; b_2 = 1/3; b_3 = 2/3 \) and \( b_4 = 1 \); and \( \beta = 1 = 4 \). The voters’ bliss points are distributed according to a beta distribution with both parameters equal to two. Figure 1 illustrates the results. The triangles represent their policies when there is no public funding, and the pentagons when there is. The Figure clearly reveals the expected result: i.e., public funding induces policy convergence.\(^9\)

5 When Parties also have Private Funds

Now let us suppose that the parties also have access to private funds. For the sake of simplicity, we shall assume that such funds are independent of the policies chosen by the parties. Political parties might have endowments or receive contributions from different social groups whose overall political

\(^8\)The Mathematica note-books are available from the authors on request.
\(^9\)It can also be shown that the policy variance is lower under the public funding case.
tendencies are more in line with the party’s. A typical example would be that of trade unions who support social-democratic parties, regardless of the precise policy they choose. There is comprehensive literature on private lobbying, and its effects on policy-making are well-understood, see e.g. Baron (1994), Grossman and Helpman (1996), or the surveys in Persson and Tabellini (1999) and Austen-Smith (1997). It is therefore not our intention to make any further contribution to this topic. Our interest, here, is placed exclusively on the interplay that exists between private and public funding of political parties.

Let us now assume, that party L has $c_{PL}$ private funds and party R has $c_{PR}$; and let $c_P = c_{PL} - c_{PR}$. The difference in funding between the two parties then is $\xi(v) = \tilde{A}(v) \cdot (1 - \tilde{A}(v)) + c_{PL} - c_{PR} = \xi(v) + c_{PL} - c_{PR}$. The vote-share of party L will be the fixed point of

$$v = \mathcal{H}(\xi(v) + c_{PL} - c_{PR}) + (1 - \mathcal{H})F(\frac{\tilde{A}(v)}{2^2} + r^2)$$

The first-order conditions for maximum are unchanged (except $H_0$ is taken in $\xi + c_{PL}$): Hence, we still have, in equilibrium, that $v = \frac{1}{2}$ and therefore that $\xi = 0$: Now let us suppose that party L has a smaller endowment than party R; so that $c_P < 0$: Then $H(\xi + c_{PR}) < \frac{1}{2}$; and since we still have $v = \frac{1}{2}$; it must be the case that $F(\frac{1 + r}{2^2}) > \frac{1}{2}$, which means $\frac{1 + r}{2} > m$: The average policy is more to the right, the reason being that the rich party R can afford to propose a more extreme policy, as it gets a larger share of the impressionable voters.

In this analysis, the relative sizes of public and private funds do not matter. This may seem unreasonable. Suppose, instead, that the impressionable votes depend on the relative campaign expenditure of the different parties. Thus, $H$ depends on $\frac{c_L}{c_L + c_R}$. Without private funding, the above analysis is
unchanged, just let \( \zeta(v) = \frac{\bar{A}(v)}{A(v) + 1} = \bar{A}(v) \): With private funds, the vote share of party \( L \) is a fixed point of

\[
v = \bar{A}H \frac{C_{PL} + \bar{A}(v)}{1 + C_{PL} + C_{PR}} + (1 - \bar{A}) F \frac{1 + r}{2}
\]

and hence

\[
\frac{\partial v}{\partial l} = \frac{1 - \bar{A}}{2} \frac{3 + r}{2} \frac{1 + C_{PL} + C_{PR}}{1 + C_{PL} + C_{PR}}
\]

The vote-gaining effect from public funding is smaller the larger private funding is. Once again, the richer party is able to propose a more extreme policy and still get half of the votes. Furthermore, in such a setting, the expression for policy polarization becomes

\[
r = \frac{1 - \bar{A}^0}{1 + C_{PL} + C_{PR}} \frac{1}{1 + C_{PL} + C_{PR}} H^0 \frac{1 + 2}{1 + C_{PL} + C_{PR}} f(m)
\]

It is obvious that the larger private funds are, (relative to the size of public funds, normalized to one), the more different the parties’ policies are. Private funding mitigates the policy convergence induced by the public funding. To summarize then, with both private and public funding, the model predicts that there should be more policy convergence in countries where public funding are relatively large in comparison to private contributions.

As discussed in the introduction, Baron (1994) considers a model with private and public funding in which parties seek to maximize plurality. With no funding whatever, the parties choose policies that are equal to the median voter’s preferred policy. In Baron’s model, private funding is provided by extreme lobby groups, and the parties therefore propose more extreme policies in order to please the lobby groups. Public funds are given in a lump sum, (as in the US system), and they mitigate the power of lobby groups, since the relative importance of private funds is reduced, and policies become more convergent. As we have seen, in our model exact opposite result
is obtained from giving parties lump-sum contributions. Such contributions, (whether private or public), decrease the relative importance of the funds earned through votes, which in turn, mitigates the incentive to modify policies. If we introduced extreme lobby groups, however, this would obviously give parties an added incentive to propose more extreme policies. The bottom line is clear: i.e., the exact way in which public and private funding is provided is crucial to the result.

6 Informative Campaigning

In this section, we assume that all voters are policy oriented and have the same preferences as those described above. However, voters are not automatically aware of the parties’ policies, and must be informed through their campaigns.

If a party spends an amount \( c \) on its campaign, a fraction \( \hat{A}(c) \) of the voters learns about its policy. Parties cannot target their campaigns, so that the probability that a particular voter becomes informed is independent of her bliss point. One can imagine that a party advertises exclusively on television\(^{10}\) or in magazines, and that only those voters who happen to see its advertising will learn about its current policy. Some voters, however, might well become informed regardless of whether they see advertisement or not. They might read newspapers, listen to radio stations or the news on television. The important feature here is that a more intensive campaign makes a larger proportion of voters informed about the party’s policy\(^{11}\), \( \hat{A}(c) > 0 \):

\(^{10}\)Television advertisements represent the most important expenditure in the electoral campaigns in many countries, see West (1993).

\(^{11}\)See Holbrook (1996) for evidence of the information generated in the electoral campaigns and Alvarez (1996), Brians and Wattenberg (1996) and Just et al. (1990) for
On the other hand, even a voter who is uninformed about a party’s policy has an expectation about it. For the usual reasons, we shall assume rational expectations. The timing is as follows: First, uninformed voters form expectations, then parties choose policies, which are seen only by informed voters, and then the election is held. Let the uninformed voters’ beliefs about parties L and R’s policies be denoted by $l^e$ and $r^e$.

We assume that both parties run electoral campaigns. For the time being we shall not concern ourselves with just how the campaign funds are raised. Let $\hat{A}_L$ be the fraction of voters who learn about party L’s policy and let $\hat{A}_R$ be the fraction who learn about party R’s. Given policies $l; r$, and fractions $\hat{A}_L; \hat{A}_R$; the vote-share for party L is

$$V(l; r; l^e; r^e; \hat{A}_L; \hat{A}_R) = \hat{A}_L \hat{A}_R F \frac{\hat{A} + r}{2} + \hat{A}_L (1 \hat{A}_R) F \frac{\hat{A} + r^e}{2} \tag{12}$$

This reduces it to

$$\frac{\hat{A}_L \hat{A}_R F \frac{\hat{A} + r}{2} + \hat{A}_L (1 \hat{A}_R) F \frac{\hat{A} + r^e}{2}}{1 + r + r^e} \tag{13}$$

Using rational expectations $r^e = r$ this reduces it to

$$\frac{\hat{A}_L \hat{A}_R}{2} \frac{\hat{A} + r}{2} \tag{13}$$

We see that $\frac{\hat{A}_L \hat{A}_R}{2}$ depends on $\hat{A}_L$: This is the crucial aspect of the analysis: only informed voters will respond to a policy change. The larger is the fraction of evidence that television advertising increases voter information.
informed voters, the more responsive the vote-share of a party is to changes in policy. We also have

\[
\frac{\partial V}{\partial \theta_L} = \lambda_L + r \frac{1}{2} + \left(1 + \frac{\lambda_R}{\lambda_L} \right) r e + \frac{1}{2} r e \frac{1}{2} = 0:
\]

since, under rational expectations, \( l^e = l \) and \( r^e = r \). Similarly, \( \frac{\partial V}{\partial \theta_R} = 0 \):

We also have that, under rational expectations,

\[
V(l; r; l; r; \lambda_L; \lambda_R) = \frac{l + r}{2}
\]

Party L’s problem is

\[
\max \ V(l; r; l^e; r^e; \lambda_L; \lambda_R) \left( j, j \right) \lambda_L \left( X_L \right) + \left(1 + \lambda_R \lambda_L \right) \left( j, j \right) X_L:
\]

When we focus on interior equilibria, party L’s first-order condition is

\[
\frac{\partial V}{\partial l} + \frac{\partial V}{\partial \lambda_L} \lambda_L + \frac{\partial V}{\partial \lambda_R} \lambda_R \right) \left( r^e, l^e \right) j = 0
\]

Inserting (13), (14) and (15) gives

\[
\left( r^e, l^e \right) = \frac{1}{2} \frac{l^e + r^e}{2} = \frac{1}{2} \lambda_L + \lambda_R
\]

In the Appendix we show that the second-order condition for maximum is fulfilled under our assumption \( \lambda < 4 \):

Similarly, party R’s first-order condition yields

\[
\left( r^e, l^e \right) = \frac{1}{2} \frac{l^e + r^e}{2} = \frac{1}{2} \lambda_L + \lambda_R
\]

Using (18) and (19) gives us

\[
\frac{\lambda_L + \lambda_R}{2} = \frac{\lambda_L}{\lambda_L + \lambda_R} = \frac{1}{1 + \frac{\lambda_R}{\lambda_L}}
\]
using (15), we finally get

\[ V(l^n; r^n; l^n; r^n; \hat{\alpha}_L; \hat{\alpha}_R) = \frac{\hat{\alpha}_L}{\hat{\alpha}_L + \hat{\alpha}_R} = \frac{1}{1 + \frac{\hat{\alpha}_R}{\hat{\alpha}_L}} \]  \hspace{1cm} (21)

The vote shares are determined by the relative fraction of voters informed about each party’s policy. If the fractions are equal, then party L gets exactly one half of the votes. If \( \hat{\alpha}_L > \hat{\alpha}_R \), party L receives more than half of the votes. Since more voters are informed about L’s policy than about R’s policy, L’s vote share is more responsive to policy changes. This implies that party L gains more votes from moderating its policy towards the middle than party R does. Hence, in equilibrium, party L’s policy will be more moderate than party R’s and party L will get a larger vote-share.

Equation (21) indicates the value of a good information technology. If we suppose that the parties receive equal funding, but that party L has better access to mass media, (perhaps for historical, personal or other reasons) it can more easily convert money into information, so that \( \hat{\alpha}_L > \hat{\alpha}_R \), and thus, obtains a larger vote-share. Good relations with the mass media makes for high vote-shares in our model.

Inserting equation (20) into (19), we get that policy polarization is given by

\[ r \| l = \frac{2}{(\hat{\alpha}_L + \hat{\alpha}_R) f \left( \frac{1 + r}{2} \right)} \]  \hspace{1cm} (22)

Let us now increase \( \hat{\alpha}_L \) and \( \hat{\alpha}_R \) in such a way that \( \hat{\alpha}_L = \hat{\alpha}_R \) is constant. From (20), \( \frac{1 + r}{2} \) is unaffected, (22) directly gives a decrease in policy polarization. In the symmetrical case, where \( \hat{\alpha}_L = \hat{\alpha}_R \); this is particularly clear. Here, (22) reduces to

\[ r \| l = \frac{1}{\hat{\alpha} f (m)}; \]  \hspace{1cm} (23)
Policy polarization decreases when more voters become informed. Hence, when campaigns are informative, policies converge if the funds available for campaigning increase.

Consider briefly a case in which expectations are not formed rationally but depend on past policy proposals. Let us also assume that time is discrete and runs from zero to infinity. Each period is as described above. First, the parties propose policies, then the elections are held, and finally, the winner implements the promised policy. Suppose that the expectations of period $t$ depend on the policy proposals of period $t - 1$; so $l_t^e = l_{t-1}$ and $r_t^e = r_{t-1}$. Suppose, furthermore, that parties are myopic, and seek to maximize the expected utility of one period only. In a stationary state, one has $l_t^e = l_{t-1} = l_t$ and $r_t^e = r_{t-1} = r_t$ for all $t \geq 0$. The rational expectations solution studied above corresponds, therefore, to a stationary state of the dynamic game outlined here.

7 Public Funding

We will now introduce the public funding system into the informative campaigning model. Party $L$'s vote share is $V(l; r; l^e; r^e; \hat{A}_L; \hat{A}_R)$ as given by equation (15). The derivations above took into account that parties realize that their choice of policy will affect funding and therefore $\hat{A}_L$ and $\hat{A}_R$; so that these derivations are still valid, see equation (17) and the related discussion. As is clear, rational expectations simplify matters considerably here. Using equation (20), we get

$$V(l; r; l^e; r^e; \hat{A}_L; \hat{A}_R) = \frac{\hat{A}_L(c_L)}{\hat{A}_L(c_L) + \hat{A}_R(1 - c_L)}; \quad (24)$$
where we have explicitly written the $A^0$s as functions of the funds allocated
to party $L$: As previously, $c_L = \tilde{A}(v)$; so

$$c_L = \tilde{A} \frac{A_L(c_L)}{A_L(c_L) + A_R(1 - c_L)}$$  \hspace{1cm} (25)

which determines $c_L$: Depending on the $\tilde{A}$ and $\tilde{A}$ functions, there may be one
or more fixed points.

We now assume that the parties are equally efficient in informing voters, so $A_L(c) = A_R(c) = \tilde{A}(c)$: In this case equation (25) becomes

$$c_L = \tilde{A} \frac{\tilde{A}(c_L)}{\tilde{A}(c_L) + A(1 - c_L)}$$  \hspace{1cm} (26)

As the public funding system is fair, $\tilde{A}(1/2) = 1/2$, and we see that $c_L = 1/2$ is a solution. Hence, a symmetric equilibrium exists, in which each party receives half of the votes.

Advertising by parties is not the only way voters receive information
about policies. Some (many) voters read newspapers, listen to radios, watch
TV etc., thus, some voters will be informed about a party’s policy even if
the party does not advertise. This means that it is reasonable to assume
that $\tilde{A}(0) > 0$: Consequently, the vote-share for party $L$, as given by (24), is
positive in equilibrium. Similarly, the vote-share for party $R$ is positive. By
assumption, a party receives funds if its vote-share is positive, so that $c_L = 0$
or $c_L = 1$ cannot be compatible with equilibrium.

Whether multiple equilibria exist or not depends on the right-hand side
of equation (26). Since it is positive at $c_L = 0$ and less than one at $c_L = 1$,
a sufficient (but not necessary) condition for multiple equilibria is that the
slope evaluated at $c_L = 1/2$ is larger than one. Using $\tilde{A}(1/2) = 1/2$; this condition
can be written

$$\frac{\frac{1}{2}\tilde{A}0 \frac{1}{2}}{\tilde{A}(1/2)} - 2 \frac{\frac{1}{2}\tilde{A}0 \frac{1}{2}}{\tilde{A}(1/2)} > 1.$$  \hspace{1cm} (27)
The first term is the elasticity of public funding with respect to a change in vote-share. The second term is the elasticity of the fraction of informed voters with respect to campaign funds. When both elasticities are large, multiple equilibria exist. There will then be a symmetric equilibrium, where \( c = 1 \) and (at least) two asymmetric equilibria. In an asymmetric equilibrium, one of the parties will be large and will therefore receive a larger fraction of public funds, and by virtue of it, continue to be large. If such an equilibrium exists, there then exists another, similar, equilibrium, where the roles are reversed. Hence, in this case, one can conclude that public funding - coupled with advertising technology - may be the cause of asymmetric support for the two parties.

In the asymmetric equilibrium, the small party remains small exactly because it is small. The reason for this is the public funding system. This phenomenon has been termed "petrification" in political science. See Nassmacher (1989, p 248).

### 7.1 On the \( \Delta_i \) function, two examples

In this section we will propose a foundation for the \( \Delta_i \) function, which generalizes the foundation proposed by Grossman and Shapiro (1984) in their work on informative advertising in monopolistic competition (building on Butters, 1977). We will show how existence of multiple equilibria depends on the specific assumptions on the advertising technology. Imagine that parties put ads in magazines. A magazine has a readership equal to a fraction \( r \) of the population, so that with a probability \( r \) a given voter will see a party’s ad in a specific magazine. We assume that in order to become informed about a party’s policy a voter has to see more than \( s \) ads from the party, where \( s \geq 0 \). The idea is that “repetition is reputation”! Hence, if a party advertises in n
magazines the fraction of agents informed about its policy is equal to,

$$\tilde{A}_s = 1 \sum_{x=0}^{\infty} \frac{\tilde{A}_x}{r^x(1-r)^n} \times$$

where $s \geq 0$.

Suppose that the cost of an ad in a magazine is a per reader of the magazine, so that an ad costs $ar$: If the party has funds $c$, it can therefore obtain the fraction of informed voters

$$\tilde{A}_s(c) = 1 \sum_{x=0}^{\infty} \frac{c^x}{r^x(1-r)^n} \times$$

The more expensive advertising is, the lower $\frac{c}{ar}$ is and the lower $\tilde{A}_s(c)$ will also be. Note that this advertising technology implies $\tilde{A}_s(0) = 0$ and $\tilde{A}_s'(0) > 0$.

Grosmann-Shapiro and Butters assumes that an agent gets informed if she sees at least one ad, in our framework this corresponds to the case where $s = 0$: One can easily show that the elasticity of $\tilde{A}_0$ evaluated at $c = 1/2$ is

$$\frac{\tilde{A}_0(1/2)}{\tilde{A}_0(1/3)} < 1$$

Hence, the elasticity of the $\tilde{A}$- function has to be greater than 1 for the sufficient condition for multiple equilibria to be fulfilled: Inserting this $A_0$ function in equation (25) yields

$$c_L = \tilde{A} \times$$

Let us consider the specific case $\tilde{A}(x) = x$, so that we can focus our analysis on the role played by the advertising technology on the existence of multiple equilibria. It is obvious that, $c_L = 1/2$ is a solution to (30). Similarly,

---

12 This derivative doesn't need to be positive for $c < ar$, i.e. for values of $n$ less than 1. We will disregard such values.
by insertion, it is readily seen that $c_L = 0$ and $c_L = 1$ also are solutions, if we assume that a party that receives no votes, receives no funds $\hat{A}(0) = 0$: These are the only solutions in the interval $[0,1]$. Multiple equilibria, therefore, exist,$^\text{13}$ but the corner equilibria depend crucially on the fact that $\hat{A}_0(0) = 0$ and $\hat{A}_0(1) = 1$; which may seem unreasonable as already discussed above.

If we modify our framework slightly and assume that a fraction $\bar{s}$ of the voters becomes informed about a party’s policy regardless of whether it advertise or not, the asymmetric equilibria disappear. The remaining fraction $1 - \bar{s}$ are as described above. If a party receives funds equal to $c$, its policy will be learned about by $\bar{s} + (1 - \bar{s})\hat{A}_0(c)$ voters. We then get

$$c_L = \frac{\bar{s} + (1 - \bar{s}) }{1 + (1 - \bar{s})\frac{1}{r}}$$

Here $c_L = \frac{1}{2}$ is the only solution in the interval $[0;1]$.

The advertising technology proposed by Grossman and Shapiro has the feature that the marginal value of advertising is overall decreasing, $A_0'(c) < 0$: Suppose, instead, that voters have to see at least two adds in order to be informed about a party’s policy, i.e. that $s \geq 1$: This corresponds to the case where a certain amount of advertising is required to “get the message across” and yet, on the other hand, it is very hard to reach all of the voters. In this case, it is easy show that for low values of $c$ we can have $A_0'(c) > 0$: The intuition to understand this possibility of increasing returns is simple: Let us consider the case $s = 1$ (the same idea applies to the advertising technologies with $s > 1$). An agents needs to see at least two ads to become informed about the party’s policy. Therefore, the ads in the .rst magazine inform none. The ads in a second magazine, however, inform a positive fraction of agents.

$^\text{13}$Remember the elasticity condition was a su¢ cient condition, so there is no contradiction here.
namely \( r^2 \). Eventually, the effect of an ad in the \( n \) magazine, for \( n \) large enough, is similar to the one in the Grossman-Shapiro technology since most people already saw an ad in the previous magazines. Thus, the advertising technology shows first increasing returns and then decreasing returns.

The possibility of increasing returns for some values of \( c \) allows for the existence of multiple equilibria all of them with both \( c_L \) and \( c_R \) positive. When the price of an ad, \( a \), is very low the function \( \dot{A}_1(c) \) looks, in the relevant range, very much as the Grossman-Shapiro technology \( \dot{A}_0(c) \). For high values of \( a \), however, the two functions might behave quite differently. Thus, we have that for low values of \( a \) the only solution (beside the two “corner” solutions) to

\[
c = \frac{\dot{A}_1(c)}{\dot{A}_1(c) + \dot{A}_1(1-i-c)}
\]

is \( c = 1=2 \); that is the same one we had in the case \( s = 0 \). We might have, however, that for large values of \( a \) the slope of \( \frac{\dot{A}_1(c)}{\dot{A}_1(c) + \dot{A}_1(1-i-c)} \) at \( c = 1=2 \) is greater than one. Let us modify our framework and assume that a fraction \( \bar{\gamma} \) of the voters is informed regardless of whether the party advertises or not. In this case, the “corner” equilibria, as explained above, disappear and two new asymmetric equilibria appear. Moreover, with this function \( \dot{A}_1(c) \); one can also generate non-corner multiple equilibria without assuming the existence of a fraction \( \bar{\gamma} \) of informed agents. Thus, it can easily be checked that there is a whole range of intermediate values of \( a \) for which the slope of \( \frac{\dot{A}_1(c)}{\dot{A}_1(c) + \dot{A}_1(1-i-c)} \) at \( c = 1=2 \) is less than one, as in the Grossman-Shapiro case, and still the system presents multiple, non-corner equilibria. Say, for example, that \( r = 0:01 \), so that a magazine reaches 1% of the population, and the cost of an ad is \( a = 0:13 \). In this case, one can show, by computing simulations, that the three equilibria are \( c_L = 1=2 \), \( c_L = 0:26 \) and \( c_L = 0:74 \), see Figure 2. Thus, the existence of increasing returns in the advertising technology might
lead to an equilibrium where a party obtains a much greater voter’s support than the other party.

8 Concluding Remarks

Public funding of political parties tends to moderate partisan policies when such funding depends on vote-share. This conclusion holds equally well whether campaign spending is uninformative and directly affects the votes of impressionable voters or whether campaign spending contributes to inform a policy-oriented electorate about the different policies of the parties.

In the first case, campaign money buys votes. If public funding depends on a party’s vote-share, it increases a party’s incentive to moderate its policy. A moderate policy gives more votes and, therefore, higher public funds, which in turn, can be used to buy even more votes. The parties, however, face a trade-off in deciding whether to maintain a policy they like or to moderate it so that it will be more likely to be implemented. With public funding the trade-off changes, making moderation a more attractive option.

If campaigning is informative the result is the same, but the channel is different. When deciding on a policy, a party takes into account that only a fraction of the electorate will be informed about its choice of policy. Uninformed voters will not learn about the precise policy choice of a given party, but rather, will rely on their expectations. Hence, the increase in a party’s vote-share from moderating its policy depends on the fraction of voters who will become informed about its policy. The larger this fraction is, the more attractive a moderation of the policy will be. Since public funding ensures that a larger fraction of the electorate is informed, it induces policy moderation. Note that what is important here is the level of funding, while, in
the case of impressionable voters, it is the responsiveness of public funding to changes in vote-share that is important. This is where the mechanism differ in the two cases.

We have assumed that voters are risk-neutral. While this makes for simplicity, it also implies that we disregard a potentially important feature of informative campaigning. Namely, that information reduces uncertainty about a party’s policy. If voters are risk-averse, this is a positive feature of campaigning (see Brock and Magee 1978, Austen-Smith 1987 and Cameron and Enelow 1992 for theoretical models with risk-averse voters, uncertainty about parties’ platforms and campaign spending financed by private contributors).

In our model, a voter who is uninformed about a party’s policy has a point expectation about it. In equilibrium, that expectation is correct. If the voter was uncertain about the policy, such that she had a non-degenerate probability distribution over possible policies, and if, furthermore, she were risk-averse, she would then tend to dislike the party, simply because of such uncertainty. A risk-averse voter who is indifferent between $l^e$ and $r^e$ would prefer party $L$ if she were not completely sure about $R$’s policy. In such a case, campaigning would have a positive effect, from the party’s point of view. Even though uninformed voters may have correct expectations on average, more voters would vote for $R$ if they were sure about its policy.

We conjecture that adding uncertainty and risk-aversion to our model would simply reinforce our results. Parties would be more eager to raise campaign funds, since reducing the uncertainty about their policies would attract more votes, which, in turn, would give them a further incentive to modify their policies towards the middle, in order to enjoy larger public funds.

As it stands, our model shows that policy convergence results even if voters are risk-neutral and there is no uncertainty about any party’s policy.
9 Appendix

9.1 With impressionable voters

Proof of Proposition 1.

The first-order conditions for the parties are

\[
\begin{align*}
\frac{\partial H(q)}{\partial l} + \frac{(1 - H(q))}{2} f \frac{l + r}{2} (r \mid l) &= 0 \\
\frac{\partial H(q)}{\partial r} + (1 - H(q)) F \frac{l + r}{2} &= 0
\end{align*}
\]

(31)

and

\[
\begin{align*}
\frac{\partial H(q)}{\partial r} + \frac{(1 - H(q))}{2} f \frac{l + r}{2} (r \mid l) &= 0 \\
\frac{\partial (l \mid H(q)) + (1 - H(q))}{2} F \frac{l + r}{2} &= 0
\end{align*}
\]

(32)

Now

\[
\frac{\partial q}{\partial l} = q v(l; r) \frac{\partial v(l; r)}{\partial q}
\]

(33)

By inserting (5) and (33), and simplifying, we can rewrite the first-order condition for maximum for party L; (31)

\[
\frac{1}{(1 - H(q) \mid l) (1 - H(q) \mid r)} f \frac{l + r}{2} (r \mid l) \frac{\partial H(q)}{\partial l} + \frac{\partial H(q)}{\partial r} + F \frac{l + r}{2} = 0
\]

(34)

Similarly, the first-order condition for R; (32) can be written as

\[
\frac{1}{(1 - H(q) \mid l) (1 - H(q) \mid r)} f \frac{l + r}{2} (r \mid l) \frac{\partial H(q)}{\partial l} + \frac{\partial H(q)}{\partial r} + 1 F \frac{l + r}{2} = 0
\]

(35)

Thus, since the first parts of (34) and (35) are identical,

\[
\frac{\partial H(q)}{\partial l} + F \frac{l + r}{2} = \frac{\partial (1 - H(q))}{2} F \frac{l + r}{2}
\]
which implies that
\[ v = \mathcal{H}(\phi) + (1 \cdot \mathcal{H}) F \left( \frac{l + r}{2} \right) = \frac{1}{2}. \]  (36)

Each party gets half of the votes. Since, \( \phi \left( \frac{1}{2} \right) = 0 \); and \( H(0) = \frac{1}{2} \); this implies \( F \left( \frac{l + r}{2} \right) = \frac{1}{2} \) so that
\[ \frac{l + r}{2} = m \]  (37)

Inserting into (34) and rearranging yields
\[ (r, l) = \frac{1}{2} \cdot \frac{i \cdot H(0) \cdot \phi \left( \frac{1}{2} \right)}{f(m)} \]  (38)

Using \( \frac{l + r}{2} = m \); we finally get
\[ l = m + \frac{1}{2} \cdot \frac{i \cdot H(0) \cdot \phi \left( \frac{1}{2} \right)}{f(m)} \]  (39)

and
\[ r = m + \frac{1}{2} \cdot \frac{i \cdot H(0) \cdot \phi \left( \frac{1}{2} \right)}{f(m)} \]  (40)

thus, the policies are symmetrical around \( m \):

Derivation of the second-order condition.

We derive the sufficient conditions for the second-order condition for maximum in the model with impressionable voters. For party \( L \) the second-order condition for maximum is
\[ \frac{\mathcal{A}}{2} \frac{d^{2} \phi}{dl^{2}} + \frac{1}{2} \cdot \frac{i \cdot \mathcal{H}}{f} \quad \frac{d^{2} \phi}{dl^{2}} + \frac{1}{4} \cdot \frac{i \cdot \mathcal{H}}{f} \cdot 0 \]  (41)

Since \( r, l \cdot 1 \); this is fulfilled if
\[ \frac{\mathcal{A}}{2} \frac{d^{2} \phi}{dl^{2}} + \frac{1}{2} \cdot \frac{i \cdot \mathcal{H}}{f} \quad \frac{d^{2} \phi}{dl^{2}} + \frac{1}{4} \cdot \frac{i \cdot \mathcal{H}}{f} \cdot 0 \]  (42)
or

\[(1 \circ \circ) \frac{\mathcal{A} f 0}{4} \frac{1}{f} + H \frac{d^2 \xi}{dl^2} + \frac{d \xi}{dl} (H \psi i) 2H 0 \cdot 0\]

assuming that \(\frac{d \xi}{dl} > 0\); (which is the case if \(\mathcal{A} 0 > 0\)), we can rewrite it as

\[(1 \circ \circ) \frac{\mathcal{A} f 0}{4} \frac{1}{f} + H \frac{d \xi}{dl} \frac{d^2 \xi}{dl^2} + \frac{d \xi}{dl} (H \psi i) 2A \cdot 0\]

We can see that a sufficient condition for this to be fulfilled is that

\[f 0 \frac{1}{f} < 4; \text{ and } \frac{d \xi}{dl} \frac{d^2 \xi}{dl^2} + \frac{d \xi}{dl} \frac{H \psi i}{H 0} < 2: \tag{42}\]

The latter condition is a joint condition on the responsiveness of the government funding system and the responsiveness of the impressionable voters. It is difficult to define general conditions that would ensure that this is fulfilled. If, however, \(H 0 = 0\) and \(\mathcal{A} 0 = 0\); so that \(\psi 0 = 0\); the condition is fulfilled. To verify this, we can calculate

\[\frac{d \xi}{dl} = \psi \frac{v(l; r)}{\mathcal{A}}\]

and

\[\frac{d^2 \xi}{dl^2} = \psi \frac{v(l; r)}{\mathcal{A}} + \psi \frac{\partial^2 v}{\mathcal{A}^2}\]

Hence

\[\frac{d^2 \xi}{dl^2} \frac{d \xi}{dl} = \frac{\psi \frac{v(l; r)}{\mathcal{A}} + \psi \frac{\partial^2 v}{\mathcal{A}^2}}{\psi \frac{v(l; r)}{\mathcal{A}}} = \psi \frac{v(l; r)}{\mathcal{A}} + \psi \frac{\partial v}{\mathcal{A}^2} \frac{v(l; r)}{\mathcal{A}}\]

Using (5) we get

\[\frac{\partial^2 v}{\mathcal{A}} = \frac{\frac{1}{4} \psi f 1 l \circ H 0 i 0 + \frac{1}{2} \psi f 0 H \psi i 0 + (\psi 0^2 H \psi i 0)}{(1 l \circ H 0 i 0)^2}\]

\[= \frac{\frac{1}{4} \psi f 0}{(1 l \circ H 0 i 0 < (1 l \circ H 0 i 0 \cdot 2)}\]

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where the first equality follows from the assumption that $H(0) = 0$ and $\bar{A}(0) = 0$; so that $\zeta(0) = 0$. The inequality follows from the assumption that $f(0) < 4$:

We now have that
\[
\frac{\partial^2 V}{\partial l^2} \frac{\partial V(l; r)}{\partial A} < 2
\]

The inequality follows from the assumption that $f(0) = f < 4$:

We now have that
\[
\frac{\partial^2}{\partial l^2} \frac{\partial V}{\partial A} = 2 \bar{A} f \frac{\bar{A} + r}{2} + 2 \bar{A} f \frac{\bar{A} + r_e}{2} = 0
\]

Together with $\zeta(0) = 0$; this gives that $\frac{\partial^2 V}{\partial l^2} \frac{\partial V(l; r)}{\partial A} < 2$; Under the assumption that $H(0) = 0$; this implies that the second inequality in (42) is fulfilled as claimed. As is clear from this deviation, it is not necessary that $H(0) = 0$ and $\zeta(0) = 0$. They should simply not be too large. Party R is in a symmetric situation, its second-order condition being fulfilled under the same assumptions. Finally, we should note that if $\frac{\partial V}{\partial A} = 0$; the second-order condition is fulfilled if $f(0) < 4$;

9.2 With informative campaigning

We shall now show that the second-order condition is also fulfilled in the model with informative campaigning.

Equation (12) implies
\[
\frac{\partial^2 V}{\partial A^2} = \bar{A}_R f \frac{\bar{A}_R + r}{2} \frac{\bar{A}_R + r}{2} = 0
\]

when $r = r_e$: Similarly, and using rational expectations again, (12) also implies
\[
\frac{\partial^2 V}{\partial A_R^2} = 0
\]

Party L's second-order condition is
\[
\frac{\partial V}{\partial A} + \frac{\partial V}{\partial A_L} + \frac{\partial V}{\partial A_R} = (r^n_x \mid l^n) V = 0
\]

Hence, its second-order condition for maximum is
\[
\frac{\partial^2 V}{\partial^2 A} + \frac{\partial V}{\partial l} \frac{\partial^2 V}{\partial l^2} = A(r^n_x \mid l^n) V = 0
\]
Using (14), (43) and (44) it reduces to

\[ \frac{\partial^2 V}{\partial^2 (r^n | l^n)} i \cdot 2 \frac{\partial V}{\partial \mathbf{a}} \cdot 0 \]

Inserting (13) yields

\[ \frac{\bar{A}_n}{4} f \cdot 0 \frac{\bar{A}_1 + r}{2} \cdot (r | l) i \cdot 2 \frac{\bar{A}_n}{2} f \frac{\bar{A}_1 + r}{2} \cdot 0 \]

which is clearly fulfilled under our assumption \( f \mathcal{O} f < 4 \):
Figure 1

Figure 2

$\frac{\phi_1(c)}{\phi_1(c) + \phi_1(1-c)}$

$a = 0.13, \; r = 0.01$
References


