IS REGIONALISM BETTER FOR ECONOMIC INTEGRATION?
NATIONS, REGIONS, AND RISK SHARING

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ABSTRACT

Our analysis yields some conclusions about the political role of regions in the formation of supranational economic areas, which turns out to be quite different from the role of nations. The claim that regions have more incentives than nations to attain a fiscal agreement implying full economic integration is likely to be correct when nations are economic stable arrangements, i.e. when the rich region of a nation is not "exploited" by the poor region. When, on the other hand, it is not in the interest of a rich region to be part of a nation, attempts to achieve full economic integration among a group of nations is more likely to be successful if nations, instead of regions, are the decision makers.

Keywords: Federalism; Fiscal coinsurance; Migration.
JEL classification: H11, H77, H87.
1 Introduction

In Europe increased integration among nations is coexisting with increasing autonomy of regions within nations. That more integration can induce more autonomy is a phenomenon that has been studied in the literature (see, for instance, Alesina and Spolaore, 1997). A theoretical question that has not been studied is whether increasing autonomy of the regions can reinforce or not the integration process. In other words, whether giving the regions enough fiscal autonomy would result in a fiscal agreement implying different levels of integration than those resulting in economies where only nations decide.

In this paper we focus on how the incentives of regions differ from those of nations when choosing a type of common fiscal arrangement. We will consider a very simple but, we hope, relevant and clarifying environment with only two nations. Each nation will consist of two regions. A unitary fiscal arrangement would bring full insurance against local shocks for all the nations (and for the regions forming the nations). Under a federal system, however, risk-sharing is achieved by means of migration from poor regions to richer regions. Since migration is costly a federal system provides only partial insurance against local risks. One might think that independent regions, being smaller economic areas than nations, are exposed to higher risks and as a consequence have stronger incentives, as compared to nations, to form a union. A nation, on the other hand, might prefer the partial insurance mechanism provided by the fiscal system since it faces a lower risk and the full insurance associated with a unitary state might be "too expensive."

We will show, however, that the previous intuition might be quite misleading. Suppose that one of the two nations is richer than the other in expected terms and, in the same way, within each nation one region is richer than the other. Thus, suppose that different nations, and different regions, each face different idiosyncratic risks. It is true that the regions, as independent economic areas, might face higher risks than they would if they were part of a nation. But now the income dispersion among the four regions is also higher than the income dispersion among the two nations. In this case, the richest region might end a partial insurance arrangement more profitable.

\footnote{We abstract from the political risk discussed, for instance, in Alesina and Perotti (1995).}
than a unitary state and at the same time, were nations the players, both nations prefer the full insurance associated with the unitary agreement to a partial insurance system.

The relevance of our analysis rests on the assumption that nations (or regions) cannot obtain full insurance against idiosyncratic shocks in the market. Thus, in our approach the union and the federation can be seen as institutions that offer risk-sharing that is not provided by the market. Obstfeld (1994), Shiller and Athanasoulis (1995), Athanasoulis and van Wincoop (1998), and van Wincoop (1999) provide empirical estimations of potential welfare gains from international and interregional risk-sharing above those not provided by the market. Sala-i-Martin and Sachs (1992) and Asdrubali, Sorensen, and Yosha (1996) also provide empirical estimations of the channels for interregional risk sharing and of the regional risk that remains uninsured within the United States. Sorensen and Yosha (1998) estimate than a lot less risk sharing is achieved within countries in the European Union than within the United States. Forni and Reichlin (1999) provide some measures of the potential insurable risk for the European countries.

One remaining question to assess the merit of the analysis provided in this paper is whether migration is, in reality, an important way to share risks among nations or regions. Barro and Sala-i-Martin (1991 and 1992) provide evidence of the relationship between migration flows to US states and per capita income. Blanchard and Katz (1992) also show that migration is an important insurance device against regional business cycle shocks. Eichen-green (1993) finds a strong relationship between migration and the lagged growth-rate of wages in the US.

Closely related to this paper is the approach taken by Bucovetsky (1998), who compares the incentives for two regions to choose a federal state agreement versus a unitary state. In this paper, regions suffer stochastic idiosyncratic shocks and so the motive for the agreement is to provide insurance. The important parameters to take into account are related to risk aversion, differences in expected income, and migration costs. Alesina and Perotti (1995) also analyze the risk-sharing motive for achieving a fiscal agreement. They discuss the tradeoff between more economic risk sharing and more political risk. Persson and Tabellini (1996a and 1996b) analyze the perverse incentives that the insurance contract fiscal agreement create for the local governments.

Other discussions about federal versus unitary agreements are provided
by Quian and Roland (1999) where the gains from decentralization are given in terms of competition between local governments and decreased ability to bail out inefficient firms. Alesina and Spaolare (1997), and Alesina and Warcziarg (1998, 1999) discuss the optimal size and number of nations. The degree of openness of the economies, increasing returns in the provision of public goods, and the diseconomies of taking decisions for larger communities determine the optimal size of nations. Bolton and Roland (1997) study how the redistribution policies are affected by the incentives to secede. Fidrmuc (1998) explores how the nature of the stochastic shocks affect the incentives to secede.

Obviously, closely related to our paper, as we also study migration as a form of insurance, are Wildasin (1995 and 2000). However there the issue is more how more risk sharing provided by the possibility of migration changes the distribution of risk among the members of a region and the implied effects.

The paper is organized as follows. Section 2 presents the model. Section 3 presents the different type of agreements to be considered. Section 4 shows some results that will be useful for the proof of the main proposition. Section 5 discusses our main result. Section 6 concludes the paper with some final remarks. Finally an Appendix presents the proofs of our results.

2 The Model

Our model shares its basics features with the model developed in Bucovetsky (1998) with the difference that we will consider two possible levels of decision. The national level is modeled exactly as in Bucovetsky's, but in this paper each nation consists of two regions which face idiosyncratic regional shocks.

We will consider two nations, A and B. A consists of two regions A1 and A2, and B consists of regions B1 and B2 (sometimes we write Rj to denote the region j of nation R, j ∈ {1, 2}; R ∈ {A, B}). We normalize population so that the number of people in each region is 1/2. So each nation's total population is 1.

There is uncertainty about the national production level (which will be also given in per capita terms, given our normalization). It can be either 1/2 if the good state of nature happens in that nation, or 1 if the bad state occurs, where 1/2 > 1. We assume that national production levels are negatively correlated, i.e. if one nation gets the good state of nature then the other
nation obtains the bad state of nature. We make this assumption in order to concentrate our analysis on the possible risk sharing advantages of forming a union. The good state of nature occurs in nation A with probability \( \frac{1}{4} \) (so the good state of nature occurs in B with probability \( \frac{1}{4} \)). We assume that \( \frac{1}{4} > 1 = 2 \) so country A is richer than country B in expected terms.

We also assume that regional idiosyncratic shocks can happen. These shocks are such that they add the amount \( \delta = 2 \) to the production level in a lucky region and reduce the production level by \( \delta = 2 \) in an unlucky one. We assume that within either country there is a perfect negative correlation between the regional shocks so that when region \( R_1 \) gets a positive (negative) shock region \( R_2 \) gets a negative (positive) shock. This implies that a lucky region in a lucky nation (from now on, in the state HH) will have the total production

\[
Y_{HH} = \frac{1 + \delta}{2}
\]

A unlucky region in a lucky nation (state HL) would have

\[
Y_{HL} = \frac{1}{2}
\]

A lucky region in an unlucky nation (state LH) would have

\[
Y_{LH} = \frac{1 + \delta}{2}
\]

And, finally, an unlucky region in an unlucky nation (state LL) would have

\[
Y_{LL} = \frac{1}{2}
\]

Region \( R_1; R_2 \) in A; B is lucky with probability \( p \) (so region \( R_2 \), \( R_2 \) in A; B is lucky with probability \( 1 - p \)). We assume that \( p > 1 = 2 \) so A1 and B1 are the rich regions (in expected terms) in nations A and B, respectively. We will interpret A1 and B1 as the North in each nation and A2 and B2 as the South. Note that the probability \( p \) is the same in both nations. These

\[\text{Obvious a more realistic assumption would be that shocks can go in any direction. However if we do not consider negatively correlated shocks there would be no point of talking about risk sharing. A more general assumption which gives the same results would be that on top of the more general shocks there are important shocks that are negatively correlated.}\]
regional production levels are consistent with the national ones defined previously. For simplicity we will assume:

\[(A.1) \ \frac{1}{2} + \theta = 1 + \theta.\]

So that the unlucky region in the lucky nation will have the same income (and per capita income) as the lucky region in the unlucky nation. Our results will not depend crucially on (A1), as long as the difference between \(\frac{1}{2} + \theta\) and \(1 + \theta\) is not large\(^3\). This assumption implies a particular form of the regional shocks. By solving the equation in (A.1) we have that

\[\theta = \left(\frac{1}{2} - 1\right) \Rightarrow 2\]

Our assumption (A1) allows for an alternative interpretation of the model: one region, the lucky one, gets a positive shock of \(\theta\) and the unlucky region gets the negative shock \(-\theta\). The other two regions do not suffer shocks and each of them obtains the average per capita income \(\frac{1+\theta}{2}\).

When we talk of the economy where nations are the agents it is assumed that nations are unitary states so that their governments make the transfers needed to equalize income within regions in the same nation. Thus in the economy with nations the regional shocks will not be relevant.

We assume that all agents of a given nation or region are identical to each other. In this case, the preferences of a nation \(R\); and the preferences of a region \(R_j\), coincide with the individual preferences of their members.

\[(A.2) \text{ All regions and nations share the same von Neuman Morgenstern}\]

\(^3\) Obviously the empirical relevance of the assumption will depend on how the North and South are aggregated within each nation. For instance, (see graph 5.3 in Esteban, 1994), in 1989, (taking 100 as the average per capita income in the European Union), 40% of the population in Spain lived in regions having a per capita income between 110 and 80. This would be the North of Spain. 55% of the population of France live in regions that had per capita income above 100. This could be the North of France in our model. The South of France would be the 45% living in regions that have a per capita between 100 and 80 (thus providing a high degree of overlap with the North of Spain). The South of Spain would be the 60% living in regions having a per capita income lower than 80. Similar considerations could be used to construct the North and South of the respective Northern and Southern countries. The outliers are Greece and Portugal where all regions have per capita incomes lower than the poorest region in several Northern countries. This would not be very important for any of our results as (A.1) could be relaxed as far as lemma 1 remains true for a relevant set of the parameters.
concave utility function $U$ with per capita income as the argument. This utility function presents constant relative risk aversion. Thus we can write $U(x) = \frac{x^{1-\gamma}}{1-\gamma} ; \gamma > 0$, where $x$ is per capita income$^4$.

The total resources in a region are distributed equally among all the residents so that the per capita income is the same for all of them. Thus, an agent that migrates from a poor region to a richer region would obtain a higher income. There is, however, a positive migration cost.

(A.3) There is a constant individual cost $c, c > 0$ of migrating from one region to another. This cost is the same whether the migration takes place within a country or from a region in a country to a region in a different country.

This is clearly a strong assumption but our results are robust to small changes allowing for lower migration cost within a country than across countries$^5$. In fact we could allow for large differences in the migration costs as long as the income dispersion between regions is large enough. Nevertheless, in order to keep things simple, we will assume throughout the paper the three previous axioms.

Following Bucovetsky (1998) we define a Federal fiscal agreement as one in which there is free migration among the nations or regions involved, but no transfers to equalize per capita income among the different regions. A Unitary fiscal agreement (we also call it a Union) is the one in which a central authority uses transfers to equalize per capita income in the different locations and, consequently, there is no migration. Reality is, no doubt, more complicated than what we assume here. The main feature we want to capture, however, is that under a federal arrangement there are less regional transfers and, as a consequence, more migration than under a unitary arrangement.

We will consider two types of environments depending on whether the decision makers are the regions or the nations. The four types of agreements we analyze are: (i) a union of nations $A$ and $B$ (UN); (ii) a federation of nations $A$ and $B$ (FN); (iii) a federation of regions $A_1;A_2;B_1$ and $B_2$ (FR);

$^4$This assumption is also made in Bucovetsky.

$^5$If the cost of migrating to other region within a nation is very small then there is no point in comparing the economy with regions with the economy with nations. Trivially, free migration with very low migration cost between regions in a nation will make each of the regions in each nation share (almost) the same income and therefore they will behave as nations.
and (iv) a union of regions $A_1; A_2; B_1$ and $B_2$ (UR). In cases i) and ii) the decision makers are the nations meanwhile in cases iii) and iv) the decision makers are the regions. We also consider the case in which the two nations are separated (S).

3 Description of Agreements

3.1 Separated nations

When the two nations are separated, we assume that there are neither international migration flows nor transfers from one nation to the other nation. The expected utility nation $A$ would obtain in this case is

$$E^S_A = \frac{1}{2}U(\frac{1}{2}) + \frac{1}{2}U(1)$$

and the expected utility of nation $B$ is

$$E^S_B = \frac{1}{2}U(\frac{1}{2}) + \frac{1}{2}U(1)$$

These are the minimum utility levels that nations should obtain in order for them to be willing to participate in any other possible arrangement.

3.2 Global Union

Given our assumptions, in a unitary state (consisting of the union of nations $A$ and $B$ or of the union of regions $A_1; A_2; B_1$ and $B_2$) the level of transfers is such that each region would end up with the same per capita income, $(\frac{1}{2}+1)=2$; with certainty. Therefore the expected utility for each region, or nation, would be

$$U^U = U^{\frac{1}{2}+\frac{1}{2}+1}$$

Thus, under the global union there is complete sharing of resources and all agents obtain the same income.

3.3 Federation of Nations

The analysis in this section follows directly from Bucovetsky's analysis of a federation between two nations. The players are nation $A$ and nation $B$. 

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Since a nation is the union of two regions, the per capita income within a nation is the same for all agents, regardless of their residency. Thus, all agents in an independent nation in the good state of nature (state H) would obtain the income level \( \frac{1}{2} \) while in the bad state of nature (state L), they would obtain the income level 1. The two nations might decide to form a federation. We assume that they must decide before knowing which nation gets the good state of nature. The preferences of a nation coincide with the expected utility of a representative agent and, since all agents end up with the same income, there is no aggregation problem: a nation seeks to maximize expected utility where the von Neuman-Morgenstern utility function, \( U \), is the one given in our Axiom 2.

If a federation of nation A and nation B is formed, then agents will be able to migrate within the federation. Some agents from the nation in the bad state of nature will migrate to the other nation, where the residents, regardless of where they came from, will equally share the total income \( \frac{1}{2} \). The equilibrium migration condition (equation (4) in Bucovetsky) is

\[
\frac{\frac{1}{2}}{1 + n_n} \cdot c = \frac{1}{1 + \left(1 - \nu \right) n_n} \quad (2)
\]

where \( n_n \) is the net flow of migrants from the unlucky to the lucky nation. This condition means that per capita income in the lucky nation net of migration costs should equal that of the unlucky nation. This should hold for the marginal migrant. We will denote by \( C_j \) per capita income, after migration takes place, for residents in a nation in the \( j \) state of nature, \( j \in \{H; L\} \). This means that the above condition could be written as

\[
C_H - c = C_L \quad (2')
\]

Expected utility under the federation of nations for the rich nation A is

\[
EU_A^{FN} = \frac{1}{2} U\left(C_H\right) + \left(1 - \frac{1}{2}\right) U\left(C_L\right) \quad (3)
\]

and for B, the poor nation, expected utility is

\[
EU_B^{FN} = \left(1 - \frac{1}{2}\right) U\left(C_H\right) + \frac{1}{2} U\left(C_L\right) \quad (4)
\]

Note that existence of a positive migration cost prevents income levels from equalizing across nations. Original residents of a nation in state of nature H...
end up with an income level greater than the one obtained by agents from
the other nation. And, clearly, all agents from the nation with the state of
nature $L$ obtain the same net income, i.e. the ones that migrate to the rich
nation obtain the same (net) income as the ones that do not migrate. In the
extreme case of no migration costs, $c = 0$, the income of all agents would be
equalized and a federation would coincide with a union of nations.

3.4 Federation of Regions

Now we suppose that the decision makers, or players, are the regions. The
agreement to be analyzed here is the federation of the all four regions (FR).
We don't consider the possibility of a partial federation of two or three re-
gions. It is also important to notice that our analysis of a federation of
regions is not equivalent to the analysis of a federation of four \"smaller\" na-
tions, because the two regions of a nation share a \"national shock\" in their
resources.

Under a federation, agents are free to migrate from one region to another.
There are no transfers so that the vector of total income levels obtained,
after the realization of the national and regional shocks, by the regions is
$fY_{HH};Y_{HL};Y_{LH};Y_{LL}g$. In this case the migration equilibrium is more di±cult
to characterize. The difficulties come from the fact that migration could take
place among any combination of regions.

The next lemma, however, shows that under our assumptions, for all
positive migration costs, migration occurs only from the unlucky region in
the unlucky nation to the lucky region in the lucky nation. In this case the
condition for migration equilibrium is

$$\frac{Y_{HH}}{1-n_r} = \frac{Y_{LL}}{1-n_r}$$

where $n_r$ is the net flow of migrants from the unlucky region in the unlucky
nation to the lucky region in the lucky nation. Denoting by $C_{ij}$ per capita
consumption in the region with state of nature $ij$, we could rewrite the above
expression as $C_{HH} - c = C_{LL}$.

Lemma 1: Let (A.1),(A.2), and (A.3) hold. In the federation of regions the
only equilibrium migration flows occur from the region in state (LL) to the
region in state (HH).
Note that Axiom 1 implies that $C_{HL} = C_{LH}$. Then the expected utility under the federation of regions for the rich region in the rich nation, i.e. region $A1$, is
\[
EU^R_{A1} = \frac{1}{4}pU(C_{HH}) + [(1_i \frac{1}{4}p + \frac{1}{4}1_i \cdot p)]U(C_{HL}) + (1_i \frac{1}{4}(1_i \cdot p)U(C_{LL})
\]
The expected utility for region $A2$ is
\[
EU^R_{A2} = \frac{1}{4}1_i \cdot pU(C_{HH}) + [\frac{1}{4}p + (1_i \frac{1}{4}(1_i \cdot p)]U(C_{HL}) + (1_i \frac{1}{4}pU(C_{LL})
\]
The expected utility for region $B1$ is
\[
EU^R_{B1} = (1_i \frac{1}{4}pU(C_{HH}) + [(1_i \frac{1}{4}(1_i \cdot p) + \frac{1}{4}p]U(C_{HL}) + \frac{1}{4}1_i \cdot pU(C_{LL})
\]
And the expected utility for region $B2$ is
\[
EU^R_{B2} = (1_i \frac{1}{4}(1_i \cdot p]U(C_{HH}) + [(1_i \frac{1}{4}p + \frac{1}{4}1_i \cdot p)]U(C_{HL}) + \frac{1}{4}pU(C_{LL})
\]
One can show that under a federation of regions the expected utility for $A1$ is higher than the expected utility for any other region. More precisely
\[
EU^R_{A1} > EU^R_z; z \in \{A2; B1; B2\}
\]
This is an important inequality which will be used when comparing the federation and the union of regions and it is easily obtained by a standard application of first order stochastic dominance.

4 The National versus the Regional Economy

In this section we start comparing the economy with regions with the economy with nations. The first of our results implies that the lucky region in the lucky nation receives more immigration in the federation of regions (FR) than in the federation of nations (FN). The intuition behind this result is that the dispersion of per capita incomes is larger in the regional economy and that this yields more migration. Recall from condition (5) that $n_r$ is the equilibrium migration flow from the region in state $LL$ to the region in state $HH$ under the arrangement (FR). In the (FN) case, $n_n$, the migration flow from the nation in the bad state of nature to the nation on the good state
of nature, is given by (2). Since a nation always equalizes the income across its regions it is natural to assume that the immigration flow $n_h$ is equally shared by the two regions conforming the nation\textsuperscript{6}. Thus, under the scenario (FN) each region of the lucky nation receives the migration flow $n_h$. Let $c^m$ be the lowest value of the migration cost such that none wants to migrate under the (FN) arrangement. We want to consider cases in which migration flows are positive, so we will assume migration costs lower than $c^m$: We have

Lemma 2: Let (A.1), (A.2) and (A.3) hold. Let $c < c^m$. Then $n_r > \frac{n_h}{2}$.

The result of the previous lemma implies that per capita income for the region in state HH, after the migration flows have taken place, is smaller in the federation of regions than in the federation of nations. That is:

Lemma 3: Let (A.1), (A.2) and (A.3) hold. Let $c < c^m$. Then, i) $C_{hh} < C_h$; ii) $C_{ll} < C_l$.

Our third result regarding the economy with regions is that the expected utility of the richest region, $A_1$, in (FR) is increasing with the difference between its expected income and the expected income of region $A_2$. That is, it is increasing in $p$.

Lemma 4: Let (A.1), (A.2) and (A.3) hold. Let $0 \cdot c < c^m$. Then $E U_{A_1}^{FR}$ is strictly increasing in $p$.

This is very important for our main results as it implies that increasing regional income dispersion increases the expected utility of the richest region in a federation of regions. However, the expected utility of that region in a federation of nations remains constant upon changes in regional income dispersion.

Bucovetsky (1998) shows that, for a degree of relative risk aversion $\gamma > 2$;\textsuperscript{7} the expected utility of a nation under a federation (FN) is a quasi-concave function of the migration cost $c$. The fourth result states that the expected utility of a region under (FR) is also quasi-concave on $c$, for $c \geq 0$. Thus we

\textsuperscript{6}This assumption is introduced to simplify the analysis and the main results of the paper don't depend on it.

\textsuperscript{7}Following Shiller and Athanasoulis (1995), $\gamma = 3$ represents a \textbackslash consensus by many who work in this topic. This is also the average of the estimates reported in Friend and Blume (1975). Therefore assuming $\gamma > 2$ is not an unrealistic assumption.
have:

**Lemma 5**: Let (A.1), (A.2) and (A.3) hold. Let $0 \cdot c < c^m$ and $\gamma > 2$; then $E U^{R}_{A}$ is a quasi-concave function of $c$.

Given that by Lemma 1 there will be no migration in the intermediate regions, the problem is formally similar to the one in Bucovetsky. The proof of this lemma is just a translation to our problem of the proof of Lemma 1 of Bucovetsky.

5 When Will Regions Prefer More Integration than Nations?

We want to analyze under which conditions regions would choose to form a union whereas nations would choose to form a federation. The poor nation, $B$, always prefers a union of nations (UN) to a federation of nations (FN). This is due to the fact that under (UN) each nation gets the per capita income $\frac{1+\sqrt{2}}{2}$ for sure whereas in the (FN) regime $B$ faces a lottery with expected value lower than $\frac{1+\sqrt{2}}{2}$. Nation $A$, however, might or might not prefer the union to the federal regime depending on the value of the different parameters of our economy. It might even be the case that $A$ prefers separation to both (FN) and (FR). To rule out this possibility, i.e. to guarantee what Bucovetsky calls Individual Rationality of the Union, we impose the following condition

(A.4) $\frac{\gamma}{2^l} \leq \frac{1}{1+\frac{1}{2}}$

It is easy to show that the above inequality implies $E U^{UN}_{A} \geq E U^{S}_{A}$ in this case, both nations prefer the Union to Separation.

If nation $A$ prefers (UN) to (FN) (and (A.4) holds so that (UN) is Individually Rational) we conclude that the Union of Nations is a Pareto dominant arrangement and, consequentially, both nations should be in favor of it. If, on the contrary, $A$ prefers (FN) to (UN) then the two nations have different interests and the Union of Nations is less likely to be implemented than in the previous case.

The goal is to characterize for which values of $c$ regions would unanimously agree on forming a Union, while, were nations the players, only the

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$^8$see Bucovetsky (1998).
poor nation would be in favor of the Union. From now on we explicitly write
the expected utility as a function of \( c \) and \( p \), for example we write \( \text{EU}^{F\,N}_A(c) \) and \( \text{EU}^{F\,R}_{Rj}(c, p) \).

Let the migration cost take the value \( c \) and the probability that a rich re-
gion gets the lucky state be \( p \). We write \( \text{UR}(c, p) \geq \text{UN}(c) \) \( \text{UN}(c) \geq \text{UR}(c, p) \) if the following two conditions hold Simultaneously:

i) \( \text{EU}^{F\,N}_A(c) > \text{EU}^U \quad \text{EU}^{F\,N}_A(c) \cdot \text{EU}^U \),

ii) \( \text{EU}^{F\,R}_{Rj}(c, p) \cdot \text{EU}^U; \) for all \( Rj \) \( \text{EU}^{F\,R}_{Rj}(c, p) > \text{EU}^U \) for some \( Rj \).

Given inequality (6), relation \( \geq \) can be simpliﬁed to

\[ \text{EU}^{F\,N}_A(c) > \text{EU}^U, \quad \text{EU}^{F\,R}_{A1}(c, p); \]

We write \( \text{UR}(c, p) \geq \text{UN}(c) \) if

either

\[ \text{EU}^{F\,R}_{Rj}(c, p) \cdot \text{EU}^U \quad \text{for all} \ Rj \ \text{and} \ \text{EU}^{F\,N}_A(c) \cdot \text{EU}^U \]

or

\[ \text{EU}^{F\,R}_{Rj}(c, p) > \text{EU}^U \quad \text{for some} \ Rj \quad \text{and} \quad \text{EU}^{F\,N}_A(c) > \text{EU}^U; \]

Thus \( \text{UR}(c, p) \geq \text{UN}(c) \) means that at \( (c, p) \) a union is Pareto superior to a federal system for the regions, but not for the nations. In the case in which the union is Pareto superior to a federation for the regions and for the nations we write \( \text{UR}(c, p) > \text{UN}(c) \). We also write \( \text{UR}(c, p) > \text{UN}(c) \) to denote the case in which a union is not Pareto superior for either the nations or for the regions. We write \( \text{UR}(c, p) \preceq \text{UN}(c) \) when either \( \geq \) or \( > \) holds or, equivalently, when \( \text{UN}(c) \geq \text{UR}(c, p) \) does not hold. This motivates the following deﬁnition

Deﬁnition: We say that, for a given value of \( p \), a union is more likely to
be implemented when the regions are the players than when the nations are
the players, and write \( \text{UR}(p) \succeq \text{UN}, \text{whenever} \ \text{UR}(c, p) \succeq \text{UN}(c) \) for all \( c \)
(in the opposite case we write \( \text{UN} \succeq \text{UR}(p) \)).

Therefore, when the economy is such that \( \text{UR}(p) \succeq \text{UN} \) we can say that
the regions are more favorable to form a union than nations. Note that

\( ^9 \)The ﬁrst expression doesn’t contain \( p \) since the expected utility for nations is independent of such probability.
UR(p) $\circ$ UN might hold for some, but not all, values of p.

**Proposition:** Let (A.1), (A.2), (A.3) and (A.4) hold. Let $0 \cdot c < c^m$ and $\bar{c} > 2$. Then there exists a level of $p^\circ$; $1=2 \cdot p^\circ < 1$; such that we have UR(p) $\circ$ UN for $p < p^\circ$ and UN $\circ$ UR(p) for $p > p^\circ$. Moreover, if $\frac{1}{1+\gamma} > \frac{1}{2}$ we know that $p^\circ > \frac{1}{2}$. The cut-off value $p^\circ$ is independent of c.

**Sketch of the Proof:** Consider the case in which (UN) is not always a Pareto dominant regime (the general case is analyzed in the formal proof provided in the appendix). In figures 1 and 2, we show expected utilities of the richest nation and the richest region, as function of the migration costs, for the federation regime and the global union. In the formal proof we show that for $p$ close to $1=2$ (case represented in figure 1), expected utility of A1 in the (FR) intersects the expected utility level $EU^U$ at a lower value of c (we call it $c^g$) than the value $c$ at which expected utility of A in the (FN) intersects the level $EU^U$. Therefore, for all $c$ in $(c^g, c)$ we know that $EU^{FN}_{A1}(c) > EU^U$ and $EU^{FR}_{A1}(c, p) > EU^U$ (for $c < c^g$) or $EU^{FN}_{A1}(c) < EU^U$ and $EU^{FR}_{A1}(c, p) < EU^U$ (for $c > c^g$).

We know, by lemma 4, that expected utility of the richest region in (FR) is increasing with $p$. However, expected utility of the richest nation does not depend on $p$. Therefore one could intuitively think that, as $p$ increases, $EU^{FR}_{A1}(c, p)$ shifts and that for some level of $p$ large enough the situation in figure 1 could be reversed, so that for all $c$ in an interval we have $EU^{FN}_{A1}(c) < EU^U \cdot EU^{FR}_{A1}(c, p)$. We show in the formal proof that this reversal happens for $p$ smaller than 1.

Given that $EU^{FR}_{A1}(c, p)$ is strictly increasing in $p$ it is also easy to show that there is a cut-off value $p^\circ$ such that this reversal happens. This cut-off value is the one in figure 2 in which $EU^{FN}_{A1}(c, p^\circ) = EU^{FN}_{A1}(c) = EU^U$ holds for one $c$. From our reasoning, it is easy to see that the cut-off value $p^\circ$ lies between $1=2$ and 1.

Thus when the regions of a nation are similar enough to each other in expected terms (small values of $p$) we claim that a global union is more likely to be achieved when the players are the regions than when the players are the nations. However, if the degree of diversity between regions in a nation is large enough (high values of $p$), then the opposite result is true. This result could be interpreted as that regions have more incentives than
nations to form a full global union when the rich region of a nation is not \"exploited\" by the poor region (p is low). In this case, regional risk-sharing is the main reason for a "scal agreement and the associated cost in terms of redistribution among regions is not that important. When, on the other hand, the rich region is \"exploited\" by the poor region in a nation (high values of p), achieving more integration is easier if nations are the decision makers. Notice that our concept of being \"exploited\" just means that there is more redistribution than risk sharing between regions in a nation.

We cannot exclude the possibility that both nations and regions consider instrumenting transfers (see Bucovetsky (1998) for a good discussion of transfers in this type of economy) that replicate an agreement closer to a union than to a federation. Given that migration consumes resources, in case separation is not preferred to federation and union, there will always be a transfer scheme in which there are enough transfers to prevent any migration and some extra resources (the cost of migration) are distributed among the regions or nations.

Consider the case $c < c < c$ and $p < p$, (a similar argument can be made for $p > p$), and suppose that a union is not always a Pareto dominant regime. Here all regions prefer a unitary agreement to a federation whereas only the poor nation prefers the union to the federation. One can consider the possibility of an intermediate agreement where the two nations form a \"union\" but with some additional transfers. In this case some transfers have to be made from the poor to the rich country (which prefers a federation to a union) and so a \"full union\" would not be achieved. Thus our results are robust to the introduction of additional transfers between nations in that case.

In the case $c < c$ both the poor regions or nations prefer a situation in which a union is achieved even though they have to make a positive transfer to the rich region or to the rich nation, respectively. In that case, the amount of the transfer needed would be negatively correlated with the degree of integration achieved. We may consider the minimum transfer that would make a rich nation to agree in forming a union.

Define $C$ as the solution to $U(C) = E U^{F \not A}$ (i.e. the certainty equivalent to a federation from the point of view of nation A). Then, nation B would need to pay to nation A a \"bribe\" of at least $C$ to convince it to form a union. In that case, nation A is exactly as well off as in a federation.
Then to compare the maximum integration achieved in the economy with nations with the achieved in the economy with regions, we can calculate the certainty equivalents to a federation in each case. The economy with larger certainty equivalent to federation will result in less integration, as it requires a higher bribe. Just looking at figure 1 one can see that which bribe is higher depends on the parameters of the model. For migration costs high enough (though lower than \( c \) one has \( EU_{AN}^f > EU_{AR}^f \) (this is a general result that is provided in the proof of the proposition), so in the economy of nations a higher bribe is needed and consequently less integration is obtained. However, in the example of figure 1 for low enough migration costs, the opposite result would be obtained. This means that for low enough migration costs our results may not be robust to introduction of transfers.

6 Final Comments

We have analyzed the circumstances under which two nations would choose a scal agreement implying less integration than the scal agreement that would have been chosen by the regions forming those nations. Our analysis yields some interesting political conclusions about the role regions versus nations play in the formation of supranational economic areas. The claim that regions have more incentives than nations to form a full global union is likely to be correct when nations are economic stable arrangements, i.e. when \( p \) is low so that the rich region of a nation is not "exploited" by the poor region. In this case, regional risk-sharing is the main reason for a scal agreement and the associated cost in terms of redistribution among regions is not that important. When, on the other hand, it is not on the interest of a rich region to be part of a nation (high values of \( p \)), achieving a full supranational union will be easier if nations are the decision makers.

An interesting situation that has not been explicitly analyzed in this paper is the one in which A1, the richest region, prefers separation to a federation of regions and to a union. In this case, it might also happen that region A1 would be better off on its own than as a member of nation A. It is easy to see that this is more likely to happen when \( p \) is very high. Thus, since the union with A2 was not in its interest, one can think of region A1 as been "forced", by non-economic reasons, to be member of nation A. Therefore when the original nations are sustained by non-economic reasons, were their regions
asked, the richest regions could even choose separation to any other "scal agreement. We shouldn't conclude, however, that whenever the nations are more in favor of the union than the regions, i.e. whenever UR(p) \( \leq \) UN happens, the richest region is "exploited" by the poor region, since it is easy to provide numerical examples for which \( p \) is high and the rich region is better off being part of its country than on its own and still UR(p) \( \leq \) UN:

We have not considered the possibility of a partial union or federation of one, two, or three regions. If this type of agreements were considered, a federation or a union of the three richer regions, excluding the poorest, would always be preferred by the three regions to a federation or union of the four regions.

We have used a very simple model with a representative agent in each region. It is true that with heterogeneous agents migration and transfers have very different effects on welfare. It could happen that migration redistributes risk among the population in a nation (or region) in such a way that it is always a worse risk sharing device that the one provided by transfers (see Wildasin, 1995). The analysis of a model with heterogenous agents is left for future research.

7 Appendix

Proof of Lemma 1:

We have to show that no net migration flows occur in regions in states (LH) or (HL) for \( c \geq 0 \). There are two cases in which migration can occur in those regions. The first happens when the migration flow from the region in state (LL) has been so big as to render the per capita income in such a region equal to the per capita income in regions in states (LH) and (HL). Then there could be, consistently with the above condition, some migration from the regions in states (LH) and (HL) to the region in state (HH). This limit case implies that

\[
\frac{Y_{LL}}{1+2} r = \frac{Y_{LH}}{1+2}.
\]

Solving for \( r \), we have that the implied migration flow is

\[
 r = \frac{Y_{LH}}{2Y_{LL}}.
\]
Now we want to solve for the lowest migration cost that would imply no migration from regions in states (LH) and (HL) when the migration flow from region in state LL to region in state HH is $n_r$. Such cost has to solve the equation

$$\frac{Y_{HH}}{1-2} + n_r = \frac{Y_{LL}}{1-2}.$$ 

That is

$$c^a = 2Y_{LL} \left( \frac{Y_{HH}}{2Y_{HH} i Y_{LL} i 1} \right).$$

By substitution of the values for the respective regional productions implied by Axiom 1, we obtain $c^a = 0$.

The second case in which migration could happen to regions in states (LH) and (HL) is when the level of migration to the region in state (HH), $n_r$, has been so big as to equalize per capita income in such a region to the per capita income in regions in states (LH) and (HL). Then there could be some migration from the region in state (LL) to the regions in states (LH) and (HL). This case implies that

$$\frac{Y_{HH}}{1-2} = \frac{Y_{LL}}{1-2}.$$ 

The implied migration flow is

$$n_r = \frac{Y_{HH} i Y_{LL}}{2Y_{LL}}.$$ 

The lowest migration cost that implies no migration to regions in (LH) and (HL) solves

$$\frac{Y_{LL}}{1-2} i n_r = \frac{Y_{LL}}{1-2} i c^{cm}.$$ 

This implies

$$c^{cm} = 2Y_{LL} (1 i \frac{Y_{LL}}{2Y_{HH} i Y_{HH}}).$$

By substitution of the values for the respective regional productions implied by Axiom 1, we obtain $c^{cm} = 0$.

We conclude that for every $c > 0$ no net migration flows will occur in regions in states (LH) and (HL). Q.E.D.
Proof of Lemma 2:
The equilibrium migration under (FN) is given by condition (2)

\[ \frac{\gamma}{1 + \eta} \]

By Lemma 1 the equilibrium migration under (FR) is given by (5)

\[ \frac{(\gamma + \theta)}{1 + \eta} \]

Assuming our particular value of \( \theta = \frac{1}{2} \), we could write this second condition as

\[ \frac{(3\frac{1}{2} + 1)}{1 + 2\eta} \]

By simply comparing the numerators of condition (2) and the modified condition (5), we conclude that \( 2\eta > \eta \). Q.E.D.

Proof of Lemma 3:
i) In our particular economy, the total per-capita income to be distributed between the region in state of nature HH and the region in state of nature LL (and also in nations H and L) is (1 + \( \frac{1}{2} \)). Thus, we have the identity

\[ C_{HH}N_{HH} + C_{LL}(1 - N_{HH}) = (1 + \frac{1}{2}) \]

where 1 is the total population of the two regions and \( N_{HH} \) the percentage of that total living in the region in state of nature HH, 0 \( \cdot \) \( N_{HH} \) \cdot 1. Since \( C_{HH} \cdot c = C_{LL} \) we also have

\[ C_{HH}N_{HH} + (C_{HH} \cdot c)(1 - N_{HH}) = (1 + \frac{1}{2}) \]

so \( C_{HH} = (1 + \frac{1}{2}) + c(1 - N_{HH}) \). The same identity holds for \( C_{H} \), \( C_{L} \) and \( N_{H} \). Thus, per-capita income in the region in state HH increases as the population of the other region increases. That is, it decreases with migration. This implies that in the federation of regions, where migration to the region in state HH is higher than under the federation of nations, the region in state HH ends up with a lower level of per capita income than under the federation of nations. The statement in ii) follows easily from \( C_{HH} \cdot c = C_{LL} \).
Q.E.D.

Proof of Lemma 4:

\[
\frac{dE_{\text{A}1}^{FR}}{dp} = \frac{1}{2}U(C_{HH}) + (1 \cdot 2\frac{1}{2}U(C_{HL}))_i (1 \cdot \frac{1}{2}U(C_{LL}))
\]

Simplifying the right-hand side, and recalling from Lemma 1 that 

\[C_{HH} > C_{HL} > C_{LL}\]

we can conclude that

\[
\frac{dE_{\text{A}1}^{FR}}{dp} > \frac{1}{2}U(C_{HH}) \cdot \frac{1}{2}U(C_{HL}) > 0
\]

as we wanted to show. Q.E.D.

Proof of Lemma 5:

From the definitions of \( E_{\text{A}1}^{FR}, C_{HH}, C_{HL} \) and \( C_{LL} \)

\[
\frac{\partial E_{\text{A}1}^{FR}}{\partial c} = n_0 \frac{1}{i} pU^q(C_{HH}) \frac{Y_{HH}}{\left(\frac{1}{2} + n_r\right)^2} + (1 \cdot \frac{1}{2}(1 \cdot p)U^q(C_{LL}) \frac{Y_{LL}}{\left(\frac{1}{2} \cdot i \cdot n_r\right)^2}
\]

Differentiating again with respect to \( c \)

\[
\frac{\partial^2 E_{\text{A}1}^{FR}}{\partial c^2} = \frac{\partial E_{\text{A}1}^{FR}}{\partial c} \cdot n_0 \cdot \frac{1}{i} pU^q(C_{HH}) \frac{Y_{HH}^2}{\left(\frac{1}{2} + n_r\right)^4} + (1 \cdot \frac{1}{2}(1 \cdot p)U^q(C_{LL}) \frac{Y_{LL}^2}{\left(\frac{1}{2} \cdot i \cdot n_r\right)^4}
\]

from the fact that \( \bar{v} = \frac{U^q(x)}{U^q(c)} \) for all \( x \)

\[
\frac{\partial^2 E_{\text{A}1}^{FR}}{\partial c^2} = \frac{\partial E_{\text{A}1}^{FR}}{\partial c} \cdot n_0 \cdot \frac{1}{i} pU^q(C_{HH}) \frac{C_{HH}}{\left(\frac{1}{2} + n_r\right)^2} + (1 \cdot \frac{1}{2}(1 \cdot p)U^q(C_{LL}) \frac{C_{LL}}{\left(\frac{1}{2} \cdot i \cdot n_r\right)^2}
\]

By assumption \( \bar{v} > 2 \), hence \( \frac{\partial E_{\text{A}1}^{FR}(c)}{\partial c} \cdot 0 \) at any \( c \) such that \( \frac{\partial E_{\text{A}1}^{FR}(c)}{\partial c} = 0 \).

Q.E.D.
Proof of the Proposition:

If the value of $\frac{1}{2}$ is too large it might happen that, for all positive values of the migration cost $c$, a federation is a worse insurance device for nation $A$ than a union so that $EU^U_A < EU^F_N$. The following condition implies that there exists values of $c$ for which such inequality is not true so that (UN) is not always a Pareto dominant regime$^{10}$,

$$\frac{1}{4} \leq \frac{1}{4}, \frac{1}{2}$$

Thus we will divide the proof of the theorem in two cases. The first will be the one in which this condition is satisfied. The second will consider the case in which (UN) is always Pareto dominant for the nations.

1) Assume $\frac{1}{4} \leq \frac{1}{4}, \frac{1}{2}$ is satisfied.

We will show that for small values of $p$, $UR(p) \geq UN$ is satisfied whereas for large values of $p$, $UN \geq UR(p)$ is satisfied. Then a continuity argument will close the proof.

1.a) Consider the extreme case with $p = 1 = 2$:

We first show that there exists a $c$ and a $\xi$ so that for every $c > (\xi, c)$ we have $UR(c; \frac{1}{2}) \geq UN(c)$: Recall that $EU^U = U(1 + \frac{1}{2})$: Thus $UR(c; \frac{1}{2}) \geq UN(c)$ if the inequalities

$$EU^F_N(c) > U(1 + \frac{1}{2})$$

and

$$EU^F_{Rj}(c; \frac{1}{2}) \cdot U(1 + \frac{1}{2})$$

hold simultaneously.

By the inequalities given in (6) we know that (8) is equivalent to

$$EU^F_{A1}(c; \frac{1}{2}) \cdot U(1 + \frac{1}{2})$$

Let $\xi > 0$ be such that

$$EU^F_A(\xi) = U(1 + \frac{1}{2})$$

$^{10}$See Bucovetsky for the proof of this claim.
Existence of $c$ follows from: a) $EU^F_A(0) = EU^U$; b) for $c^m$ we have $EU^F_A(c^m) = EU^S_A$; c) by (A.4) $EU^U > EU^S_A$; d) by continuity of $EU^F_A(c)$ and; e) since $\frac{\nu}{1-\gamma} > \frac{1}{2}$ there exists $e$ such that $EU^F_A(e) > EU^U$ (see figure 1).

If such $e$ is not unique take the infimum of them. Also realize that condition $\frac{\nu}{1-\gamma} > \frac{1}{2}$ guarantees that $e > 0$.

We can write

$$EU^F_A(c) = \frac{1}{2}U(C_H) + \frac{1}{2}U(C_L)$$

where $C_H$ and $C_L$ are the equilibrium per capita income levels when the migration cost is $c$. For that $c$ and for $p = \frac{1}{2}$ we write the expected utility for region $A$ in (FR) as

$$EU^F_{A1}(c, \frac{1}{2}) = \frac{1}{2}U(\frac{1 + \frac{\nu}{2}}{2}) + \frac{1}{2}U(C_{HH}) + (1 - \frac{1}{2})U(C_{LL})$$

so that inequality $(8')$ holds, for $c$ and $p = \frac{1}{2}$ if

$$\frac{1}{2}U(C_{HH}) + (1 - \frac{1}{2})U(C_{LL}) < \frac{1}{2}U(\frac{1 + \frac{\nu}{2}}{2})$$

Lemma 3 states that $C_L > C_{LL}$ and $C_H > C_{HH}$. It follows that

$$\frac{1}{2}U(C_{HH}) + (1 - \frac{1}{2})U(C_{LL}) < \frac{1}{2}[\frac{1}{2}U(C_H) + (1 - \frac{1}{2})U(C_L)]$$

and from (9) and (10) we have

$$\frac{1}{2}[\frac{1}{2}U(C_H) + (1 - \frac{1}{2})U(C_L)] = \frac{1}{2}U(\frac{1 + \frac{\nu}{2}}{2})$$

Thus (14) and (13) imply that (12) holds and, as a consequence, $(8')$ and (8) also hold for $c$ and $p = \frac{1}{2}$.

By continuity of $EU^F_{A1}(c, \frac{1}{2})$, for a set of values of $c$ smaller than $e$ we have that (8) also holds. It only rests to show that for values of $c$ close enough to $c$ and $c < c$ we have that (7) is true, i.e. $EU^F_A(c) > U(\frac{1 + \frac{\nu}{2}}{2})$. Since $EU^F_A(c)$ is continuous at $c$ we only need to show that $EU^F_A(c)$ is decreasing at $c$. But this follows from observations a)-e) above. Summing up: we have shown that there exists a set $(c^0, e)$ of values of $c$ that satisfy (7) and (8) simultaneously, i.e. for all $c \in (c^0, e)$ we have $UR(c, \frac{1}{2}) \geq UN(c)$. 

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Let $c$ be defined as follows

$$c = \inf \{ E U_{A_1}^R (c; \frac{1}{2}) = U(\frac{1 + \frac{1}{2}}{2}) \}$$

(15)

By the argument given above we know that $E U_{A_1}^R (c; \frac{1}{2}) < U(\frac{1 + \frac{1}{2}}{2})$ and we also have that $E U_{A_1}^R (0; \frac{1}{2}) = U(\frac{1 + \frac{1}{2}}{2})$: Then, quasi-concavity of $E U_{A_1}^R (c; \frac{1}{2})$ implies that $c < c$: Quasi-concavity also implies that for all $c 2 (c; c)$ we have $E U_{A_1}^R (c; \frac{1}{2}) \cdot U(\frac{1 + \frac{1}{2}}{2})$: Remember that we already showed that $E U_{A}^N (c) > U(\frac{1 + \frac{1}{2}}{2})$ for all $0 < c < c$: Therefore we have $UR(c; \frac{1}{2}) \lhd UN(c)$ for all $c 2 (c; c)$

Next, we show that $UR(c; \frac{1}{2}) > UN(c)$ for all $2 (c; c)$: Quasi-concavity of the functions $E U_{A_1}^R (c; \frac{1}{2})$ and $E U_{A}^N (c)$ and the fact that both of them are decreasing at $c$ imply that $E U_{A_1}^R (c; \frac{1}{2}) < U(\frac{1 + \frac{1}{2}}{2})$ for all $c > c$ and $E U_{A}^N (c) \cdot U(\frac{1 + \frac{1}{2}}{2})$ for all $c > c$: In this case a union is Pareto efficient for both the regions and the nations. Hence we have $UR(c; \frac{1}{2}) \lhd UN(c)$ for all $c > c$.

The definition of $c$ and the equalities $E U_{A_1}^R (0; \frac{1}{2}) = E U_{A}^N (0) = U(\frac{1 + \frac{1}{2}}{2})$ and quasi-concavity of these functions imply that $E U_{A_1}^R (c; \frac{1}{2}) > U(\frac{1 + \frac{1}{2}}{2})$ for all $c < c$ and $E U_{A}^N (c) > U(\frac{1 + \frac{1}{2}}{2})$ for all $c < c$: Hence, a union is not Pareto superior to a federation neither for the regions nor for the nations and we have $UR(c; \frac{1}{2}) \lhd UN(c)$ for all $c < c$.

A continuity argument can be used to show that for some values of $p > \frac{1}{2}$ we still have $UR(p) \lhd UN$.

1.b) Now consider the limit case in which $p = 1$. We have

$$E U_{A_1}^R (c; 1) = \frac{1}{2} U(C_{HH}) + (1 - \frac{1}{2}) U(C_{HL})$$

By the reasoning in Lemma 1, we know that $C_{HH} > C_{HL} = \frac{1 + \frac{1}{2}}{2}$. So trivially $E U_{A_1}^R (c; 1) > E U$ for every $c > 0$. Thus, when $p = 1$, a union is never Pareto superior to a federation for the regions. However $E U_{A}^N (c)$ is independent of $p$, and we know that $E U_{A}^N (c) \cdot E U$ for all $c > c$ and $E U_{A}^N (c) > E U$ for all $c < c$ where the value $c$ is the one given in part 1.a) above.

1.c) $E U_{A_1}^R$ is continuous in $p$ and, by Lemma 4, is also strictly increasing in $p$. Then, existence of $p^* > \frac{1}{2}$ easily follows. It is also easy to check that $p^*$ is such that $E U_{A_1}^R (c; p^*) = E U_{A}^N (c) = E U$ (see Figure 2).
2) Assume now that (UN) is always Pareto dominant for the nations.

For \( c = 0 \) a union will give the same expected utility as that of any kind of federation. Therefore, for costless migration \( E\ U_{\alpha_1}^{FR}(0; p) = E\ U_{\alpha_1}^{FN}(0) = E\ U^U \). Since \( E\ U_{\alpha_1}^{FN}(c) \cdot E\ U^U \) for all \( c \geq 0 \), quasi-concavity of \( E\ U_{\alpha_1}^{FN}(c) \) implies that \( E\ U_{\alpha_1}^{FR}(c; p) \) is increasing (decreasing) in \( c \) at \( c = 0 \); by quasi-concavity, we have \( UN \supset UR(p) \) \( (UN \supset UR(p)) \). Thus, we need to show that there exists \( p^* \) such that for all \( p < p^* \) the function \( E\ U_{\alpha_1}^{FR}(0; p) \) is increasing in \( c \): From the definitions of \( E\ U_{\alpha_1}^{FR}, C_{HH}; C_{HL} \) and \( C_{LL} \);

\[
\frac{\partial E\ U_{\alpha_1}^{FR}(0; p)}{\partial c} = n^0_r(c) [\frac{1}{2} p U^Q(C_{HH}) + (1 - \frac{1}{4} p) U^Q(C_{LL})] + \frac{Y_{HH}}{2 + n_r} \frac{1}{2} + \frac{Y_{LL}}{2} \frac{1}{2} n_r
\]

However when \( c = 0 \) we know that free migration leads to

\[
C_{HH} = \frac{Y_{HH}}{2 + n_r} = \frac{Y_{LL}}{2} \frac{1}{2} n_r = C_{LL}
\]

This implies that we could write

\[
\frac{\partial E\ U_{\alpha_1}^{FR}(0; p)}{\partial c} = n^0_r(c) U^Q(C_{HH}) C_{HH} [\frac{1}{2} + \frac{1}{4} p] + (1 - \frac{1}{4} p) U^Q(C_{LL})] + \frac{Y_{HH}}{2 + n_r} \frac{1}{2} + \frac{Y_{LL}}{2} \frac{1}{2} n_r
\]

Given \( A.1 \), the implied migration flow is

\[
n_r = \frac{2^{1/4} \frac{1}{2} - 2}{2^{1/2} - 2}
\]

Consider the case where \( p = 1 \):

\[
\frac{\partial E\ U_{\alpha_1}^{FR}(0; 1)}{\partial c} = n^0_r(c) U^Q(C_{HH}) C_{HH} [\frac{1}{2} + \frac{1}{4} p]
\]

Given that \( n^0_r(c) < 0 \), this derivative is always positive. Now consider the extreme case where \( p = 1 = 2 \), so

\[
\frac{\partial E\ U_{\alpha_1}^{FR}(0; \frac{1}{2})}{\partial c} = n^0_r(c) U^Q(C_{HH}) C_{HH} [\frac{1}{2} + \frac{1}{4} p] + (1 - \frac{1}{4}) \frac{1}{4}
\]

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The sign of the above derivative is not determined. However, we know that
\[
\frac{\bar{\mathbb{E}} U^{F}_A(0;1)}{\bar{c}} > \frac{\bar{\mathbb{E}} U^{F}_A(0;\frac{1}{2})}{\bar{c}}.
\]
Lemma 4, and that fact that the utility function is continuous as a function of \( p \) and \( c \) are enough to guarantee the existence of a \( p^* \) such that: i) if \( \frac{\bar{\mathbb{E}} U^{F}_A(0;\frac{1}{2})}{\bar{c}} \geq 0 \) we have \( p^* = \frac{1}{2} \) and \( U^E \preceq U^R(p) \) for all \( p \); ii) if \( \frac{\bar{\mathbb{E}} U^{F}_A(0;\frac{1}{2})}{\bar{c}} < 0 \) then \( p^* > \frac{1}{2} \) and for \( p \cdot p^* \) we have \( U^R(p) \succ U^E \) and for all \( p > p^* \) we have \( U^R(p) \preceq U^E \). Q.E.D.
Figure 1
Figure 2
8 References


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