THE IMPORTANCE OF FIXED COSTS IN THE DESIGN OF TRADE POLICIES: AN EXERCISE IN THE THEORY OF SECOND BEST*

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WP-AD 94-16

* We are grateful to seminar audiences in Alicante, Toulouse and Viena and to C. Herrero, I. Jimenez-Raneda, F. Marhuenda, A. Mas-Colell, I. Ortuño-Ortíñ, J.A. Silva and I. Steedman for helpful comments. We are solely responsible for any remaining error. The first author acknowledges support from DGICYT under project PB91-0756 and the hospitality from Royal Complutense College were the last version of the paper written.

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ABSTRACT

In this paper we study the optimal trade policy in an oligopolistic market with a given number of quantity-setter firms. It is shown that under no fixed costs, the optimal trade policy displays certain characteristics but under fixed costs most of these characteristics no longer hold.

Keywords: Optimal Trade Policy; Tariffs; Quotas.
1. INTRODUCTION

There is a sizable literature studying the effects of economic policy in a static oligopolistic market subject to foreign competition (see for instance Dixit (1984), Helpman (1984), Krishna (1984), Brander & Spencer (1985), Buffett & Spiller (1986), Eaton & Grossman (1986), Laussel & Montet & Peguin-Feissolle (1988), Markusen & Venables (1988), Das & Donnenfeld (1989), Krugman (1989), Krishna & Thursby (1991) and Barros & Cabral (1992)). Such effects and the characterization of the optimal policy look different from those under perfect competition as it usually happens in a Second-Best framework (in this case the additional restriction is that firms are not perfectly competitive). Another important insight obtained in this literature is that conclusions depend very much on the solution concept. The basic technique which is used to prove most of these results is differential calculus.

In this paper we want to focus our attention on two issues. First, that under no fixed costs (i.e. when the reaction of firms to market conditions is smooth) it is possible to say something about the effect of certain trade policies on domestic welfare and about the shape of the optimal trade policy. This contradicts the belief that second-best theory is of not much practical help. Second, under fixed costs many basic insights on the optimal economic policy obtained in the smooth case do not carry through to the discontinuous case. In other words, there is a clear distinction between the case in which domestic firms respond to market conditions in a smooth way (and thus the use of differential calculus is warranted) and the case in which, because of the existence of fixed costs, the reaction of domestic firms is discontinuous. Even though there are papers in which the fixed cost case is studied none of them focus on the crucial impact of the above discontinuity on the design of economic policy. In order to make our point we take the simplest solution concept, namely Cournot equilibrium with a given number of firms. Of course, a complete picture of the differences mentioned above will only emerge when other solution concepts have been analyzed but this is outside the scope of the present paper.
We first analyze the smooth case. We remark that some of our results here are just generalizations of results that already appeared in the literature. Specifically, we show that, under some conditions the following holds:

1) In the case in which the Government has quotas as the only policy instrument, second-best is neither a small quota nor a quota for which almost no domestic production is undertaken since in both cases domestic welfare can be raised by lowering (resp. increasing) the quota (see Proposition 1). The second best is attained either under free trade or under complete protection (see Proposition 2) and in such a way that if domestic firms are relatively efficient (resp. inefficient) with respect to the foreign firm they should be protected (resp. not protected). If domestic and the foreign firms are identical, we find reasonable conditions under which complete protection is second-best (see Proposition 3). This findings are summarized in Theorem 1.

2) If tariffs are available the protectionist argument is considerably weakened. We consider two kinds of tariffs: a tariff on profits of the foreign firm (which we call an $\alpha$-tariff) and a tariff on the output of the foreign firm (which will be called simply a tariff)\(^{(1)}\). We first show that there is an $\alpha$-tariff such that domestic welfare is locally increasing with the quota (see Proposition 4 which however requires that the foreign firm is more efficient than the domestic ones. If this is not the case the Proposition does not necessarily hold). Moreover under non increasing marginal costs there is an $\alpha$-tariff such that domestic welfare is locally increasing with the quota and such that very little knowledge on the economy is needed in order to determine the value of this tariff (in particular no knowledge on the demand function is needed, see Proposition 5). It is also shown that an $\alpha$-tariff always yields more domestic welfare than a tariff (see Proposition 6) provided the latter is positive. A summary of all these findings is stated as Theorem 2 which says that no quotas with an $\alpha$-tariff always yields more domestic welfare than

\(^{(1)}\) The $\alpha$-tariff can be interpreted either as a tax on profits of the foreign firm or as a legal requirement that forces the foreign firm to produce in the home country and to raise a fraction $\alpha$ of domestic founds in order to enter into the domestic market (i.e., a partnership with domestic investors).
tariffs (which in turn can be proved to be better than quotas). Thus, the optimal economic policy requires neither quotas nor tariffs, only an $\alpha$-tariff.

3) In the case of a tariff on output we show that domestic welfare is increasing in the tariff at the point where domestic firms produce zero output and it is decreasing at the point where the foreign firm is just inactive (see Proposition 7). With respect to the optimal tariff (i.e. the tariff which maximizes social welfare) we can show that it exists and that under some additional conditions it is unique. We also identify sufficient conditions for this optimal tariff to be positive (see Proposition 8). Theorem 3 summarizes these results.

Next, we switch to a model in which the technology for the (unique) domestic firm is characterized by a fixed cost and a constant marginal cost. We show that under quotas alone, the optimal policy may be complete protection, free trade or a level of imports such that profits of the domestic firm are zero. Moreover the connexion of these possibilities with the relative efficiency of the domestic firm is weaker than in the smooth case (see Proposition 8). If tariffs are available, there might be no $\alpha$-tariff for which an increase in the quota increases domestic welfare (see Example 1) and under conditions equivalents to those in Proposition 8, the optimal tariff may be negative (see Example 2). Moreover, an $\alpha$-tariff may yield less welfare than a linear tariff on imports (see Example 2) and the optimal tariff may imply autarky (See Example 3). Thus, most conclusions about the shape of the optimal policy obtained in the smooth case no longer hold when fixed costs are considered (see Theorem 4).

The rest of the paper goes as follows. The next Section presents the basic model. In Section 3 we study the smooth case and in Section 4 the fixed cost case. Finally, Section 5 gathers our final comments.
2. THE MODEL

There is a unique consumer with a utility function $u(x) - px$, where $x$ is the consumption of the homogeneous good and $p$ is the price. We will assume that $u(0) = 0$, $u'(x) > 0$ and $u''(x) < 0 \forall x$. There are $n$ domestic firms with increasing cost functions denoted by $c_i(x_i)$, $i = 1, ..., n$ where $x_i$ is the output of firm $i$. Let $x_d = \sum_{i=1}^{n} x_i$ be domestic output and $\pi_d = px_d - \sum_{i=1}^{n} c_i(x_i)$ be the aggregate domestic profit. We will assume that there is a unique foreign firm with an increasing cost function denoted by $c_f(x_f)$ where its output is denoted by $x_f$ (and thus $x = x_d + x_f$). The cost functions of both domestic and the foreign firm will be assumed to be twice continuously differentiable for strictly positive outputs.

The instruments in the hands of the government are tariffs and quotas. We will assume that the decision on them is irreversible and prior to the decision of firms on outputs, i.e., we assume that the government acts as a Stackelberg leader. Notice that we do not consider the possibility of a subsidy to domestic firms: in this case it is clear that the government can enforce an equilibrium with domestic firms as Stackelberg leaders and the foreign firm as a follower (different assumptions on the timing of the game and thus on the solution concept are considered in González-Maestre (1992). See also Collie (1993)).

Domestic welfare, denoted by $S$, will be the sum of producers’ and consumer’s surpluses plus the rents captured by the government via tariffs, denoted by $R$, i.e. $S = u(x) - px + \pi_d + R$. Assuming that the consumer is perfectly competitive we have that $p = u'$ and therefore

$$S = u(x) - u'(x) x_f - \sum_{i=1}^{n} c_i(x_i) + R \quad (1)$$

We will assume that firms are quantity-setters and that they maximize profits given the quota and the tariff. Assuming interiority for domestic
firms, first order conditions of profit maximization in a Cournot equilibrium, relative to a quota Q and a tariff schedule \( R(\cdot) \) read

\[
p(x_i + x_d) + x_l p'(x_i + x_d) - c_i'(x_i) = 0 \quad i = 1, \ldots, n, \tag{2}
\]

\[
p(x_f + x_d) + x_f p'(x_f + x_d) - c_f'(x_f) - R'(x_f) \geq 0, \quad Q \geq x_f \tag{2'}
\]

We will assume that for sufficiently large outputs, costs are not recovered. This allows us to restrict attention to a compact interval in which any possible equilibrium must lie. All the subsequent assumptions must be understood as referring to outputs lying in this interval. We now assume

**Assumption 1.**

a) \( \forall x_i, x_j', p'' x_j + p' < 0, \quad j = 1, \ldots, n, f \)

b) \( p' - c_j'' < 0, \quad j = 1, \ldots, n, f \)

This assumption (A.1 in the sequel) is standard when dealing with the Cournot model (see e.g. Friedman (1982) assumption 3 p.496 and Brander and Spencer (1985)). It implies the strict concavity of the profit function with respect to the own output (and thus under this assumption (2) and (2') are also sufficient conditions of profit maximization) and the existence and uniqueness of the Cournot equilibrium (see Friedman (1982) Theorem 1 p.496 and Collie (1992)).

Another implication of A.1 is that if \( R' \) is non increasing on \( x_f \) (and this is true in the cases we will consider) the foreign firm is willing to supply at least the quota as far as it is not larger than its equilibrium output with no quotas. This follows from the fact that if \( x_f \) is less than the quota, aggregate output is also less than in equilibrium with no quotas -see the proof of Proposition 1 below- and thus marginal profit is positive. Given that the profit function is concave the profit maximizing output equals the quota. Thus, the quota will also be denoted by \( x_f \).

Let us denote the output of firm \( j \) in the Cournot equilibrium with no quotas by \( \tilde{x}_j \), \( j = 1, \ldots, n, f \). We will first consider the case in which cost curves are differentiable everywhere, i.e. no fixed costs.
3. THE CASE OF NO FIXED COSTS

The purpose of this Section is to characterize the optimal policy when the behavior of firms is continuous. We begin by the case in which no tariff is available. Let us introduce some notation. Let \( \frac{dS}{dx_f}(A) \) be evaluated at the point of autarky (i.e. zero quota) and \( \frac{dS}{dx_f}(ZDO) \) be evaluated at a quota, assumed to be feasible, for which domestic output is zero. Then, we have our first result:

**Proposition 1.** Suppose that A.1 holds. Then \( \frac{dS}{dx_f}(A) < 0 \) and \( \frac{dS}{dx_f}(ZDO) > 0 \).

**Proof:** Using (1), it is easy to compute

\[
\frac{dS}{dx_f} = u'(\frac{dx_d}{dx_f} + 1) - p'(\frac{dx_d}{dx_f} + 1)x_f - p - \sum_{i=1}^{n} c_i \frac{dx_i}{dx_f} = u' \frac{dx_d}{dx_f} - p' x_f (\frac{dx_d}{dx_f} + 1) - \sum_{i=1}^{n} c_i \frac{dx_i}{dx_f}.
\]

Now, since for all firms for which the first order condition is fulfilled with inequality \( dx_1 / dx_f = 0 \), using (2) we get that

\[
\frac{dS}{dx_f} = u' \frac{dx_d}{dx_f} - p' x_f (\frac{dx_d}{dx_f} + 1) - \sum_{i=1}^{n} (p + x_i p') \frac{dx_i}{dx_f} = -p' x_f (\frac{dx_d}{dx_f} + 1) - p' \sum_{i=1}^{n} x_i \frac{dx_i}{dx_f}.
\]

Next, we will show that A.1 implies that \( \frac{dx_d}{dx_f} > -1 \) and that \( \frac{dx_i}{dx_f} < 0, i = 1, \ldots, n \) (for the case of identical firms see Buffie & Spiller (1986) p.68). First, first order conditions of profit maximization imply that if \( x_f \) increases, neither \( x \) nor \( x_i, i = 1, \ldots, n \) can be constant. Then, for a particular firm, say i, we have four possible cases.
\begin{align*}
  (a) & \quad \text{x increases and } x_i \text{ increases} \\
  (b) & \quad \text{x increases and } x_i \text{ decreases} \\
  (c) & \quad \text{x decreases and } x_i \text{ increases} \\
  (d) & \quad \text{x decreases and } x_i \text{ decreases}
\end{align*}

It is easy to see that Assumption 1 implies that cases (a) and (d) above are impossible. Suppose that (c) is true. Then it must be that case (c) holds for every \( i = 1, \ldots, n \). But since \( x = x_f + x_d \) this is impossible, so case (b) holds for all firms and therefore \( \frac{dx_f}{dx} > -1 \) and \( \frac{dx_i}{dx} < 0 \), \( i = 1, \ldots, n \). Thus, the Proposition follows from these inequalities and equation (3) above.

The intuition behind Proposition 1 is that the effect of an increase in quotas on social welfare can be decomposed into two elements (see equation (3)). On the one hand we have a positive effect on welfare which comes from the fact that the increase in aggregate output caused by the increase in quotas, decreases both market price and expenditure on imports. On the other hand, production of domestic firms falls, which is socially inefficient since price is higher than marginal cost. Thus if \( x_f \) is zero the first effect vanishes so that only the second effect remains and if domestic output is zero only the first effect remains.

It is worth to notice two implications of Proposition 1. First neither a very small quota, nor a quota for which the domestic industry is almost producing zero can be optimal. Second, there exists a quota for which equation (3) above equals to zero –so the first order condition of welfare maximization is fulfilled- but total welfare is minimized. Therefore, the optimal quota can not be found by calculus. In order to be more specific about the optimal quota will have to make more assumptions on preferences and technology. First, let us define \( \beta = \beta(x) \equiv p''(x) x / p'(x) \). Thus, \( \beta \) measures the degree of concavity of the inverse demand function. Then, we have the following

**Proposition 2.** Let us assume A. 1 and that domestic firms are identical, the technology displays constant returns to scale and \( \beta(x) \) is non increasing on \( x \). Then, \( S(x_f) \) is quasi-convex on \( x_f \).
Proof: Let $q = x_f / x_d$. Then

$$\frac{dS}{dx_f} = -p'x_d \left( q(\frac{dx_d}{dx_f} + 1) + \frac{dx_1}{dx_f} \right)$$

$$\frac{dx_i}{dx_f} = -\frac{p' + x_i p''}{n(p' + x_i p'') + p'} = -\frac{n(q + 1) + \beta}{(n+1)(q + 1)n + n\beta}$$

Thus

$$\frac{dS}{dx_f} = -p'x_d \frac{q^2 - 1 - \beta/n}{(n + 1)(q + 1) + \beta}$$

By A.1 $\beta > -n$ and thus $(n + 1)(q + 1) + \beta > 0$. So disregarding the case in which $x_d = 0$ (in which we know that $\frac{dS}{dx_f} > 0$), $\frac{dS}{dx_f} = 0$ if $q = \sqrt{(1 + \beta/n)}$ ($q > 0$ by A.1). The last equation implies that the $x_f$ for which $\frac{dS}{dx_f} = 0$, denoted by $x'_f$, is unique since $s$ is increasing on $x_f$ and $\beta$ is non increasing on $x$ (which in turn is increasing with $x_f$). Finally if $x_f > x'_f$, $q$ increases and $\beta$ decreases so $\frac{dS}{dx_f} > 0$ and if $x_f < x'_f$, by identical reasoning $\frac{dS}{dx_f} < 0$.

Proposition 2 has been proved by Laussel & Montet & Peguin-Feissolle (1988) in the case of one domestic firm and linear inverse demand (they allow for some product differentiation which can be easily introduced in our model). Notice that our assumption on $\beta$ allows for linear inverse demand functions.

Proposition 2 has two important implications. a) under our assumptions, first order conditions will never yield a maximum and the problem of finding the optimal quota reduces to compare welfare levels at autarky and Cournot equilibrium with no quotas (i.e. free trade). b) if $\frac{dS}{dx_f}(x_f) \leq 0$, the quasi-convexity of $S(\cdot)$ implies that domestic welfare is maximized in autarky. Therefore we turn to study sufficient conditions for $\frac{dS}{dx_f}$ evaluated at the Cournot equilibrium with no quotas to be non positive.

Proposition 3. Suppose that A.1 holds and all firms (including the foreign) are identical. Then, under constant returns to scale
a) If \( n = 1 \), \( \frac{dS}{dx_f}(\bar{x}_f) \geq 0 \) if and only if \( p(\cdot) \) is concave (i.e. \( \beta \geq 0 \)).

b) If \( n > 1 \), \( \frac{dS}{dx_f}(\bar{x}_f) < 0 \).

**Proof:** It is easily calculated that

\[
\text{Sign} \left( \frac{dS}{dx_f}(\bar{x}_f) \right) = -\frac{n(p' + x_i p'') - p' + c_i'}{n(p' + x_i p'') + p - c_i''}
\]  

(4)

Under constant returns the previous expression simplifies to

\[
\text{Sign} \left( \frac{dS}{dx_f}(\bar{x}_f) \right) = -\frac{n \beta + n^2 - 1}{n \beta + (n + 1)^2}
\]  

(5)

where the denominator is positive (by A.1). Thus if \( n = 1 \)

\[
\text{Sign} \left( \frac{dS}{dx_f}(\bar{x}_f) \right) = -\text{Sign } \beta
\]

If \( n > 1 \), A.1 implies that the numerator of (4) is positive, so a) and b) above are proved.

An interpretation of Proposition 3 is that if domestic firms have low marginal cost relative to the marginal cost of the foreign firm, i.e. if domestic firms are relatively efficient, the output of the foreign firm in a Cournot equilibrium with no quotas will be small and it is likely that it will be located in the decreasing part of \( S(\cdot) \). Thus, the best policy is autarky. Conversely, if the domestic firm is relatively inefficient it is likely that the optimal policy is free trade. In other words, it is likely that the optimal policy agrees with the commandment "Do not protect the inefficient" (or "do not allow foreign mediocrities to enter").

Propositions 1, 2 and 3 imply the following

**Theorem 1.** If A.1 holds,
a) Then the optimal quota can not be a very small one or such that the domestic industry produces almost nothing.

b) If all firms (including the foreign) are identical, the technology displays constant returns to scale, \( \beta(x) \) is non increasing on \( x \), and either \( n = 1 \) and the inverse demand function is concave or \( n > 1 \), then domestic welfare is maximized in autarky. (2)

We now consider the case in which the government can impose tariffs. We begin our analysis by considering the case in which the tariff is proportional to foreign profits so it does not create distortions in the output decision of the foreign firm. Thus, in this case, \( R = \alpha \Pi_f \) where \( \Pi_f \) is the profit of the foreign firm and \( 0 < \alpha < 1 \). We will call these tariffs \( \alpha \)-tariffs. There are several possible interpretations of an \( \alpha \)-tariff. It can be understood as a tax on profits contingent to the entry of the foreign firm (i.e. an entry fee) or that the foreign firm is forced to raise a fraction \( \alpha \) of domestic founds in order to enter into the domestic market (however joint partnership with some domestic firms already in the market is not allowed see Barros & Cabral (1992) for this case). Notice that if \( \alpha \) is close to one, \( \alpha \Pi_f \) measures the greatest possible rent which can be captured by the domestic government without creating a distortion in the decision of the foreign firm.

Let \( x_i(x_f) \) be the equilibrium output for the domestic firm \( i \) relative to the quota \( x_f \) and let \( x(x_f) \) be total output as a function of the quota \( x_f \). Thus

(2) The conclusion obtained in Proposition 3 may be reversed in the case of many identical foreign firms with constant returns to scale. In this case the numerator of (5) above becomes \( h^2 - n\beta^2 - n \), where \( h \) is the number of foreign firms. Easy calculations show that if \( h > n \) and \( \beta < 2 + 1/n \), \( \frac{dS}{dx_f} (x_f) > 0 \) if \( h = n \) \( \frac{dS}{dx_f} (x_f) < 0 \) Iff \( p(\cdot) \) is concave, and if \( h < n \) and \( \alpha \cdot 1 \rightarrow \frac{dS}{dx_f} (x_f) < 0 \). Thus, the number of foreign firms is a critical variable in the determination of the optimal tariff. The more foreign firms are prepared to enter into the domestic market, the less likely is that autarky is the best policy since with many foreign firms in a market the total output produced by them is large. Thus, if \( S(\cdot) \) is quasi-convex and the foreign output is sufficiently large, domestic welfare will be maximized with free trade.
we can define $S(x, \alpha) = u(x(x_f)) - u'(x(x_f)) x_f - \sum_{i=1}^{n} c_i(x_i(x_f)) + \alpha \Pi_i$. Now we will show that if $\alpha$-tariffs are available, there is one for which an increase in the quota increases domestic welfare.

**Proposition 4.** Let $x_f (\leq \bar{x}_f)$ be a quota and let be $i$ the domestic firm whose marginal cost is the lowest among domestic firms. Then if A.1 holds and $c_i'(x_i(x_f)) \geq c_f'(x_f)$ there is an $\alpha$-tariff $\alpha'$ such that $\frac{dS}{dx_f}(x_f, \alpha) > 0 \ \forall \alpha \geq \alpha'$.

**Proof:**

$$\frac{dS}{dx_f} = -p'x_f \left(\frac{dx_d}{dx_f} + 1\right) - p' \sum_{i=1}^{n} x_i \frac{dx_i}{dx_f} + \alpha (p - c_f' + p'\left(\frac{dx_d}{dx_f} + 1\right) =$$

$$-p'x_f \left(\frac{dx_d}{dx_f} + 1\right)(1 - \alpha) - p' \sum_{i=1}^{n} x_i \frac{dx_i}{dx_f} + \alpha (p - c_f').$$

Since the firm $i$ has the lowest marginal cost, from the first order condition of profit maximization we have that $x_i \geq x_j \ \forall \ j = 1, ..., n$. Also $\alpha < 1$ and $\frac{dx_d}{dx_f} + 1 > 0$. Thus, we have that

$$\frac{dS}{dx_f} > -p' x_i \frac{dx_d}{dx_f} + \alpha (p - c_f'). \quad (6)$$

Now define $\alpha' = \frac{p(x(x_f)) - c_i'(x_i(x_f))}{p(x(x_f)) - c_f'(x_f)} - \varepsilon$, with $\varepsilon > 0$ but $\varepsilon \equiv 0$.

Taking into account the first order condition of profit maximization for firm $i$, if $\varepsilon$ sufficiently close to zero the right hand side of (6) becomes approximately $-p'x_i \left(\frac{dx_d}{dx_f} + 1\right) > 0$. For $\alpha \geq \alpha'$ the same reasoning applies. \(\blacksquare\)

The condition on the marginal cost of the foreign firm can be weakened somewhat. But if the marginal cost of the foreign firm is greater than the marginal cost of any domestic firm, an easy adaptation of the examples given
by Schmalansee (1976) and Lahiri & Ono (1988) shows that Proposition 4 fails even if \( \alpha \) is arbitrarily close to one.

Proposition 4 states that given a quota, there exist an \( \alpha \)-tariff such that domestic welfare increases with the quota, i.e. there are incentives to liberalize infinitesimally the market. However the \( \alpha \)-tariff doing the job depends on the existing quota. The next Proposition gives a sufficient condition for an \( \alpha \)-tariff to increase domestic welfare for all \( x_f \).

**Proposition 5.** Under the same conditions than Proposition 4, if \( c_{i}' \leq 0 \quad \forall \ i = 1, \ldots, n, f \), then there exists an \( \alpha \)-tariff \( \bar{\alpha} \) such that \( \frac{ds}{dx_f} (x_f, \alpha) > 0 \), for all \( x_f < \bar{x}_f \), and for all \( \alpha \geq \bar{\alpha} \).

**Proof:** First we will show that under our assumptions \( \alpha(x_f) \) as defined in the previous Proposition is decreasing on \( x_f \). Indeed

\[
\text{sign}\ rac{d\alpha}{dx_f} = \text{sign} \left( p' \frac{dx}{dx_f} (c_i' - c_f') - (p - c_f') c_i'' \frac{dx_i}{dx_f} + (p - c_i') c_f'' \right) < 0
\]

Thus, an \( \alpha \)-tariff defined as

\[
\bar{\alpha} = \frac{p(x(0)) - c_i(x_i(0))}{p(x(0)) - c_f'(0)} - \epsilon \quad (\text{with} \ \epsilon > 0 \ \text{but} \ \epsilon \ll 0) \geq \alpha(x_f) \ \forall \ x_f < \bar{x}_f
\]

will do the job since equation (6) above holds for \( \alpha \geq \bar{\alpha} \).

Several remarks are in order. First, under strictly increasing marginal costs the previous Proposition does not hold. Second, the Proposition implies that if the government knows \( p(x(0)), c_i(x_i(0)) \) and \( c_f'(0) \) (i.e. the price and the marginal cost of both the most efficient domestic firm and the foreign firm in autarky) there is an entry fee on profits \( \bar{\alpha} \) or alternatively a partnership of the foreign firm with domestic investors such that free trade is the optimal policy (notice that knowledge of the inverse demand function is
not needed). Thus, the negative results obtained in the quota only case are reversed by considering $\alpha$-tariffs.

Next we consider the case of a linear tariff on foreign output, $R = t x_f'$. It can be shown that under A.1 there is a unique Cournot equilibrium relative to $t$, i.e. a unique vector of outputs satisfying equations (2) and (2') with $R' = t$. Let $S(t)$ be the domestic welfare as a function of $t$. We first show that if $t$ is non-negative, there is an $\alpha$-tariff and a quota such that the corresponding domestic welfare equals the domestic welfare obtained with $t$.

**Proposition 6.** If $t \geq 0$, $\exists \; \hat{\alpha}, \; \hat{x}_f$ such that $S(\hat{\alpha}, \hat{x}_f) = S(t) \forall \alpha \geq \hat{\alpha}$.

**Proof:** Let $\hat{\alpha} = \frac{t \; x_f(t)}{p(x(t))x_f(t) - c_f(x_f(t))}$ where $x_f(t)$ and $x(t)$ are respectively the equilibrium output of $f$ and the aggregate output relative to $t$. It is clear that $0 \leq \hat{\alpha} \leq 1$. Take $\hat{x}_f = x_f(t)$. We will show that the equilibrium output of the foreign firm under the above $\alpha$-tariff and quota equals the quota. Under tariffs we have that $\forall x_f$

$$p(x_d(t) + x_f(t))x_f(t) - c_f(x_f(t)) \geq p(x_d(t) + x_f')x_f' - c_f(x_f') - tx_f$$

If $x_f(t)$ does not maximize profits for $f$ under an $\alpha$-tariff $\hat{\alpha}$, and a quota $\hat{x}_f$, it must be that $\exists x_f' < x_f(t)$ such that

$$p(x_d(t) + x_f(t))x_f(t) - c_f(x_f(t)) < p(x_d(t) + x_f')x_f' - c_f(x_f')$$

Combining both inequalities we obtain that since $t \geq 0$, $x_f(t) < x_f'$ which is a contradiction. The proof ends by noting that the rents extracted to the foreign firm with an $\alpha$-tariff of $\hat{\alpha}$ equals the rents obtained with a tariff $t$.

Notice that if the foreign firm makes positive profits with a non-negative tariff, in general it is possible to obtain a higher welfare with an $\alpha$-tariff and a quota.

Propositions 4-6 can be summarized in the following.
Theorem 2. If all domestic firms are identical, all firms (including the foreign) have constant returns to scale, $c_i' \geq c_f' \forall i = 1, \ldots, n$ and A.1 holds, there is an $\alpha$-tariff $\alpha*$ such that $S(\alpha*, \hat{x}_f) \geq S(t), \forall t \geq 0$.

In other words, under the above conditions no tariff can do better than an $\alpha$-tariff and no quotas. This has the implication that, when feasible, partnership of the foreign firm with domestic investors and no quota is the best possible trade policy. As we will see, things are very different when a positive fixed cost is considered. The question of the relationship between domestic welfare under tariffs and $\alpha$-tariffs for negative values of $t$ remains open. Since a negative tariff can increase the foreign output beyond the Cournot equilibrium output with no quotas (which is the maximum attainable output under an $\alpha$-tariff), in principle this effect can offset the loss in revenue. Moreover as Proposition 7 shows the optimal tariff might be negative.

We now concentrate on the characterization of the optimal tariff on output. In the following analysis, we will assume constant marginal costs -denoted by $c$- and identical firms (including the foreign one). Let us define $x_i(t), x_f(t)$ and $x(t)$, respectively, as the equilibrium output corresponding to domestic firm $i$, the foreign firm, and aggregate output as a function of $t$.

Now, let us define $\bar{t}$ and $\tilde{t}$, respectively, by the following conditions:

(I) $p(x(\bar{t})) - c = 0$.
(II) $p(x(\tilde{t})) - c - \tilde{t} = 0$.

In words, facing a tariff $\bar{t}$ the foreign firm chooses zero output, while $\tilde{t}$ is the tariff for which domestic firms will choose zero output. Then we have the following auxiliary result:

Lemma 1. Let us assume that $p(x)$ tends to 0 when $x$ tends to infinity, then
(i) $\bar{t}$ and $\tilde{t}$ exist, and
(ii) Under A.1, $x_i(t)$ is increasing, $x_f(t)$ is decreasing and $x(t)$ decreasing. Moreover $\bar{t}$ and $\tilde{t}$ are unique.
Proof: To show existence, note that $p(x(0)) - c$ is positive, since domestic and foreign output are positive under free trade. But, if $t$ is large enough, the left hand side of equation (II) above becomes negative. Therefore, by continuity $\bar{t}$ must exist. Similarly, if $t$ is small enough, profit-maximization of the foreign firm implies that $x$ is infinity, so the left hand side of that equation (I) above becomes negative and $\bar{t}$ exists.

Let us now prove part (ii) of the lemma. Under our assumptions it follows that

$$\frac{dS}{dt} = p \cdot \frac{dx}{dt} + p \cdot \frac{dx_f}{dt} - x_f p' \cdot \frac{dx}{dt} - c \cdot \frac{dx_d}{dt} + t \cdot \frac{dx_f}{dt} + x_f$$

Since $\frac{dx_d}{dt} = \frac{dx}{dt} - \frac{dx_f}{dt}$, the above derivative can be rewritten as

$$\frac{dS}{dt} = \frac{dx}{dt} \cdot [p - x_f p' - c] + \frac{dx_f}{dt} \cdot [t + c - p] + x_f$$

Profit maximization of domestic firms implies $p - c + p' x_i = 0$, so that

$$\frac{dS}{dt} = (n \cdot \frac{dx_i}{dt} + \frac{dx_f}{dt}) \cdot (-p' x_f - p' x_i) + \frac{dx_f}{dt} \cdot [t + p' x_i] + x_f =$$

$$= -p'(x_f + x_i) n \frac{dx_i}{dt} + \frac{dx_f}{dt} \cdot [t - p' x_f] + x_f$$

(7)

Now, let calculate $\frac{dx}{dt}$ and $\frac{dx_f}{dt}$ from the first order conditions of profit maximization of every firm, that is, from

$$p + x_i p' - c = 0, \quad t = 1, ..., n \text{ and}$$

$$p + x_f p' - c - t = 0,$$

By taking derivative in the two above conditions, we can obtain

$$\frac{dx_f}{dt} = \frac{p' + p' p' + p'' x_i}{p'[p'(1 + 2/n) + p''(x_i + x_f/n)]}$$

(8)

$$\frac{dx}{dt} = \frac{1}{n[p'(1 + 2/n) + p''(x_i + x_f/n)]},$$

(9)
Now, we can calculate
\[
\frac{d x_i}{d t} = \frac{d x}{d t} - \frac{d x_f}{d t} = - \frac{p' + p''x_i}{p'[p'(1 + 2/n) + p''(x_i + x_f/n)]}, \quad (10)
\]

But, according to our assumptions, (8) (9) are negative and (10) is positive, which completes the proof. Moreover, since \(x_i(t)\) is strictly increasing and \(x_f(t)\) is strictly decreasing, \(t\) and \(\bar{t}\) are unique.

The previous lemma allows us to prove our next result:

**Proposition 7.** Under A.1, constant returns to scale and identical firms (including the foreign one), then:
(i) \(\frac{dS}{dt}(t) > 0\) and (ii) \(\frac{dS}{dt}(\bar{t}) < 0\)

**Proof:** Part (i): Since \(x_f = 0\) at \(t = \bar{t}\), we have, from equation (7) above

\[
\frac{dS}{dt}(t) = -p'x_i n \frac{dx_i}{dt} + \frac{dx_f}{dt} t
\]

Moreover, since \(c_f = c\), the first order conditions of profit-maximization implies that in this case, \(p + p'x_f - t - c = p - t - c = p + p'x_i - c = 0\). Thus we obtain

\[
\frac{dS}{dt}(t) = (p - c)n \frac{dx_i}{dt} + (p - c) \frac{dx_f}{dt} = (p - c) \frac{dx}{dt}
\]

which is negative, since, our assumptions implies that \(\frac{dx}{dt}\) is negative.

Part (ii): Evaluating \(\frac{dS}{dt}\) at \(t = \bar{t}\), which implies \(x_i = 0\), we obtain from (7) the following expression,

\[
\frac{dS}{dt}(\bar{t}) = -p'x_f n \frac{dx_i}{dt} + \frac{dx_f}{dt} \left[t - p'x_f + x_f\right]
\]
but, if \( t = \bar{t} \), then first order condition for a Cournot equilibrium implies that \( p = c \), and \( p + p'x_f - c - t = 0 \), so that \( p'x_f = t \). Therefore, we can rewrite our previous derivative as

\[
\frac{dS}{dt}(t) = - p'x_f n \frac{dx_i}{dt} + x_f
\]

which is positive, since our assumptions implies that \( \frac{dx_i}{dt} \) is positive.

Our previous result implies that, under our assumptions, a prohibitive tariff (that is, a tariff which implies autarky) is never optimal (part (ii)). Also, part (i) implies that it is not optimal to set a tariff yielding a very small (but positive) level of domestic production. In some sense, this result is equivalent to the one obtained in Proposition 1 for the case of quotas. It is interesting to remark that, contrary to what happened there, our previous results implies that, under some conditions, the optimal tariff is interior.

Let \( t^* \) be the optimal tariff, that is, the tariff which maximizes the domestic welfare with the restriction that outputs will be determined as a Cournot equilibrium for a given tariff.

**Proposition 8.** Under constant returns to scale, if firms are identical (including the foreign one), the following properties hold:

(a) If \( n = 0 \), then

(a.1) \( t^* \) exists.

(a.2) Under A.1, \( t^* \) is positive. Moreover if \( \beta'(x) \leq 0 \) \( S \) is strictly quasi-concave on \( t \) and thus \( t^* \) is unique.

(a.3) If \( p' + p''x > 0 \) in the optimum, then \( t^* \) is negative.\(^{(3)}\)

(b) If \( n \geq 1 \), then

(b.1) Under A.1 \( t^* \) exists and it is not prohibitive, i.e., \( t^* < \bar{t} \).

(b.2) Under A.1, \( t^* \) is positive. Moreover, if \( \beta'(x) \geq 0 \) and \( p'' \geq 0 \) then \( t^* \) is unique.

(b.3) If \( p' + p''x > 0 \) in the optimum, then \( t^* < 0 \).

\(^{(3)}\) Moreover, it can be shown that if \( p(x) \) is isoelastic, then it satisfies this condition and the optimal tariff is unique.
Proof:

Part (a.1): Since \( n = 0 \), the domestic welfare is given by

\[
S = u(x) - px + tx
\]

Therefore,

\[
\frac{dS}{dt} = \frac{dx}{dt} (t - p'x) + x
\]

From the first order condition of profit-maximization, it follows that

\[
\frac{dx}{dt} = \frac{1}{p'/(\beta + 2)}
\]

Which is negative, since the second order condition of profit-maximization implies that \( \beta + 2 > 0 \). Thus, \( \frac{dS}{dt} \) is positive for \( t \) small enough, since \( p'x \) is bounded. Also, when \( t \) is such that \( x \) is near enough to zero, then \( \frac{dS}{dt} \) is negative. Finally, domestic welfare is zero when \( x = 0 \). Thus, \( S \) can be redefined on a compact interval and the proof of this part is completed, since \( S \) is a continuous function of \( t \).

Part (a.2): From our previous expressions of \( \frac{dS}{dt} \) and \( \frac{dx}{dt} \), we can obtain

\[
\frac{dS}{dt} = \frac{1}{p'/(\beta + 2)} [t + xp'(\beta + 1)] \quad (11)
\]

First order condition of welfare-maximization implies \( \frac{dS}{dt} = 0 \) so that the optimal tariff must satisfy \( t^* = -xp'(\beta + 1) \), which is positive under A.1. Now, we will show that if \( \beta'(x) \leq 0 \), then \( F(t) = -[t + xp'(\beta + 1)] \) is strictly decreasing which implies that \( S(t) \) is strictly quasi-concave, since \( F(t) \) has the same sign as \( \frac{dS}{dt} \) and both equal to zero just once. Thus, we compute

\[
\frac{dF}{dt} = -[1 + xp' \frac{d\beta}{dt} + (\beta + 2)(p' + xp'') \frac{dx}{dt}]
\]

But, according to our assumptions, \( \frac{d\beta}{dt} = \beta' \frac{dx}{dt} \geq 0 \), since \( \frac{dx}{dt} < 0 \). Thus, by A.1 the above derivative must be negative. Therefore, \( S(t) \) is strictly quasi-concave and the optimal tariff is unique.
Part (a.3): It can be shown, easily, that inequality $p' + p''x > 0$ is equivalent to $\beta + 1 < 0$. Thus $t^* = -xp'(\beta + 1)$ is negative under this condition.

Part (b.1): For $t$ small enough, no domestic firm is active, since $t$ is finite. Thus, according to the proof of part (a.1), $S(t)$ must be strictly increasing for $t$ small enough. On the other hand, part (iii) of Proposition 8 implies that there exists some $t < t$ yielding higher welfare than any $t \geq t$. Therefore, the continuity of $S(t)$ ensures the result.

Part (b.2): It is easy to show that, under our assumptions, $t$ must be negative. But, from part (a.1), $S(t)$ must be increasing for $t < t$. Thus, $t^*$ must imply positive domestic production. Therefore, expression (7) is the relevant derivative of $S(t)$ in order to obtain $t^*$. By substituting (8) and (10) into (7) we obtain

$$
\frac{dS}{dt} = p'(x_f + x_i) \frac{p' + p''x_i}{p'A} + \frac{p' + p'/n + p''x_i}{p'A} [t - p'x_f] + x_f
$$

$$
= \frac{1}{p'A} \left[ p'(x_f + x_i)(p' + p''x_i) + (p' + p'/n + p''x_i)t + p'x_f(p'/n + p'x_f/n) \right]
$$

(12)

where $A \equiv p'(1 + 2/n) + p''(x_i + x_f/n)$. (Note that under A.1, $A$ is negative).

The first order condition of welfare-maximization implies that the above derivative must be equal to zero, which under assumptions yield a positive $t^*$.

Now, it remains to show that if $\beta'(x) \geq 0$, then $t^*$ is unique. We will show that $S(t)$ is single picked, which ensures that $t^*$ is unique. Note that sign $\left( \frac{dS(t)}{dt} \right)$ is the same as the sign of $H(t)$ where this is defined as

$$
H(t) = p'(x_f + x_i)(p' + p''x_i) + (p' + p'/n + p''x_i)t + p'x_f(p'/n + p'x_f/n)
$$

But, from the first order condition of a Cournot equilibrium, it follows that $t = p'(x_f - x_i)$. Thus, we obtain

$$
H(t) = p'x_f(2p' + 2p''x_i + 2p'/n + p''x_f/n) - (p')^2x_i/n
$$

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By using the definitions of $\beta(x)$ and $q = x^d/x$, we can rewrite the above as

$$H(t) = (p')^2 \{ [2 + 2\beta q/n + 2/n + \beta(1-q)/n]x_f - x_i/n \}$$

$$= (p')^2 \{ [2 + 2/n + \beta(1+q)/n]x_f - x_i/n \}$$

According to Proposition 7, $x_f(t)$ is decreasing, $x_i(t)$ is increasing and $x(t)$ is decreasing. Also, the second order conditions of a Cournot equilibrium implies that $[2 + 2/n + \beta(1+q)/n] > 0$. Thus, a sufficient condition to ensure that $H(t)$ is strictly decreasing is that $(1 + q)\frac{d\beta}{dt} + \beta\frac{dq}{dt} < 0$, which is ensured by our assumptions, since $\frac{dq}{dt}$ is positive and $\frac{d\beta}{dt} = \beta'(x) \frac{dx}{dt} \leq 0$. Therefore, $H(t)$ is single-peaked, as well as $S(t)$, so that $t^*$ is unique.

**Part (b.3):** As in part (b.2), the optimal tariff is obtained from equalizing expression (12) to zero, which, under our assumptions, implies a negative $t^*$ in this case $\blacksquare$

Part (a) of Proposition 8 has been previously analyzed by Brander & Spencer (1985). However they did not study the second order condition, so that they did not prove neither existence nor uniqueness. Part (b) is, at the best of our knowledge, new. Notice that conditions under which $t^*$ is positive (or negative) do not depend on the existence of domestic competitors.

The next Theorem summarizes our findings in the case of tariffs.

**Theorem 3.** If all domestic firms are identical, all firms (including the foreign) have constant returns to scale, then:

a) Under A.1, an optimal tariff exists and it is interior, i.e. $\bar{t} > t^* > \underline{t}$.

b) Under A.1, $t^* > 0$, and if $p' + p'' x_i > 0$, then $t^* < 0$. 

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4. THE CASE OF POSITIVE FIXED COSTS

In this Section we will assume that domestic firms have to incur in a fixed cost in order to obtain a positive output. We will see that the conclusions obtained in the previous Section change dramatically. Throughout this Section we will maintain the following assumption.

**Assumption 2.** There is a unique domestic firm with a cost function given by $c(x_i) = K + cx_i$ if $x_i > 0$ and $c(0) = 0$.

This assumption is made in order to keep the analysis tractable. Our qualitative conclusions still valid with several domestic firms and more general forms of technology. Let $x_i^*(x_f)$ be the profit maximizing output of the domestic firm. Under A.1 a) this correspondence is well defined and single valued as long as domestic profits are positive. If domestic profits are zero, there are two outputs -one positive, the other zero- which maximize profits. In that case we will assume that the domestic firm produces a positive output so the relevant point is the maximum output belonging to this correspondence (since by the maximum theorem this correspondence has a closed graph this maximum always exists). Thus $x_i^*(x_f)$ can be taken as single valued. Let $x(x_f) = x_i^*(x_f) + x_f$. Define

$$
\hat{x}_f = (x_f \mid p(x_f) x_i^*(x_f) - c(x_i^*(x_f)) = 0, x_i^*(x_f) > 0)
$$

In words, $\hat{x}_f$ is the output of the foreign firm for which the maximum profits of the domestic firms are zero, i.e. the output corresponding to limit pricing of the foreign firm. We now show the following

**Lemma 2.** $\hat{x}_f$ exists and if $x_i^*(\hat{x}_f) > 0$, it is unique.

**Proof:** If $\forall y, 0 \geq p(y)y - c(y)$, then $\hat{x}_f = 0$. If $\exists y$ such that $p(y)y - c(y) \geq 0$ the median value theorem and our boundness assumptions guarantee the existence of $\hat{x}_f$. Now let us prove uniqueness. Suppose there are two different
\( \x_f \), denoted by \( \x_f \) and \( \x_f' \) and without loss of generality let us assume that \( \x_f < \x_f' \). Then we have

\[
0 = p(x(\x_f')) x_i(\x_f) - cx_i(\x_f) \geq p(x_i(\x_f') + x_f) x_i(\x_f) - c(x_i(\x_f')) \quad \text{and} \quad p(x(\x_f')) x_i(\x_f') - cx_i(\x_f') = 0. \text{But since } p(\cdot) \text{ is decreasing}
\]

\[
p(x_i(\x_f') + x_f) x_i(\x_f') - c(x_i(\x_f')) > p(x(\x_f')) x_i(\x_f') - cx_i(\x_f') = 0
\]

which contradicts the first equality. \( \blacksquare \)

Lemma 2 simplifies the task of finding the optimal quota since it implies that domestic welfare as a function of quotas can be decomposed in two parts. If \( 0 \leq x_1 < x_f \), \( S(x_1) \) looks as in the no fixed cost case. When \( x_1 = x_f \), \( S(x_1) \) is multivalued and in this case (accordingly with what we assumed about the selection of \( x_i(x_f) \)) we will take the maximum value. Since in this case the domestic firm is obtaining zero profits this allocation implies Ramsey pricing for the domestic firm and the corresponding welfare will be denoted by \( S(R) \).

If \( x_1 < x_f \), \( S(x_1) = u(x_1) - p(x_1)x_1 \). Lastly let us introduce some notation. Let \( x_{f0} \) be the minimum output for which \( S(A) = S(x_1) \), \( x_f \neq 0 \) (\( x_{f0} \) might be \( \infty \)) and let \( x_{f1} \) be the maximum output for which \( S(R) = S(x_1) \) (\( x_{f1} \) might be \( \infty \)) (see Figures 1-2).

**Proposition 9.** If A.1 a) and A.2 hold, A.1 b) holds for positive outputs and \( \beta(x) \) is non-increasing in \( x \), the optimal quota exists and it is characterized by

(a) If \( S(A) > S(R) \) we have two possible cases

(i) \( x_f = 0 \) if \( \x_f \leq x_{f0} \)

(ii) \( x_f = \x_f \) if \( \x_f \geq x_{f0} \)

(b) If \( S(A) < S(R) \) we have four possible cases

(i) \( x_f = 0 \) if \( \x_f \leq x_{f0} \)

(ii) \( x_f = \x_f \) if \( \x_f \leq x_{f0} \)

(iii) \( x_f = x_f \) if \( \x_f \leq x_f \)

(iv) \( x_f = \x_f \) if \( \x_f \geq x_{f1} \)

**Proof:** Existence of the optimal quota follows from the closed graph of \( S(\cdot) \) and \( x_f \in [0, \x_f] \). The characterization follows from the fact that Proposition
2 implies that $S(\hat{x}_f)$ is quasi-convex on $[0, \hat{x}_f]$, that $S(\hat{x}_f)$ is strictly increasing on $(\hat{x}_f, \omega)$ (since $\frac{\partial S}{\partial x_f} = u' - p - p' x_f = - p' x_f > 0$) and a graphical argument (see Figures 1 and 2).

![Figure 1](image1)

![Figure 2](image2)

Notice that Proposition 1 b) does not hold, i.e. an increase in the quota evaluated at the point of zero profits of the domestic firm will make this firm to run out of business and will imply a decrease in welfare.

The case in which $S(A) > S(R)$ works like the non-fixed cost case: if the foreign firm has relative high marginal cost, total protection is the optimal
policy (case ai in Proposition 9) and if the foreign firm has relative low marginal cost, free trade is the optimal policy (case aii). In other words, the choices are total protection or free trade and the desirability of any of them depends monotonically on the relative efficiency of the foreign competitor (but notice that here, in contrast with the zero fixed cost case, free trade implies necessarily zero domestic production). As in the no fixed cost case the underlaying philosophy is "do not protect the inefficient".

However the case in which $S(A) < S(R)$ contains new features, namely that a positive quota less than the Cournot equilibrium output of the foreign firm might be optimal and that the desirability of free trade as the optimal policy is not directly related to the relative efficiency of the foreign firm: for high values of the marginal cost of the foreign firm total protection is the optimal policy (case bi in Proposition 9), for high-intermediate values of the marginal cost free trade is optimal (case bii), for low-intermediate values of the marginal cost it is optimal to set up a quota equal to $x_f^*$ (case biii) and for low values free trade is optimal again (case biv). In other words, if $S(A) < S(R)$, the fixed cost breaks the quasi-convexity of $S( )$ and the optimal policy is difficult to characterize. The bottom line is now "do not protect the very inefficient, protect the very efficient and be careful with the intermediate cases!

Next we show that an analog of Proposition 4 fails in the case of positive fixed costs even if $\alpha = 1$.

**Example 1.** Let us assume that Assumption 2 holds and that $\alpha = 1$. Let $u(\cdot) = ax - \frac{\chi^2}{2}$. Then we have that if the foreign firm is not active $S = (a - c)^2 / 3 - K$ and when it is active, $S = 4(a - c)^2 / 9 - 2K$. Profits are positive as long as $(a - c)^2 > 9K$. Thus when $(a - c)^2 5/72 < K < (a - c)^2 / 9$ profits are positive in duopoly but domestic welfare decreases with the entry of the foreign firm.

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(4) With respect to the question about the occurrence of case a) or b) it can be shown that if the inverse demand function is linear, $S(A) > S(R)$ iff the fixed cost is relatively high.
The intuition behind this example is that when a new firm enters into the market it produces two effects. On the one hand the competitive effect, i.e. prices are driven downwards because of increased competition and on the other hand the technological effect, i.e. existing firms contract their output and, under economies of scale, they produce less efficiently. Example 1 shows that the second effect might be larger than the first one. Again the conclusion is that under positive fixed cost things are very different to zero fixed cost since here α-tariffs are not enough to imply the optimality of free trade.

Next, we show that the analog of Proposition 6 does not hold under fixed costs.

Example 2. The basic functions are as in Example 1 but with \( n = 0 \). Thus A. 1 a) holds and A.1 b) holds for positive outputs. Suppose that the fixed cost \( K \) is such that \( \frac{(a - c)^2}{4} < K < \frac{(a - c)^2}{2} \). This implies that a positive output yields more domestic welfare than zero output, but for any output profits of the foreign firm are non positive. Thus an α-tariff can never induce the foreign firm to produce. However, if \( K \) is sufficiently close to \( (a - c)^2/4 \) a negative tariff will induce a positive output from the foreign firm and will yield more domestic welfare.

Let us close this Section by considering the case of a tariff on output. We will show that the analog to Propositions 8-9 do not hold.

Example 3. The market is identical to the one described in the Example 1 but with \( c = 0 \) and \( a = 1 \). Profits in a Cournot equilibrium relative to a tariff \( t \) can be shown equal to \( \Pi_1 = (1 + t)^2/9 - K \) and \( \Pi_f = (1 - 2t)^2/9 - K \). Let us define \( \bar{t} \) and \( \tilde{t} \) respectively as the value of \( t \) for which \( \Pi_1 \) and \( \Pi_f \) are zero. It is easily calculated that domestic welfare reads:

\[
S = \begin{cases} 
(1 + 2t - 3t^2)/8 & \text{if } t < \bar{t} \\
(2 + 2t - 3t^2)/6 - K & \text{if } \bar{t} < t < \tilde{t} \\
3/8 - K & \text{if } t > \tilde{t}
\end{cases}
\]
If \( \frac{1}{2} < \bar{t} < \frac{1}{6} \), the optimal tariff is greater or equal than \( \bar{t} \), implying that autarky is the best policy.

Proposition 9 and examples 1, 2 and 3 can be summarized in the following

**Theorem 4.** Under fixed costs and even if A1 holds for positive outputs:

a) the optimal quota can be different from autarky or free trade (Proposition 9 part b)).

b) A quota on output might yield higher welfare than any \( \alpha \)-tariff (example 1).

c) An \( \alpha \)-tariff may yield less welfare than a tariff on output (example 2)

d) The optimal tariff may be negative (see example 2).

e) The optimal tariff may imply autarky (See example 3).

Summing up: The basic insights obtained in the no fixed cost case and summarized in Theorems 1, 2 and 3 do not apply in the positive fixed cost case.

5. FINAL COMMENTS

In this paper we have shown that when designing the trade policy, fixed costs matter: Many insights on the form of the optimal policy when there are no fixed costs do not carry to the case where fixed costs are not negligible. In order to make our point we had chosen a particularly simple, static, model in which a given number of firms compete in a homogeneous market in quantities and in which the government has tariffs and quotas as the only available instruments. Thus, important topics like product differentiation, interaction of domestic and foreign markets, price-setting firms, dynamic models, free entry, and the possibility to subsidize domestic firms are not covered by our model. We hope that our paper will stimulate similar research to ours on these areas.
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