TERMS-OF-TRADE AND THE CURRENT ACCOUNT:
A TWO-COUNTRY/TWO-SECTOR GROWTH MODEL*

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ABSTRACT

This paper examines the equilibrium relation, within the non-specialized area, between the current account and the terms-of-trade in a two-country/two-sector growth model. Along a convergent equilibrium path, this relation can have any sign depending on the trading sectors' relative factor intensities and on the steady state relative price slopes of demand for and supply of investment goods. It is also shown that a negative association between capital accumulation and the terms-of-trade for the exporter of capital intensive goods does not require an import sector growing faster than the export sector.

Keywords: Terms-of-trade; Overlapping generations; Two-sector growth.
1. INTRODUCTION

The aim of this paper is to analyze in a general equilibrium context the evolution of a country's terms-of-trade and the capital stock along convergent equilibrium paths emphasizing the role of production technology restrictions and their implications for the external adjustment of an economy. The effects of terms-of-trade changes on the current account have been extensively studied in the literature for the small economy case although only a few works have developed a two-country approach to the problem. A feature of all these models is that they do not focus on the role played by production technology restrictions in determining the equilibrium comovements between capital accumulation and the evolution of the terms-of-trade, despite their natural implications for the current account. In contrast, the relationship between economic growth and terms-of-trade is an old issue that has received considerable attention in international trade theory but in a context of partial equilibrium with balanced trade. Our work integrates these two bodies of the theory contributing to the two-country literature in two directions. One complements the current account models considering different aspects of production, and the other extends the growth models considering the relation between terms-of-trade and capital in a context of optimizing consumers who can lend and borrow in the international financial market.

The concern about the effects of terms-of-trade changes on the current account dates back to Harberger (1950) and Laursen and Metzler (1950), who predicted that, given investment and real income measured in units of exportables, a deterioration of the terms-of-trade will lead to a deterioration in the current account balance due to a decrease in savings. Obstfeld (1982) questions the validity of this proposition when the saving behavior is derived from an intertemporal optimizing framework, giving rise to an extensive literature. This literature includes Dornbusch (1983), Svensson and Razin (1983), Persson and Svensson (1985), Bean (1986), Matsuyama (1987, 1988), Sen and Turnovsky (1989) and Mansoorian (1993) among others. These authors develop different intertemporal frameworks to assess the effect of exogenous terms-of-trade deteriorations on the spending behavior of optimizing consumers and the current account of a small open economy. An important feature of these models is that the results are very sensitive to the persistence of the shock and the assumptions about preferences. As pointed out by Backus (1993, p.381), 'Transitory shocks typically lead the terms-of-trade and trade balance to move in opposite directions, but the effect of permanent shocks depends on the behavior of the discount factor'. Different aspects of the production side are stressed by Dornbusch (1983), who includes a non-traded goods sector, and Matsuyama (1988), who focuses on the
role of relative factor intensities. A different perspective is undertaken by Stockman and Svensson (1987) and Backus (1993) who develop two-country dynamic stochastic models with homothetic preferences and complete markets. They show that the sign of equilibrium comovements between the terms-of-trade and the trade balance depends on the degree of intertemporal substitution in consumption. Of these two works, only Stockman and Svensson (1987) consider a production economy but where only the home country can produce using a very simple technology.

The links between terms-of-trade and capital accumulation in two-sector growth models is a question that has received no attention in the above literature but that was extensively studied in the sixties (see Findlay (1973) for a survey); a common feature of these models is that capital accumulation for a country that diversifies production and exports the capital intensive good will result in a deterioration of its terms-of-trade unless the import sector expands fast enough relative to the export sector.

It seems natural then to connect these two bodies of the theory and to analyze the implications of the trading sectors' relative factor intensities for the evolution of the terms-of-trade and the current account. With this aim, we propose a two-period overlapping generations version of Oniki and Uzawa (1965)'s two-country/two-sector growth model, where capital has a positive country-specific external effect on each sector's scale of production. The analysis demonstrates that if the economy evolves within the non-specialized region, an equilibrium trajectory can exhibit periods of positive capital accumulation and terms-of-trade improvements (deteriorations) when the export sector is more capital (labor) intensive. Furthermore, it also shows that, along a convergent equilibrium path, terms-of-trade improvements (deteriorations) can come along with current account deficits (surpluses). These two outcomes arise when the investment good's sector is more labor intensive and (i) Rybczynski's theorem does not hold and the external effect dominates the price effect, or (ii) the steady state ratio between the price slope of demand and the price slope of supply in the investment goods market is not too small. Otherwise, (1) the terms-of-trade of a growing country exporting the capital intensive good will deteriorate and (2) current account surpluses (deficits) accompany terms-of-trade improvements (deteriorations).

The paper is organized as follows: section II describes the structure of the model and characterizes the dynamic competitive equilibrium over the world diversified production area. Section III analyzes the relationship between terms-of-trade and capital accumulation along a convergent equilibrium trajectory, and section IV studies the
implications for the current account dynamics of this equilibrium relation. Section V summarizes the results and presents some of the potential extensions of the model. The mathematical proofs as well as some technical results are relegated to the appendix.

2. THE MODEL

Consider a world economy consisting of two countries, country H and country F, where economic activity takes place over an infinite discrete time horizon, \( t = 1, 2, ..., \infty \). International trade involves two kinds of commodities, consumption goods and investment goods, and claims to real capital. The consumption good \( c \) is perishable but the investment good \( l \) can be accumulated as capital. Production of any good requires the use of two factors, capital \( (K) \) and labor \( (L) \). Capital and goods can freely move between countries but labor is not internationally mobile. At every period, each country is populated by two overlapping generations in the line of Diamond (1965), the size of each generation is constant and normalized to one. Countries differ in the production technology through an scale or efficiency factor, and they also may have different tastes over intertemporal consumption plans. All markets are perfectly competitive and agents have perfect foresight about the future.

Notation

- \( K^j_t \) = capital stock in country \( j \) at period \( t \)
- \( K^j_{it} \) = capital stock employed in sector \( i \) in country \( j \) at period \( t \)
- \( L^j_{it} \) = labor employed in sector \( i \) in country \( j \) at period \( t \)
- \( k^j_i = K^j_i / L^j_i \), \( k^j_t = K^j_t / L^j_t = K^j \)
- \( Y^j_{it} \) = output of good \( i \) in country \( j \) at period \( t \)
- \( A^j_t \) = scale factor in country \( j \) at period \( t \), it affects equally both sectors
- \( f^i_i \) = production function per unit of labor and efficiency in sector \( i \), i.e.: \( Y^j_{it} = A^j_t L^j_{it} f^i_i (k^j_i) \)
- \( w^j_t \) = wage in consumption units in country \( j \) at period \( t \)
- \( q_t^j \) = rental price of capital in consumption units in country \( j \) at period \( t \)
- \( P^j_t \) = price of investment goods in consumption units in country \( j \) at period \( t \)
- \( R^j_t \) = gross rate of return in consumption units on claims to real capital in country \( j \) at period \( t \)
- \( C^j_{ht} \) = consumption in country \( j \) by generation \( t \) in its h-period of life
- \( \bar{s}_t^j \) = savings in units of investment goods by generation \( t \) in country \( j \)
- \( U^j \) = intertemporal utility function of generation \( t \) in country \( j \)
\( b_t^j \) = foreign claims to real capital owned by generation \( t \) in country \( j \) at the beginning of period \( t \), measured in units of investment goods.

**Momentary Equilibrium**

The single-period or momentary equilibrium of the world economy is summarized in equations (1)-(11):

\[
\begin{align*}
\max_{c_{1t}^j, c_{2t}^j} & \quad u^j(c_{1t}^j, c_{2t}^j) \quad \text{s.t.} \quad \text{(P.1)} \\
& \quad c_{1t}^j + p_t^j \pi_t^j \leq w_t^j \quad \text{(1)} \\
& \quad c_{2t}^j \leq R_{t+1}^j p_t^j (\overline{s}_t^j - b_{t+1}^j) + R_{t+1}^j p_t^j b_{t+1}^j \quad \text{(2)} \\
& \quad c_{1t}^j, c_{2t}^j \geq 0; \quad j, j' = H, F; \quad j \neq j' \quad \text{(3)} \\
& \quad w_t^j = A_t^j [f_i^j(k_{ct}^j) - k_{ct}^j f_i'(k_{ct}^j)], = p_t^j A_t^j [f_i^j(k_{ct}^j) - k_{ct}^j f_i'(k_{ct}^j)] \quad \text{for } j = H, F \quad \text{(4)} \\
& \quad q_t^j = A_t^j L_t^j f_i^j(k_{ct}^j) = p_t^j A_t^j f_i'(k_{ct}^j) \quad \text{for } j = H, F \quad \text{(5)} \\
& \quad L_{ct}^j + K_{ct}^j = K_t^j \quad \text{for } j = H, F \quad \text{(6)} \\
& \quad L_{ct}^j + K_{ct}^j = 1 \quad \text{for } j = H, F \quad \text{(7)} \\
& \quad K_{t+1}^H + K_{t+1}^F = Y_{It}^H + Y_{It}^F \quad \text{(8)} \\
& \quad \overline{s}_t^H - K_{t+1}^H = -[\overline{s}_t^F - K_{t+1}^F] \quad \text{(9)} \\
& \quad p_t^H = p_t^F \quad \text{(10)} \\
& \quad R_{t+1}^H = R_{t+1}^F \quad \text{(11)} 
\end{align*}
\]

(P.1) represents the optimization program of a generation-\( t \) consumer in country \( j \). Generations within a given country are identical and consist of identical individuals who live for two periods; in the first period they supply inelastically their endowment of labor and in the second, they retire. Equations (1) and (2) are, respectively, the budget constraints in periods \( t \) and \( t+1 \) of a generation-\( t \) consumer, and expression (3) represents the corresponding non-negative restrictions for consumption. The utility function, \( u^j \), is twice differentiable, strictly quasi-concave, increasing in \( c_{1t}^j \) and \( c_{2t}^j \), and satisfies the Inada conditions. The assumptions on the utility function imply that the budget constraints will hold with equality and that a unique interior solution to (P.1) will exist.

Equations (4) and (5) come from the firms' first order conditions at an interior solution and state that factor markets are competitive and that capital and labor can freely move between sectors within a given country. Equations (6) and (7) represent, respectively, the market clearing conditions for those factors in each country. The
function $f_j$ is twice continuously differentiable, strictly increasing, strictly concave and satisfies $f_j(0) = 0$ plus the Inada conditions. The scale factors are different across countries but common to both sectors; they are exogenous to the firm and given by the function $A_j^j = A^j\left( k_t^j \right)$, where $A^j$ is strictly increasing and such that $A_j^j(k) f_j(k)$ is a strictly concave function satisfying $A_j^j(0) f_j(0) = 0$ and the Inada conditions. The production technology is also characterized by the absence of factor intensity reversals. Combining equations (4), (5), (6) and (7) we can derive the region of prices $p_j^j$ at which country $j$ will diversify production for given $k_t^j > 0$ (see Figure 1).

(a) \hspace{1cm} (b)

**Figure 1:** Capital/Labour Ratios and Relative Prices

(a) $k_c > k_I$, (b) $k_c < k_I$

Equation (8) states that the world supply of investment goods at period $t$ must equal the world demand for capital at period $t+1$, where full depreciation is assumed. That is, investment goods can be produced locally or can be imported but the world net imports must be zero. Equation (9) states that the world market for claims to real capital must
clear. In other words, the value of worldwide savings at period \( t \) in units of investment goods must equal the worldwide capital stock at the beginning of period \( t+1 \).

Finally expressions (10) and (11) follow respectively from the free trade and international capital mobility assumptions. Taking into account that, in equilibrium, the rates of return on investment goods and claims to capital must be the same, \( R_{t+1} = q_{t+1}^j/p_c \), it is clear that the rental price of capital will be the same in both countries.

The analysis will focus on solutions to (1)-(11) where both countries diversify production. In that case, and under the assumptions about the technology, the capital-labor ratio in sector \( i \) at period \( t \) will be a strictly increasing function of the relative factor price \( \omega_{ct}^j = w_c^j/q_c^j \). Then, it is easy to show that sector \( i \)'s capital/labor ratio will be uniquely determined by the level of prices \( p_c \) regardless of its specific country location:

\[
k_{ct}^j = k_i(p_c), \quad k_{ct}^j(p_c) > 0 \Leftrightarrow k_{ct} < k_{tc}, \quad j = H, F, \quad i = c, I
\]  

(12)

Thus, a sector's capital-labor ratio will be a strictly increasing (decreasing) function of the price level if the production of consumption (investment) goods is the most capital intensive activity. In this case, the wage rate will also be the same across countries and therefore, if there is no specialization, factor price equalization will hold. It follows that the countries' capital/labor ratios are related to each other through a monotone relationship given by:

\[
k_c^F = \left( A^F \right)^{-1} A^H \left( k_c^H \right)
\]  

(13)

Expression (A.1) below is both a simplifying assumption and a sufficient condition for a non-empty diversified production region. It states that technologies are 'similar' for a sufficiently small \( \varepsilon \), and that the scale factor is a constant elasticity function of the capital stock:

\[
(A.1)
\]

(i) \( A^j(k_c^j) = \gamma^j A(k_c^j), \quad \gamma^j > 0, \quad j = H, F, \quad \gamma^H = \gamma^F - \varepsilon, \quad \varepsilon > 0 \)

(ii) \( \frac{A'(k_c^j)}{A(k_c^j)}k_c^j = \eta \quad \forall k_c^j > 0, \quad \eta \in (0, 1) \)
Under assumption (A.1) expression (13) becomes

$$k_t^F = \hat{\theta}k_t^H, \quad \hat{\theta} = A^{-1}\left(\frac{\gamma^H}{\gamma^F}\right)$$ (13)'

That is, the countries' capital/labour ratios are proportional to each other and country \( F \) will be relatively richer in capital at an interior solution if and only if the parameter \( \hat{\theta} \) is greater than one.

The interior solution to the consumer's program can be represented by the savings function \( \bar{s}_t^j = \bar{s}_t^j(R_{t+1}, w_t, p_t) \). Assuming that \( c_{1t}^j \) and \( c_{2t}^j, \) are normal goods and gross substitutes, and that the utility function is homothetic, it is well known that the properties of the savings function are summarized by

$$\bar{s}_t^j(R_{t+1}, w_t, p_t) = s_t^j(R_{t+1}) \frac{w_t}{p_t}, \quad s_t^j(R_{t+1}) \in (0, 1), \quad s_t'(R_{t+1}) > 0 \quad (14)$$

The proportionality of savings with respect to labor income, measured in units of investment goods, follows from the homotheticity of the utility function; the second and third expressions derive, respectively, from the normality and gross substitutability of current and future consumption.

Combining equations (8) and (9), and taking into account (4), (5), (6), (7), (10), (11), (12), (13)' and (14), the set of equilibrium conditions can be reduced to the following pair of difference equations in the level of prices \( p \) and the capital stock of a given country, say country \( H, k^H \). Henceforth \( k^H = k \).

$$k_{t+1} = \gamma^H A(k_t) \psi(k_t, p_t) = \Psi(k_t, p_t)$$ (15)

$$s(k_{t+1}, p_{t+1}, p_t) = \theta \psi(k_t, p_t)$$ (16)

where:

$$\theta = 1 + \hat{\theta}$$ (17)

$$\psi(k_t, p_t) = \left[ k_t - 2\theta^{-1}k_t(p_t) \right] f_I(k_t(p_t))$$ (18)

$$s(k_{t+1}, p_{t+1}, p_t) = \left[ s^H(R_{t+1}) + s^F(R_{t+1}) \right] w_I(p_t)$$ (19)

$$R_{t+1} = \gamma^H A(k_{t+1}) \frac{f'_\sigma(k_{t+1}(p_{t+1}))}{p_{t+1}}$$ (20)

$$w_I(p_t) = f_I(k_t(p_t)) - k_I(p_t) f'_I(k_t(p_t))$$ (21)
Thus, the evolution of the capital stock is governed by equation (15) which follows directly from the clearance of the investment goods in the world market, equation (9). On the other hand, if technologies are similar, there always will be a solution to equation (16) (see appendix) and hence, the resulting expression of substituting (15) into (16) will define the law of motion of the price level provided that savings depend on the interest rate:

$$\rho_{t+1} = \Phi(k_t, p_t)$$  \hspace{1cm} (22)

The characteristics of both the savings and production functions imply that $\Phi$ will be a single valued continuously differentiable function.

Lemma 1 (Stolper-Samuelson): Suppose both goods are produced, then:

(i) if $k_{xt} > k_{ct}$ \forall t, $\frac{\partial w_t}{\partial p_t} < 0$ and $\frac{\partial q_t}{\partial p_t} > 1$;

(ii) if $k_{xt} < k_{ct}$ \forall t, $\frac{\partial q_t}{\partial p_t} < 0$ and $\frac{\partial w_t}{\partial p_t} > 1$.

Lemma 2: Suppose both goods are produced, then:

(i) if $k_{xt} > k_{ct}$ \forall t, $\frac{\partial Y_{ct}}{\partial k_t} << 0$ and $\frac{\partial Y_{xt}}{\partial k_t} \frac{k_t}{Y_{xt}} > 1$;

(ii) if $k_{xt} < k_{ct}$ \forall t, $\frac{\partial Y_{ct}}{\partial k_t} << 0$ and $\frac{\partial Y_{xt}}{\partial k_t} \frac{k_t}{Y_{ct}} > 1$.

The content of Lemma 1 is well known: an increase in the price of a good will imply an increase (decrease) in the price of the factor used more (less) intensively in its production, the proportion of the increase in the intensive factor price being more than one. On the other side, the effect of increases in the capital stock in neoclassical two-sector models is that the production of the capital intensive good will increase in a proportion greater than one, and the production of the labor intensive good will fall (Rybczynski's theorem). Due to the presence of a positive external effect of capital on the scale of production, the second part of Rybczynski's theorem will not necessarily hold (Lemma 2); it will if externalities are very weak (i.e.: $\eta$ close to zero).

**Intertemporal Equilibrium**

An intertemporal equilibrium where both countries diversify production will be a sequence of solutions to equations (1)-(11) at every period $t$. Formally:

12
Definition 1: A diversified production equilibrium is a sequence \( \{ k_t, p_t \}_{t=0}^{\infty} \), given \( k_0 > 0 \), such that:
\[
\begin{align*}
k_{t+1} &= \Psi(k_t, p_t) \\
p_{t+1} &= \Phi(k_t, p_t)
\end{align*}
\]

(i) Steady States

The steady state equilibria in the non-specialized region will be determined by the stationary solutions to equations (15) and (22) or, equivalently, as stated in Definition 2, by the fixed points of the dynamic system.

Definition 2: A steady state diversified production equilibrium is a pair \((\bar{k}, \bar{p})\) such that:
\[
\begin{align*}
\bar{k} &= \Psi(\bar{k}, \bar{p}) \\
\bar{p} &= \Phi(\bar{k}, \bar{p})
\end{align*}
\]

Let \((p_{\min}(k_t), p_{\max}(k_t))\) be the interval of prices at which country \( H \) diversifies production at period \( t \), given its stock of capital \( k_t \). It follows from Figure 1 that \( p_{\min} \) and \( p_{\max} \) are single-valued functions that are increasing (decreasing) in the capital stock if the consumption (investment) sector is more capital intensive. A proof of this statement can be found in Galor (1992). In our model, \( p_{\min} \) and \( p_{\max} \) do not depend on the efficiency parameters, \( \gamma^H \) and \( \gamma^F \), and therefore they are common to both countries.

Let
\[
\begin{align*}
\hat{p}_{\min}(k_t) &= \max \left\{ p_{\min}(k_t), p_{\min}(\hat{\theta} k_t) \right\} \\
\hat{p}_{\max}(k_t) &= \min \left\{ p_{\max}(k_t), p_{\max}(\hat{\theta} k_t) \right\}
\end{align*}
\]

It follows from assumption (A.1) that if technologies are similar, for any given \( k_t > 0 \), the region of prices at which both countries diversify production at period \( t \), \((\hat{p}_{\min}(k_t), \hat{p}_{\max}(k_t))\) is non-empty (see Figure 1).

It is convenient to define the worldwide excess demand for capital in the non-specialized region. In equilibrium, this excess demand, \( E(p_{t+1}, k_t, p_t) \), can be written as the excess of world savings over the world production of investment goods. That is:
\[
E(p_{t+1}, k_t, p_t) = S(\Psi(k_t, p_t), p_{t+1}, p_t) - \theta \psi(k_t, p_t)
\]

(23)
It is easy to check that, for any given $k_t > 0$, worldwide savings are greater (smaller) than worldwide production of investment goods for prices close to the infimum (supremum) of the world diversified production region and hence, by the continuity of $E$, there could be more than one stationary price clearing the market. To avoid this problem, let us assume that this excess demand is decreasing in the relative price of investment goods when evaluated at a stationary price solution:

$$\frac{\partial E}{\partial p_t} = \left[ \frac{\partial E(p_t, k_t, p_t)}{\partial p_{t+1}} + \frac{\partial E(p_t, k_t, p_t)}{\partial p_t} \right] < 0$$

The loci of stationary solutions for the capital stock and for the relative investment good price within the diversified production region can be represented, respectively, by monotone single valued functions provided there are no factor intensity reversals. Specifically, it can be shown that $p_{t+1} = p_t$ if and only if $p_t = \phi(k_t)$, and $k_{t+1} = k_t$ if and only if $p_t = \rho(k_t)$, where $\phi$ and $\rho$ are strictly increasing (decreasing) functions of $k_t$ if consumption (investment) goods are more capital intensive. Thus, the set of steady state diversified production equilibria will be determined by the non-trivial intersection points, if any, of $\rho(k_t)$ and $\phi(k_t)$.

Furthermore, it is straightforward to see from Figures 2 and 3 that if the slope of $\phi$ is greater in absolute value than the slope of $\rho$ as we approximate to the origin, there will be at least one non-trivial steady state, and that if $\phi$ is also a strictly concave function, then this steady state will be unique.

\textit{(ii) Dynamics}

Our main concern in this section is to analyze the relationship between the relative prices and the capital stock along convergent equilibrium paths and, in particular, to find out whether these two variables are inversely related or not. The economic implications of this study will be analyzed in more detail in sections III and IV.

Let $(\bar{k}, \bar{p})$ be a steady state diversified production equilibrium. It is easy to show that, given the initial capital stock, the evolution of the world economy over the non-specialized region is always non-cyclical. In addition, if the investment sector is capital intensive, the economic system is also non-oscillatory but a convergent equilibrium path can display oscillatory behavior if this sector is labor intensive. The stability properties of a steady state are discussed in lemmas (3), (4) and (5) below. In the case of investment
goods being more capital intensive, a convergent path is always characterized by a saddle path, and hence there is no indeterminacy of equilibrium (lemma 3). However, if the investment sector is labor intensive, a convergent path can go towards a (locally) stable or saddle steady state and so, in this case, we may have indeterminacy. Furthermore, a convergent equilibrium path that starts above (or below) \( \phi(k_0) \) and \( \rho(k_0) \) may not stay more than two consecutive periods on the same side of the steady state because of the inverse relationship between the evolution of relative prices and the capital stock that exists over these areas. The intuition is clear when externalities are weak (i.e.: \( \partial \Psi / \partial k < 0 \), Rybczynski’s theorem holds) since the evolution of prices enhances the effect of changes in the capital stock on the labor intensive sector: an increase (decrease) in the capital stock in period \( t \) will imply a decrease (increase) in the production of investment goods and hence a decrease (increase) in the capital stock available in period \( t+1 \). The oscillatory behavior can also arise when externalities are stronger (i.e.: \( \partial \Psi / \partial k > 0 \), Rybczynski's theorem does not hold) if this positive external effect on the scale of production is dominated by the price effect; either of these two cases can occur under assumptions in lemma 4. However, if the external effect is strong enough relative to the price effect (lemma 5), a convergent path starting above (or below) \( \phi(k_0) \) and \( \rho(k_0) \) is non-oscillatory, and so it can be characterized by a sequence of decreasing prices and increasing capital stocks. These results are summarized in Corollary 1. They relay on the fact that empirically the elasticity of savings with respect to the interest rate is very close to zero; this feature implies that the total income effect of a rise in the current relative price of investment goods dominates the substitution effect (assumption (A.3)).

\[
(A.3) \quad \text{sign} \left[ \frac{\partial S_t}{\partial P_t} \right] = \text{sign}[w'(p_t)]
\]

**Lemma 3:** Let \( \overline{(k, \bar{p})} \) be a steady state diversified production equilibrium. If \( k_{ct} < k_{It} \quad \forall t \), the steady state is (locally) either a saddle or it is unstable. And if it is unique, it is a saddle.

Before stating lemma 4, let us introduce a few definitions. Let \( E_{S,p} \) and \( E_{\Psi,p} \) be, respectively, the price elasticity of \( S \) and \( \Psi \), and \( \mu \) be the ratio \( (1 - \eta)/(1 - \partial \Psi / \partial k) \), all of them evaluated at the steady state equilibrium. It is easy to check that the last expression is always positive and less than one even for an arbitrarily small value of \( \eta \).
Lemma 4: Let \( (\bar{k}, \bar{p}) \) be a steady state diversified production equilibrium. If \( k_{ct} > k_{It} \quad \forall t \) and \( \text{Det}\left[ J(\bar{k}, \bar{p}) \right] < 0 \), then:

(i) If \( E_{s, p_t} > \mu E_{\psi, p_t}, (\bar{k}, \bar{p}) \) is (locally) either stable or a saddle. And if it is unique, it is stable.

(ii) If \( E_{s, p_t} < \mu E_{\psi, p_t}, (\bar{k}, \bar{p}) \) is (locally) a saddle or it is unstable. And if it is unique, it is a saddle.

Lemma 5: Let \( (\bar{k}, \bar{p}) \) be a steady state diversified production equilibrium. If \( k_{ct} > k_{It} \quad \forall t \) and \( \text{Det}\left[ J(\bar{k}, \bar{p}) \right] > 0 \), then \( (\bar{k}, \bar{p}) \) is (locally) either stable or a saddle. And if it is unique, it is stable.

Corollary 1: Let \( (\bar{k}, \bar{p}) \) be a steady state diversified production equilibrium. Then, for some \( \delta > 0 \) and any initial condition \( k_0 \in (\bar{k} - \delta, \bar{k} + \delta) \):

(i) There is at least one convergent path that displays oscillatory behavior if assumptions in Lemma 4(i) are satisfied. And, if \( (\bar{k}, \bar{p}) \) is a saddle, the saddle path has negative slope.

(ii) Under assumptions in Lemma 4(ii), if there is a convergent equilibrium path, it is unique and non-oscillatory. Moreover, the slope of the path is positive except for the case \( \partial \Psi / \partial k > 0 \) where it can be negative.

(iii) There is at least one convergent equilibrium path and all paths are non-oscillatory if assumptions in Lemma 5 are satisfied. And if \( (\bar{k}, \bar{p}) \) is a saddle, the saddle path has negative slope.

3. TERMS-OF-TRADE AND CAPITAL ACCUMULATION

In this section we analyze the relationship between the capital stock and the terms-of-trade (relative price of importables to exportables) for the exporter of capital intensive goods along convergent diversified production equilibrium paths. In particular, we explore the possibility of having capital accumulation and improvement in the terms-of-trade for that country despite a multiplicative efficiency factor that affects equally both sectors and a stationary labor supply. Our model is essentially an overlapping generations version of Oniki and Uzawa (1965) where, in contrast to our outcome, if a country diversifies production and exports the capital intensive good, capital accumulation will lead the country to a worsening in its terms-of-trade. The common result in this area is that any increase in the capital stock of a country with these characteristics will tend to
turn the terms-of-trade in its favor only if the supply of labor grows fast enough relative to the supply of capital, or only if technological progress occurs at a higher rate in the labor intensive sector; see, for example, Bhagwati (1958), Johnson (1958), Takayama (1963) and Findlay (1973). All these models are based on the assumption of balanced trade and, except for Oniki and Uzawa (1965), on comparative statics results; an explicit expression of the relationship between growth rates of relative prices and each sector's rate of technological change along a balanced growth path is provided in Ando (1964) for the closed economy case, where prices move in the direction predicted by Takayama. We will show that this common outcome is not necessarily true when the investment sector is labor intensive and the economy is off the steady state.

Proposition 1: Higher steady state levels of capital come along with higher steady state levels of terms-of-trade for the exporter of capital intensive goods.

Proposition 2: Let \( \overline{K}, \overline{P} \) be a steady state diversified production equilibrium. Then:
(i) If \( k_{It} > k_{ct} \) \( \forall t \), any convergent path which is characterized by capital accumulation is also characterized by terms-of-trade deteriorations for the exporter of investment goods.
(ii) If \( k_{ct} > k_{It} \) \( \forall t \), there is at least one convergent path characterized by periods of capital accumulation and terms-of-trade improvements (deteriorations) for the exporter of consumption goods if conditions in Lemma 4(i) or (and) Lemma 5 do (not) hold.

Thus, there is always a positive correlation between the steady state levels of capital and the steady state levels of the terms-of-trade in the country exporting the capital intensive good and hence at the stationary state, our results are consistent with those obtained in previous studies (see Figures 2 and 3). This positive correlation between the capital stock and the terms-of-trade for the exporter of capital intensive goods also appears along any convergent path starting outside the steady state when the investment sector is capital intensive. However, if this sector is labor intensive, periods of capital accumulation can be characterized by terms-of-trade improvements for the exporter of consumption goods (declining \( p \)) if the slope of a given equilibrium trajectory is negative. This possibility arises when, at the steady state equilibrium, the price elasticity of the investment good demand relative to the supply's elasticity is large enough (lemma 4(ii)), or the capital external effect on the scale of production is sufficiently strong, \( \text{Det} \left[ J \left( \overline{K}, \overline{P} \right) \right] > 0 \) (lemma 5). In addition, an equilibrium path with these characteristics may have oscillatory behavior because if conditions in Lemma 4(i) are met, it cannot stay for two consecutive periods on the same side of the steady state since production and
prices of investment goods are positively correlated (Corollary 1(i)). In summary, when the investment sector is labor intensive, there can be convergent paths where a country exporting the capital intensive good can enjoy terms-of-trade improvements during periods of economic growth (see Figure 2.a).

Figure 2.a: \( k_e > k_i \)

A steady state diversified production equilibrium is either (locally) stable or a saddle. Trajectories marked with a dashed line indicate that there may be oscillatory behaviour.
Figure 2.b: $k_e > k_l$

A steady state diversified production equilibrium is either (locally) a saddle or unstable.

Figure 3: $k_e < k_l$

A steady state diversified equilibrium is either (locally) a saddle or unstable.
4. TERMS-OF-TRADE AND THE CURRENT ACCOUNT

The study of the effects of terms-of-trade changes on the current account in intertemporal frameworks is by no means novel. Most of the existent studies focus the analysis on the small economy case assuming complete specialization (see section I); in this case, the implications of terms-of-trade deteriorations for the external adjustment of an economy depend crucially on the savings behavior and hence on the particular specification of the preferences. If a decline in aggregate savings measured in units of exportables follows from a terms-of-trade deterioration, then we say that the H-L-M effect holds. The intuition is simple when an unexpected rise in the price of imports is analyzed in a static framework: a rise in the price of imports implies an increase in aggregate spending for any given level of income and so it leads to a reduction in aggregate savings. However, in intertemporal frameworks where temporal and intertemporal substitution effects in spending and in production can be taken into account, the H-L-M effect becomes ambiguous. Indeed, Persson and Svensson (1985) show that the response of savings can have any sign for plausible parameter values; in particular, they find that the H-L-M effect will hold if the rate of time preference is constant, preferences are homothetic and present and future real consumption are gross substitutes. Under the assumption that only home goods can be accumulated as capital, they analyze the current account dynamics when the H-L-M effect holds and show that unanticipated terms-of-trade deteriorations imply a current account deficit.

Matsuyama (1988) analyses the small economy in the non-specialized region stressing the role of relative capital intensities on the external adjustment. He considers a two-period overlapping generations framework where the production side is given by a standard two-sector model with installation costs. Assuming a logarithmic utility function he shows that an unexpected terms-of-trade deterioration leads to a current account deficit (surplus) if the export (import) sector is labor intensive. In that framework, savings are a constant fraction of labor income and hence, if the economy produces both goods, an unexpected rise in the terms-of-trade will reduce savings if and only if the export sector is labor intensive. If the change is permanent, savings jump to the new steady state level in just one period but the sign of the current account during the adjustment process will depend on the size of the installation costs.

In our two-country/two-sector growth model, the rate of time preference is constant, preferences are homothetic and current and future consumption are gross substitutes. We
will prove that the relationship between a country's terms-of-trade and savings may not depend only on the international trade pattern as in Matsuyama (1988), but also on whether there is positive capital accumulation (initial conditions). Moreover, the links between the terms-of-trade and the current account will depend on the international trade pattern as well as on the lending and borrowing position.

The international pattern of trade and credit will depend on the technology parameter \( \hat{\theta} \) and on the particular specification of preferences; it is easy to show that \( \hat{\theta} \) alone will determine the international pattern of trade and credit provided preferences are not very different. In this case, the country relatively richer in capital (labor) will be an exporter of capital (labor) intensive goods, and a borrower (lender) in the international financial market. Since the concern of the paper is not on the determinants of international flows, we will assume, for simplicity, that in this section preferences are identical across countries; this assumption will not affect the results obtained above.

The net foreign asset position of country \( H \) at the end of period \( t \), measured in units of investment goods, is then given by \( B_t = \bar{s}_t - k_{t+1} \) since only young workers hold financial assets in equilibrium. Therefore, the current account surplus in units of investment goods in that period can be written as:

\[
CA_t = B_t - B_{t-1}
\]  

(24)

or equivalently as:

\[
CA_t = (\bar{s}_t - \bar{s}_{t-1}) - (k_{t+1} - k_t)
\]  

(24)'

That is, as the difference between aggregate savings and investment.

On the other hand, the equilibrium condition in the international financial market becomes

\[
2\bar{s}_t = \left(1 + \hat{\theta}\right)k_{t+1}
\]  

(9)'

Subtracting from (9)' the same expression lagged one period, it yields

\[
2(\bar{s}_t - \bar{s}_{t-1}) = \left(1 + \hat{\theta}\right)(k_{t+1} - k_t)
\]

(25)

It follows from (24)' and (25) that the evolution of the current account along an equilibrium path is determined by the capital/labor ratios alone. That is:
\[ CA_t = \left( \frac{1 + \hat{\theta}}{2} - 1 \right) (k_{t+1} - k_t) \]  

(26)

Therefore, once we know the equilibrium relation between prices and capital, given \( \hat{\theta} \), we will have determined the links between the terms-of-trade and both savings and the current account.

It is clear from expression (25) that, in equilibrium, a deterioration of the terms-of-trade will be associated to a reduction in the level of savings if and only if the terms-of-trade deterioration is associated with a capital stock reduction. Therefore, along a convergent diversified production equilibrium path where periods of positive capital accumulation are associated with rises in the terms-of-trade for the exporter of capital intensive goods, savings by the young are rising but aggregate savings measured in units of investment (for \( t \) large enough) are falling. These implications occur when the investment sector is more capital intensive; in this case, if we consider a convergent path characterized instead by terms-of-trade deteriorations for the exporter of consumption goods (the economy converges to the steady state from the right) savings by the young in units of investment are falling in that country, and hence aggregate savings are negative and increasing towards zero, but what happens to savings in units of consumption is ambiguous and it will depend upon the specific slope of the equilibrium trajectory.

On the other side, if the consumption sector is more capital intensive and there is no oscillatory behavior, terms-of-trade deteriorations for a given country can be associated to a rise or a fall in the capital stock. In the former case, aggregate savings in units of exportables and the terms-of-trade are inversely related. In the latter, however, terms-of-trade deteriorations are associated with higher levels of aggregate savings if the export sector is more labor intensive, but the sign of this relationship is ambiguous otherwise. If the equilibrium displays oscillatory behavior, then the slope of the trajectory is negative and so periods of deteriorations in the terms-of-trade for the exporter of capital (labor) intensive goods are associated with increasing (decreasing) aggregate savings in units of exportables, and vice versa.

With respect to the relationship between the terms-of-trade and the current account along an equilibrium trajectory, it follows from (26) that the sign of it will be the same as the sign of the relationship between the terms-of-trade and the capital if and only if \( \hat{\theta} > 1, \hat{\theta} \neq 1 \). Using (13)', it is easy to check that if \( \hat{\theta} > 1 \), then country \( H \) is a lender and an exporter (importer) of labor (capital) intensive goods; and conversely, if
\( \hat{\theta} < 1 \). In contrast with Matsuyama (1988)'s result, it can be shown that terms-of-trade deteriorations are associated with current account deficits, regardless of the trade pattern, whenever positive capital accumulation is associated with a worsening in the terms-of-trade for the exporter of capital intensive goods. Therefore, this is always the case when the investment sector is more capital intensive but not necessarily otherwise (proposition 2). If this sector is labor intensive, equilibrium terms-of-trade deteriorations come along with current account surpluses (deficits) if the equilibrium trajectory is negatively (positively) sloped (proposition 3); see Figure 2.

**Proposition 3:** Suppose the economy evolves along a convergent diversified production equilibrium with identical preferences. Then:
(i) If \( k_{It} > k_{ct} \ \forall t \), terms-of-trade deteriorations are associated with periods of declining current account deficits.
(ii) If \( k_{ct} > k_{It} \ \forall t \), terms-of-trade deteriorations are associated with periods of worsening current account surpluses (deficits) if the capital/labor ratio is negatively (positively) related with the investment good's price along the equilibrium path.

In Stockman and Svensson (1987) and Backus (1993) the sign of equilibrium comovements between the terms-of-trade and the trade balance, for a net foreign asset position close to zero, depends on the degree of intertemporal substitution in consumption. In our model, increasing terms-of-trade come along with declining current account deficits if the investment sector is capital intensive, but if this sector is labor intensive, this relationship can have any sign. In particular, if Rybczynski's theorem holds or if it does not, the positive effect of capital on the production of investment goods is very small (lemma 4), an equilibrium trajectory can be characterized by periods of increasing terms-of-trade and declining current account deficits (surpluses) if, at the steady state, the ratio between the price-elasticity of savings and the price-elasticity of investment good's supply is smaller (larger) than \( \mu \).

5. CONCLUSION

The aim of this paper has been to analyze in a general equilibrium context the evolution of a country's terms-of-trade and the capital/labor ratio along convergent equilibrium paths emphasizing the role of relative factor intensities and their implications for the external adjustment of an economy. The framework that has been used for this purpose can be thought as an overlapping generations version of Oniki and Uzawa (1965)'s two-country/two-sector growth model, or as a two-country extension of Galor.
(1992), where agents can borrow and lend in the international financial market. In this framework, we have assumed that capital has a positive country-specific external effect on the scale of production but that countries share the same technology in units of efficiency; for the sake of simplicity, we have also assumed that the scale factor is a constant elasticity function of the capital stock and that this elasticity differs across countries by an arbitrary small amount. This assumption has allowed us to obtain a non-trivial current account keeping, at the same time, the tractability of the model. The analysis demonstrates that, within the non-specialized area, if preferences are homothetic, current and future consumption are gross substitutes, the income effect of a rise in the current price of investment goods dominates the substitution effect, and the excess demand for investment goods is downward sloping in prices at the stationary price solution, then: terms-of-trade improvements are compatible with periods of economic growth in the country exporting the capital intensive good in contrast to existing models, where the terms-of-trade will improve for a growing country that exports the capital intensive good only if the import sector expands fast enough relative to the export sector. Our analysis also shows that, along a convergent equilibrium path, terms-of-trade improvements (deteriorations) can be associated with improving (worsening) current account deficits (surpluses). These two outcomes arise when the investment sector is more labor intensive and (i) Rybczynski's theorem does not hold and the external effect dominates the price effect, or (ii) the steady state ratio between the current price elasticity of savings and the price elasticity of investment goods supply is bounded from below by $\mu \in (0, 1)$ (i.e., at the steady state equilibrium, investment goods supply is not too steep in prices relative to demand). Otherwise, (1) the terms-of-trade of a growing country exporting the capital intensive good will deteriorate and (2) worsening (improving) current account surpluses (deficits) accompany terms-of-trade improvements (deteriorations). These two conditions are sufficient for an equilibrium trajectory to have negative slope when the investment sector is more labor intensive; the former rules out any oscillatory behavior but the latter does not. The study also demonstrates that we may have indeterminacy of equilibria when one of these two conditions holds.

These results illustrate the role played by relative factor intensities in explaining the comovements between the terms-of-trade and capital, and hence between the terms-of-trade and the current account. A natural extension of the model will be to explore the implications of sector specific capital externalities in the process of the external adjustment as well as some more general assumptions on preferences.
APPENDIX

Dynamic System and Comparative Statics:

\[ k_{t+1} = \Psi(k_t, p_t) \]  \hspace{1cm} [1]

Equation [1] is defined in expressions (14), (16) and (17); its partial derivatives evaluated at a stationary solution are given by [1.i] and [1.ii].

\[
\begin{align*}
\left. \frac{\partial \Psi(k_t, p_t)}{\partial k_t} \right|_{ss} &< 0 \iff \bar{\eta} < -\gamma^H A(k) \frac{\partial \psi(k_t, p_t)}{\partial k} \quad \text{if} \quad k_{It} < k_{ct} \\
\left. \frac{\partial \psi(k_t, p_t)}{\partial k_t} \right|_{ss} &> 0 \quad \text{if} \quad k_{It} > k_{ct}
\end{align*}
\]  \hspace{1cm} [1.i]

\[
\left. \frac{\partial \Psi(k_t, p_t)}{\partial p_t} \right|_{ss} > 0 \quad \forall k_{ct}, k_{It}
\]  \hspace{1cm} [1.ii]

Proof:

\[
\frac{\partial \Psi(k_t, p_t)}{\partial k_t} = \gamma^H A'(k_t) \psi(k_t, p_t) + \gamma^H A(k_t) \frac{\partial \psi(k_t, p_t)}{\partial k_t}
\]

\[
\frac{\partial \psi(k_t, p_t)}{\partial k_t} = \frac{f_I(k_{It})}{k_{It} - k_{ct}} > 0 \iff k_{It} - k_{ct} > 0
\]

\[
\left. \frac{\partial \Psi(k_t, p_t)}{\partial k_t} \right|_{ss} = \bar{\eta} + \gamma^H A(k) \frac{\partial \psi(k_t, p_t)}{\partial k_t}
\]

\[
\frac{\partial \Psi(k_t, p_t)}{\partial p_t} = \gamma^H A(k_t) \frac{\partial \psi(k_t, p_t)}{\partial p_t}
\]

\[
\frac{\partial \psi(k_t, p_t)}{\partial p_t} = \left( k_t - \frac{2}{\theta} k_{ct} \right) k_{It} f_I - \left( k_t - \frac{2}{\theta} k_{ct} \right) \left( k_{It} f'_{I} + k_{ct} f'_{I} \right) k'_{I}
\]

which is positive since \( \text{sign} \left( k_t - 2 \theta^{-1} k_{It} \right) = -\text{sign} \left( k_t - 2 \theta^{-1} k_{ct} \right) \) positive iff \( k_{It} < k_{ct}, k'_{I} < 0 \) iff \( k_{It} < k_{ct} \), and the term \( f_I - k_{It} f'_{I} + k_{ct} f'_{I} \) is always positive.

\[ p_{t+1} = \Phi(k_t, p_t) \]  \hspace{1cm} [2]

Equation [2] derives from the IFT applied to \( E(p_{t+1}, k_t, p_t) = 0 \). Its partial derivatives evaluated at the steady state solution are given by [2.i] and [2.ii].
\[
\frac{\partial \Phi(k_t, p_t)}{\partial k_t} \bigg|_{ss} = \frac{-\partial E/\partial k_t}{\partial E/\partial p_{t+1}} \bigg|_{ss} \begin{cases} > 0 & \text{if } k_{It} > k_{ct} \land E_{S,R} = 0 \\ > 0 & \text{if } k_{It} < k_{ct} \end{cases}
\]

\[
\frac{\partial \Phi(k_t, p_t)}{\partial p_t} \bigg|_{ss} = \frac{-\partial E/\partial p_t}{\partial E/\partial p_{t+1}} \bigg|_{ss} \begin{cases} > 0 & \text{if } k_{It} > k_{ct} \\ < 0 & \text{if } k_{It} < k_{ct} \end{cases}
\]

Proof:
A solution to the expression \(E(p_{t+1}, k_{t}, p_{t}) = 0\) for any given \(k_{t} > 0\) always exists provided production technologies are similar enough (Lemma 2). \(\partial E/\partial p_{t+1} \neq 0\) is guaranteed by the gross substitutability of current and future consumption. In this case, IFT applies. After rearranging terms, we get:

\[
\frac{\partial E}{\partial k_t} \bigg|_{ss} = E_{S,R} \frac{\theta p \eta^2}{\gamma^B A(k)} + (E_{S,R} \eta - 1) \frac{\partial \psi(k, p)}{\partial k_t}
\]

\[
\frac{\partial E}{\partial p_{t+1}} \bigg|_{ss} = (s'_{H} + s'_{F}) \frac{\gamma^B A(k)}{p} \frac{f''_{o}(k_{c}(p))}{p} k_{o}w(p)
\]

where \(E_{S,R}\) is the elasticity of worldwide savings with respect to the interest rate. The result follows from the proof of [1.i], expression (12) in the text and \(f''_{o} < 0\), taking into account that \(s'_{H}\) and \(s'_{F}\), and so \(E_{S,R}\), are all positive.

\[
\frac{\partial E}{\partial p_t} \bigg|_{ss} = \frac{\partial S}{\partial p_t} - \theta \left( p \frac{\partial \psi}{\partial p_t} + \psi(k, p) \right)
\]

\[
\frac{\partial S}{\partial p_t} = \frac{S}{p} \left( E_{w,p} - E_{S,R} \right)
\]

where \(E_{w,p}\) is the elasticity of the wage rate with respect to the relative price of investment goods. The result follows from the proof of [1.ii], [2.ii] and from the Stolper-Samuelson Theorem (i.e.: \(E_{w,p} < 0\) if \(k_{It} > k_{ct}\), \(E_{w,p} > 1\) if \(k_{ct} > k_{It}\) ) taking into account that \(E_{S,R}\) is positive and less than one.

**Steady States:**

1. Let \(\hat{k}\) be the supreme of sustainable levels for the capital stock if each country is to diversify production. Suppose that assumption (A.1) holds, then, for every \(k_{t} \in (0, \hat{k})\):
   1. There is a unique \(p_{t} = \rho(k_{t})\), \(p_{t} \in (\hat{p}_{\text{min}}(k_{t}), \hat{p}_{\text{max}}(k_{t}))\), such that \(k_{t+1} - k_{t} = 0\).
   2. For all \(p_{t} \in (\hat{p}_{\text{min}}(k_{t}), \hat{p}_{\text{max}}(k_{t}))\), \(k_{t+1} - k_{t} \geq 0 \iff p_{t} \geq \rho(k_{t})\).
   3. \(\rho'(k_{t}) > 0 \iff k_{ct} > k_{It}\).

   **Proof:**

26
A convenient way of writing $\Psi(k_t, p_t)$ is the following: 

$$\Psi(k_t, p_t) = \theta^{-1} \left[ \gamma^R A(k_t) l_I(k_t, p_t) f_I(k_t(p_t)) + \gamma^F A(\hat{\theta} k_t) l_I(\hat{\theta} k_t, p_t) f_I(k_I(p_t)) \right]$$

where $l_I(k, p) = [k - k_0(p)]/l_I(p) - k_0(p)$ is the proportion of labor in the l-sector ($l_I = 0$ if $p = \hat{p}_{\text{min}}$, $l_I = 1$ if $p = \hat{p}_{\text{max}}$).

Let: $\overline{\Psi}(k) = \gamma^R A(k) f_I(k) + \gamma^F A(\hat{\theta} k) l_I(\hat{\theta} k, p_{\text{max}}(k)) f_I(k_I(p_{\text{max}}(k)))$ if $k_{ct} > k_{ct}$ and $\overline{\Psi}(k) = \gamma^R A(k) l_I(k, p_{\text{max}}(k)) f_I(k_I(p_{\text{max}}(k))) + \gamma^F A(\hat{\theta} k) f_I(\hat{\theta} k)$ if $k_{ct} > k_{it}$, where $p_{\text{max}}(k) = \hat{p}_{\text{max}}(k)$ if $k_{it} > k_{ct}$ and $p_{\text{max}}(\hat{\theta} k) = \hat{p}_{\text{max}}(k)$ if $k_{ct} > k_{it}$ ($\hat{\theta} \in (0, 1)$).

Let $\overline{\Psi}(k_t) = \gamma^R A(k_t) f_I(k_t) + \gamma^F A(\hat{\theta} k_t) f_I(\hat{\theta} k_t)$. Given the assumptions on the technology, there is a unique $\overline{k} > 0$ such that $\overline{\Psi}(\overline{k}) = \overline{k}$.

Moreover, $0 \leq \overline{\Psi}(k) - \overline{\Psi}(k) \leq \gamma^{\hat{\theta}} A(\hat{\theta} k) f_I(\hat{\theta} k) - f_I(k_I(p_{\text{max}}(\hat{\theta} k)) - \delta)$.

if $k_{it} > k_{ct}$, and assumption (A.1) holds. The difference $\overline{\Psi}(k) - \overline{\Psi}(k)$ can be arbitrarily small for very similar technologies. Therefore, if technologies are similar enough, there will exist a unique $\overline{k} > 0$ such that $\overline{\Psi}(\overline{k}) = \overline{k}$.

Moreover, $0 \leq \overline{\Psi}(k) - \overline{\Psi}(k) \leq \gamma^{\hat{\theta}} A(\hat{\theta} k) f_I(\hat{\theta} k) - f_I(k_I(p_{\text{max}}(\hat{\theta} k)) - \delta)$.

if $k_{it} > k_{ct}$, and assumption (A.1) holds. The difference $\overline{\Psi}(k) - \overline{\Psi}(k)$ can be arbitrarily small for very similar technologies. Therefore, if technologies are similar enough, there will exist a unique $\overline{k} > 0$ such that $\overline{\Psi}(\overline{k}) = \overline{k}$.

(i) Given $k_t > 0$, if $p_t \to \hat{p}_{\text{min}}(k_t)^+$, then $k_{t+1} \to 0$ and hence $k_{t+1} - k_t < 0$.

On the other side, if $p_t \to \hat{p}_{\text{max}}(k_t)^-$, then $k_{t+1} \to \theta^{-1} \overline{\Psi}(k_t).$ So $k_{t+1} - k_t < 0$ if $k_t > \overline{k}$, and $k_{t+1} - k_t > 0$ if $k_t < \overline{k}$.

Since $k_{t+1} - k_t$ is a continuous and strictly increasing function of $p_t$. For any given $k_t \in \left(0, \overline{k}\right)$, there exists a unique $p_t = \rho(k_t)$. $p_t \in \left(\hat{p}_{\text{min}}(k_t), \hat{p}_{\text{max}}(k_t)\right)$, such that $k_{t+1} - k_t = 0$. Moreover, $k_{t+1} - k_t \to 0$ as $k_t \to \overline{k}$.

(ii) If $p_t \in \left(\hat{p}_{\text{min}}(k_t), \hat{p}_{\text{max}}(k_t)\right)$, $k_{t+1} - k_t = \Psi(k_t, p_t) - k_t$. The result follows from [1.i.i] and Lemma 3(i).

(iii) It follows from the IFT and expressions [1.i] and [1.i.i] in this appendix.

(II) Suppose assumptions (A.1) and (A.2) hold. Then, for any given $k_t > 0$:

(i) There exists a unique $p_t = \phi(k_t)$. $p_t \in \left(\hat{p}_{\text{min}}(k_t), \hat{p}_{\text{max}}(k_t)\right)$, such that $p_{t+1} - p_t = 0$.

(ii) For all $p_t \in \left(\hat{p}_{\text{min}}(k_t), \hat{p}_{\text{max}}(k_t)\right)$, $p_{t+1} - p_t \leq 0 \iff p_t \geq \phi(k_t)$ if $k_{ct} > k_{it}$, and $p_{t+1} - p_t \geq 0 \iff p_t \geq \phi(k_t)$ if $k_{ct} < k_{it}$.
(iii) \( \phi'(k_t) > 0 \Leftrightarrow k_{ct} > k_{ft}. \)

**Proof:** Let \( E_t = s_t^H + s_t^F - (y_t^H + y_t^F) \), \( p_{t+1} = p_t. \)

\[
\hat{p}_{\text{max}}(k_t) = \max \left\{ p_{\text{max}}(k_t), k_t \right\} \quad \text{and} \quad \hat{p}_{\text{min}}(k_t) = \min \left\{ p_{\text{min}}(k_t), k_t \right\}.
\]

(i) For any given \( k_t > 0 \), if \( p_t = \hat{p}_{\text{max}}(k_t) \), both countries will specialize in the production of investment goods and hence \( E_t < 0 \), since savings are a fraction of labor income. On the other side, if \( p_t = \hat{p}_{\text{min}}(k_t) \), both countries will specialize in consumption goods \( (y_t^H = y_t^F = 0) \) and so \( E_t > 0 \). Furthermore, if technologies are similar enough (assumption (A.1)), \( E_t > 0 \) for all \( p_t \in \hat{p}_{\text{min}}(k_t), \hat{p}_{\text{min}}(k_t) \) and for all \( p_t \in \hat{p}_{\text{max}}(k_t), \hat{p}_{\text{max}}(k_t) \), \( E_t < 0 \). (A.2) implies that there exists a unique \( p_t \in \hat{p}_{\text{min}}(k_t), \hat{p}_{\text{max}}(k_t) \), \( p_t = \phi(k_t) \), for any \( k_t > 0 \) such that \( E(p_t, k_t, p_t) = 0 \).

(ii) If \( p_t \in \hat{p}_{\text{min}}(k_t), \hat{p}_{\text{max}}(k_t) \), \( p_{t+1} - p_t = \Phi(k_t, p_t) - p_t. \) The result follows from assumption (A.2), expression [2.i] and Lemma 4(i).

(iii) It follows from the IFT and expressions [2.i] and [2.ii] in this appendix.

(III) Let \( (\bar{k}, \bar{\nu}) \) be a steady state diversified production equilibrium and \( J(\bar{k}, \bar{\nu}) \) the Jacobian associated to equations (15) and (22) evaluated at that equilibrium. Under assumptions (A.1), (A.2) and (A.3), the eigenvalues of \( J(\bar{k}, \bar{\nu}) \) are always distinct and real. Moreover:

(i) If \( k_{ct} < k_{ft}, \forall t \), the eigenvalues are both positive.

(ii) If \( k_{ct} > k_{ft}, \forall t \), the eigenvalues have different sign except for the case \( \partial \Psi / \partial k_t > 0 \) where they can be both positive.

**Proof:**

Let \( (\bar{k}, \bar{\nu}) \) be a steady state diversified production equilibrium. The discriminant of the characteristic equation associated to \( J(\bar{k}, \bar{\nu}) \) is given by

\[
\Delta = \left( \partial \Psi / \partial k_t - \partial \Phi / \partial p_t \right)^2 + 4 \left( \partial \Psi / \partial p_t \right) \left( \partial \Phi / \partial k_t \right) \text{ evaluated at } (\bar{k}, \bar{\nu}).
\]

In virtue of expressions [1.ii] and [2.i] is always positive. Hence, the roots of this characteristic equation are distinct and real. Let \( D \) and \( T \) be, respectively, the determinant and trace of \( J(\bar{k}, \bar{\nu}) \). After rearranging terms, it follows from [1.ii], [1.ii], [2.i] and [2.ii] that:
\[ D = -\frac{1}{\partial E/\partial \rho_{t+1}} \left\{ \frac{\partial \psi}{\partial k_t} \cdot \frac{\partial}{\partial \bar{E}} \left[ E_{w,\rho} - E_{S,R} - 1 \right] - \frac{\partial}{\partial \rho_t} \frac{\partial \psi}{\partial \rho_t} \cdot E_{S,R} \eta^2 + \eta \frac{\partial E}{\partial \rho_t} \right\} \]

(i) \( k_{it} > k_{ct} \quad \forall t \). In this case, \( \partial E/\partial \rho_{t+1} > 0, \partial E/\partial \rho_t < 0 \) and \( E_{w,\rho} < 0 \). Taking into account that \( E_{S,R} \) is positive, [1.i] and [1.ii] imply that \( D > 0 \). On the other hand, \( \partial \Psi(\bar{K}, \bar{P})/\partial k_t = \eta + \left( \frac{k_t}{\bar{K}} - 2 \theta^{-1} \kappa_t \right) > 1 \). and using (A.2), it is easy to check that \( \partial \Phi/\partial \rho_t - 1 > 0 \); Hence, \( T > 2 \) and the roots of the characteristic equation are both positive.

(ii) \( k_{it} < k_{ct} \quad \forall t \). In this case, \( \partial \psi/\partial k_t < 0, \partial \Phi/\partial \rho_t - 1 < 0, \partial E/\partial \rho_{t+1} < 0 \) and \( E_{w,\rho} > 1 \).

(ii.1) \( \partial \Psi(\bar{K}, \bar{P})/\partial k_t < 0 \): If \( \partial \Phi(\bar{K}, \bar{P})/\partial \rho_t > 0 \), then \( D < 0 \) and so eigen-values have different sign. If \( \partial \Phi(\bar{K}, \bar{P})/\partial \rho_t < 0 \), then \( D < 0 \) for small enough \( E_{S,R} \) (assumption (A.3)). Moreover, in this case, \( T < 0 \).

(ii.2) \( \partial \Psi(\bar{K}, \bar{P})/\partial k_t > 0 \): In this case, \( \partial \Psi(\bar{K}, \bar{P})/\partial k_t < \eta < 1 \) since \( \eta \in (0, 1) \). If \( \partial \Phi(\bar{K}, \bar{P})/\partial \rho_t < 0 \), then \( D < 0 \). If \( \partial \Phi(\bar{K}, \bar{P})/\partial \rho_t > 0 \), then \( D > 0 \), and the sign of \( D \) is ambiguous. However, if \( D > 0 \), both eigen-values are positive since \( T > 0 \).

Lemma 1. Proof: See Jones (1965).

Lemma 2. Proof: Using equations (6) and (7), and taking into account that there are constant returns to scale, a country's production of good \( i \) is given by \( Y_{it} = \gamma a(k_t)[k_t - k_{t-1}/k_{t-1} - k_{t-1}]e_i(k_{t-1}), i = i', i, i' = c, d \). The results follow from assumption (A.1).

Lemma 3. Proof: Let \( P(\lambda) = \lambda^2 - T\lambda + D \) be the characteristic polynomial associated to \( J(\bar{K}, \bar{P}) \). It can be checked that \( P(-1) > 0, P'(1) < 0, P'(-1) < 0 \) and so at least one root is greater than one since both eigen-values are positive. If \( P(1) < 0, (\bar{K}, \bar{P}) \) is a saddle and if \( P(1) > 0 \), it is unstable. If \( (\bar{K}, \bar{P}) \) is unique, it must be true that \( |\phi'(\bar{K})| > |\rho'(\bar{K})| \), which holds iff \( D < T - 1 \). Hence, \( P(1) < 0 \) and \( (\bar{K}, \bar{P}) \) is a saddle.

Lemma 4. Proof: \( (1 - \eta) \partial E/\partial \rho_t > (\partial \Psi/\partial k_t - \eta) \partial S/\partial \rho_t \). iff \( P(-1) > 0 \) provided assumption (A.3) holds. Moreover, \( \rho'(\bar{K}) > \phi'(\bar{K}) \) iff \( D > T - 1 \) iff \( P(1) > 0 \).

(i) \( P(-1) > 0 \) and \( D < 0 \) imply that \( (\bar{K}, \bar{P}) \) is either (locally) stable \( (\rho'(\bar{K}) > \phi'(\bar{K})) \) or (locally) a saddle \( (\rho'(\bar{K}) < \phi'(\bar{K})) \). And if it is unique, it must be true that \( \rho'(\bar{K}) > \phi'(\bar{K}) \), so it is stable.
(ii) \( P(-1) < 0 \) and \( D < 0 \) imply that \( (\overline{k}, \overline{p}) \) is either (locally) a saddle \( (\rho'(\overline{k}) > \phi'(\overline{k})) \) or it is unstable \( (\rho'(\overline{k}) < \phi'(\overline{k})) \). And if it is unique, it must be true that \( \rho'(\overline{k}) > \phi'(\overline{k}) \), so it is a saddle.

It is straightforward that \((1 - \eta) \frac{\partial E}{\partial p_t} > (\partial \Psi/\partial k_t - \eta) \frac{\partial S/\partial p_t}{E_{S_t, p_t}} \) iff \((1 - \partial \Psi/\partial k) E_{S_t, p_t} > (1 - \eta) E_{\Psi, p_t} \). The result follows.

**Lemma 5. Proof:**

If \( D > 0 \), then \( \partial \Psi/\partial k \leq 0 \) (see proof in III.(ii)). Therefore, it must be the case that \( \partial \Phi/\partial p_t > 0 \) at the steady state and so, under (A.2), \( \partial \Phi/\partial p_t \leq (0, 1) \) and hence \( P(-1) > 0 \). Moreover, since \( \rho'(\overline{k}) > \phi'(\overline{k}) \) iff \( D > T - 1 \) iff \( P(1) > 0 \), it follows that \( (\overline{k}, \overline{p}) \) is either (locally) stable \( (\rho'(\overline{k}) > \phi'(\overline{k})) \) or it is (locally) a saddle \( (\rho'(\overline{k}) < \phi'(\overline{k})) \). And if it is unique, \( \rho'(\overline{k}) > \phi'(\overline{k}) \), it must be stable.

**Corollary 1. Proof:**

(i) By lemma 4(i), if the steady state is a saddle \((D < T - 1)\), the stable root is negative; moreover, the saddle path has negative slope. If the steady state is stable, for any given \( k_0 \) sufficiently close to it, there are infinitely many initial equilibrium prices above (below) \( \rho(k_0) \) and \( \phi(k_0) \) that make the economy to oscillate towards the steady state.

(ii) By lemma 4(ii), if the steady state is a saddle \((D > T - 1)\), the stable root is positive; moreover, the slope of the saddle, given by \( \frac{\partial \Psi/\partial k}{(\lambda - \partial \Psi/\partial k)} = \frac{\partial \Phi/\partial k}{(\lambda - \partial \Phi/\partial p)} \), is positive except for the case \( \partial \Psi/\partial k > 0 \) where it can be negative if \( \partial \Phi/\partial p > 0 \).

(iii) By lemma 5 and III(ii), both eigen-values are positive. Moreover, if the steady state is a saddle, the saddle path has negative slope.

**Proposition 1. Proof:** Steady state diversified production equilibria are given by the intersection points of \( p_t = \rho(k_t) \) and \( p_t = \phi(k_t) \), which are both strictly decreasing (increasing) (See I and II). The result follows.

**Proposition 2. Proof:** (i) The result follows from lemma 3 and inspection of Figure 3.

(ii) It follows from lemma 4, lemma 5, corollary 1 and inspection of Figure 2.

**Proposition 3. Proof:** Suppose \( \hat{\theta} > 1 \), then country H (F) exports the labor (capital) intensive good. Let TOTj be country j's terms of trade. (i) In this case, TOT_H = p and TOT_F = 1/p. The result follows from expression (26) and inspection of Figure 3. (ii) In this case, TOT_H = 1/p and TOT_F = p. The result follows from expression (26) and inspection of Figure 2. Similarly for the case \( \hat{\theta} < 1 \).
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31


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