EQUILIBRIUM DISTRIBUTION SYSTEMS UNDER RETAILERS’ STRATEGIC BEHAVIOUR

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ABSTRACT

This paper investigates what are the equilibrium distribution systems in a successive duopoly when retailers hold the power to choose the number of products they wish to market. Since they both can be multi-product sellers, the number of possible channel structures considered is larger than in previous work. Then, we study whether the resulting distribution systems obtained in earlier papers still remain. In particular, whether there are incentives to adopt exclusive distribution agreements, whether a manufacturer is foreclosed from the market and, essentially, whether there exists, at equilibrium, enough inter and intra-brand competition.

The analysis shows that provided low brand asymmetry, it is sufficient that retailers hold the power to choose the number of products they wish to distribute to obtain endogenously both inter and intra-brand competition; both retailers become multi-product sellers. However, as the profitability of brands diverges sufficiently, only the most profitable brand will be distributed by both retailers thus only arising intra-brand competition at equilibrium. Neither the exclusive distribution system nor a common distribution system analyzed in the previous literature appears at equilibrium.

Keywords: Distribution systems, retailer power.
J.E.L. Classification: L19, L42.
1 Introduction

Large chains such as Carrefour, Wal-Mart, Métro AG, Kroger and Intermarché, have considerably increased their market shares. On the one hand, concentration in the retail distribution industry seems unstoppable. On the other, private label sales are growing impressively at the expense of competing national manufacturer brands in large stores. This clearly confers retailers a stronger position vis-à-vis manufacturers in their effort to capture strategic rents. Provided that shelf space is limited, retailers may set manufacturers against each other by threatening not to carry their products. However, there remains the question whether such behaviour, which will probably translate into better terms of payment, leads to sufficient market competition.¹

This paper investigates what are the equilibrium distribution systems in a successive duopoly when retailers hold the power to choose the number of products they wish to market. It is retailers that fix their product line and since they both can be multi-product sellers, and intra-brand competition is allowed, the number of possible channel structures considered is larger than in previous work on this area. In this framework we wish to study whether the resulting distribution systems obtained in earlier papers still remain. In particular, whether there are incentives to adopt exclusive distribution agreements, whether a manufacturer is foreclosed from the market and, essentially, whether there exists, at equilibrium, enough inter and intra-brand competition.

Typically, retailers are assumed to set final prices - depending on the contractual clauses - and are bound to accept the contracts designed by the manufacturers.² Some recent contributions do consider more decision power

¹See e.g. Mills (1995) and Raju et al. (1995). Carrefour is the result of a recent merger between Carrefour and Promodes, and there are ongoing mergers of smaller size both within and across countries in the European Union. A recent headline appeared in El País (26th September '99) read: "Large distribution chains impose their conditions on suppliers", which translates into a delay in payments from retailers to manufacturers.

²Representative papers in the literature on distribution systems include Bernheim and Whinston (1985, 1998), Bonanno and Vickers (1988), Lin (1990), O’Brien and Shaffer (1993), Besanko and Perry (1994), and Rey and Stiglitz (1995), only to mention a few. Two distribution structures are studied: an exclusive distribution system and a common distribution system.
on the retailer’s side. The retailer in Bernheim and Whinston (1998) and in the common subgame in O’Brien and Shaffer (1993) chooses to represent one of the manufacturers, both of them or neither of them. The implications of more retailer power are more specifically studied by Gabrielsen and Sorgard (1999a) and by Shaffer (1991). In both these papers there is a stage at which the retailer decides how many products to carry. The former authors consider a three-stage game played by two independent manufacturers and a retailer who, in the first stage, decides whether manufacturers’ offers should be exclusive dealing offers. This decision has a commitment value in that, without commitment, the retailer would always carry both products. Shaffer (1991) analyzes a two-product monopolist who sells to one retailer. Retailer power stems from two facts: shelf space is scarce and the retailer’s decision on how many products to stock. This allows him to gain some strategic rent which is dissipated when the manufacturer imposes contractual (vertical) restraints such as maximum resale price maintenance, full-line forcing or brand discounts.

In contrast, we assume a duopolistic retailer structure which brings into the model a further element of competition, that is, intra-brand rivalry. The retailers can be multi-product sellers and this enlarges the set of possible equilibrium distribution systems. Besides, and to concentrate on the strategic motives behind retailers’ decisions on product line we do not allow manufacturers to impose any type of vertical restraints. These assumptions precisely aim at emphasizing retailer power rather than manufacturer power.\(^3\)

More specifically, the model we propose assumes two differentiated manufacturers which are asymmetric because they are differently valued by consumers, and two potential retailers who play a non-cooperative multi-stage game. In the first stage, manufacturers simultaneously and independently propose a contract to the retailers. The contract only specifies the terms of payment, a linear transfer price. In the second stage, retailers are given the

\(^3\)Another way of looking into the incentives for firms to enter into exclusive trading relations is to consider mutual vertical agreements by each manufacturer-retailer pairing. This is studied by Chang (1992) and by Dobson and Waterson (1997). In equilibrium, each manufacturer only supplies one retailer for the homogeneous good case, as in Chang (1992), or does it for low levels of product and retailer differentiation, as in Dobson and Waterson (1997).
power to decide whether they wish to be supplied by one manufacturer, by both or by none of them. Finally, and given the inherited outcome of the first two stages, retailers compete à la Cournot. Thus, we build a model where the existence of both inter and intra-brand competition is endogenously derived when retailer power is important.\footnote{As reported in Gabrielsen and Sorgard (1999a), large retailers use to invite a restricted number of suppliers to make their offers (possibly including exclusive dealership). Then, the retailers decide which offer to accept. The contracts have a limited duration at the end of which the "auction" is repeated. A similar well-known practice is employed by Spanish retailer chain Mercadona with its "always-low-prices" policy.}

The analysis shows that, despite the fact that competition between retailers is intense (since they are not differentiated), and provided low brand asymmetry, it is sufficient that retailers hold the power to choose the number of products they wish to distribute to obtain endogenously both inter and intra-brand competition; both retailers become multi-product sellers. However, as the profitability of brands diverges sufficiently, only the most profitable brand will be distributed by both retailers thus only arising intra-brand competition at equilibrium. Also, our findings suggest that the assumption of more retailer power is not irrelevant provided that the well-known exclusive and common distribution systems are among the choice set and do not arise at equilibrium. A natural question to ask is whether the retailers’ equilibrium choice of distribution systems would be selected when it is manufacturers who choose the number of retailers they wish to employ. Suppose that manufacturers choose simultaneously and independently whether to employ none, retailer one, retailer two, or both. In the second stage, they decide upon transfer prices and finally retailers compete à la Cournot. It can be shown that, the retailers’ equilibrium choice is not always an equilibrium for manufacturers had they the option to choose the distribution system. What our analysis highlights is that there is a conflict between manufacturers’ and retailers’ choices and it does matter who are the agents with more power in the trading relationship.

The analysis of how manufacturers and retailers organize their distribution systems is part of the literature on vertical restraints. Also, it is an issue with great interest for anti-trust authorities. Recently, the European approach to vertical restraints changed. There is a new Block Exemption
Regulation\textsuperscript{5} which replace the three old Block Exemption Regulations applicable to exclusive distribution, exclusive purchasing and franchising agreements respectively.\textsuperscript{6} In words of Commissioner Mario Monti, "the Commission aim with the new regulation is to simplify our rules and reduce the regulatory burden for companies, while ensuring a more effective control of vertical restraints implemented by companies holding a significant market power".\textsuperscript{7} The new Block Exemption Regulation allows companies, whose market share is below 30\% to benefit from a so-called safe harbour under the Community competition rules.\textsuperscript{8} The safe harbour offers companies the freedom to create supply and distribution arrangements best suited to their individual interests. However, the Block Exemption does not apply to two sets of restrictions. The first set concerns the so-called hard-core restrictions which companies are not allowed to use in their agreements. \textsuperscript{9}And more relevant for our purposes, the second set of restrictions not covered by the new Regulation concerns certain restrictions which are not exempted


\textsuperscript{7}The previous Green Paper on Vertical Restraints by the European Commission (January '97) and a follow-up to the Green Paper (dated September '98) were the basis for the new approach to vertical restraints embodied in the new regulation. These Communications precisely incorporate some of the ideas expressed by Caballero-Sanz and Rey (1996) and Dobson and Waterson (1996) which shift the emphasis from the regulatory approach underlying the old legislation towards a more economic approach in the assessment of vertical restraints.

\textsuperscript{8}To highlight the interest of our paper and note that there is still work to be done, the following statement can be read in the follow-up to the Green Paper: "In general, it will only be necessary to estimate the market share of the supplier. However, in cases of exclusive supply the market share of the buyer may have to be used as the relevant indicator. The guidelines will address the issue of how the Commission will take account of the buyer’s market position in the analysis of individual cases".

\textsuperscript{9}In particular: a producer may not impose on its distributors at which price to resell its products; a producer may not restrict its distributors selling to any customer if it is an unsolicited order (passive sales); a producer applying a selective distribution system, for instance in cosmetics, may neither restrict active nor passive selling by the authorised distributors to end-users or other authorised distributors; a producer buying components for incorporation in its own products may not prevent the supplier of the components from selling these as spare parts to end-users or independent repairers.
but which may under certain circumstances nonetheless be compatible with the EC competition rules. The most important concerns exclusive dealing and its variations (selective distribution, exclusive purchasing) when their duration exceeds five years. Above the 30% market share threshold, vertical agreements will not be covered by the new Block Exemption, but they are not automatically presumed to be illegal either. They may require an individual examination under Article 81 of the Treaty. Therefore, the questions addressed here are of immediate interest from an anti-trust perspective and, with the necessary qualifications, some policy implications can be extracted.

Our analysis has focused on how, in the presence of retailer decision power, some vertical restraints such as exclusive dealing would never be accepted. Note that the various types of competition intensity vary depending on the agent who holds more power on trading relationships. We will show that vertical agreements in which retailers are decisive in determining distribution systems ensure sufficient inter and intra-brand competition. In other words, and to sum up, only will exclusive dealership or vertical foreclosure appear if a) either the manufacturers are the agents who effectively impose exclusivity clauses, b) or it has jointly been agreed by manufacturers and retailers, c) or in the presence of more retailer power and sufficient brand asymmetry.

2 The Model

We set up a three-stage non-cooperative game to study the equilibrium distribution structure that will arise by the strategic decisions of two manufacturers (\(M_1\) and \(M_2\)) and two retailers (\(R_1\) and \(R_2\)). Each manufacturer produces a differentiated good that can be distributed by either one or two retailers, or not distributed at all. In the first stage of the game, the manufacturers choose and announce simultaneously and independently the transfer prices to retailers (\(w_1\) and \(w_2\)). In the second stage, the retailers, having observed the manufacturers’ choice in the first stage, decide simultaneously and independently with which manufacturer (possibly both or none) they wish to trade. Finally, and given the inherited outcome of the previous two stages, the retailers choose simultaneously and independently the quantities of each of the goods they will sell to consumers. We assume that the manufacturers
cannot enforce a given distribution structure by including clauses in the contract. In other words, the equilibrium distribution structure is the outcome of the strategic interaction between retailers given the terms of the contracts offered by the manufacturers.

More specifically, the two manufacturers, $M_1$ and $M_2$, produce their own branded good under constant returns to scale and incurring a common unit cost $c$. The retailers, $R_1$ and $R_2$, are supplied by the manufacturers at a constant unit price, the transfer price. Let $w_i$ denote the transfer price set by manufacturer $i$. Then, each retailer $k$, having observed those transfer prices, chooses the manufacturer(s) with which he wants to trade. Each retailer $k$ chooses simultaneously and independently one element, $s_k$, from the set $S_k = \{0, 1, 2, 12\} \ k = 1, 2$, where $s_k = 0$ denotes not to deal, $s_k = 1$ denotes that the retailer will deal with $M_1$, likewise for $s_k = 2$, and finally $s_k = 12$ means that the retailer will deal with both manufacturers. Then, sixteen different distribution schemes may result from the retailers’ strategic choice of brands. Finally, each retailer selects the quantity for each branded good they have decided to deal, denoting by $q_{ik}$ the quantity of brand $i$ that retailer $k$ sells to consumers. As the brand produced by each of the manufacturers can be sold by one, both or none of the retailers, consumers, at least initially, would be able to distinguish between brands and the place where they are sold. However, we assume that retailers are not differentiated in the sense that consumers get for brand $i$ the same utility no matter which retailer $k$ is selling the brand $i$ to them. To close notation, $Q_i$ stands for the total amount of brand $i$ produced and distributed to the retailers which is $Q_i = q_{i1} + q_{i2}$ when both retailers distribute it. The relevant element(s) of that sum are set to zero, according to the retailers’ choice of distribution system for brand $i$. As well as paying the transfers, the retailers incur common retailing costs at constant per unit level $r$ which, for the sake of the exposition and without loss of generality, are assumed to be zero.

The retailers face a continuum of consumers of the same type. The representative consumer maximizes $U(Q_1, Q_2, y)$ subject to the budget constraint $I = y + p_1Q_1 + p_2Q_2$, where $I$ is the income, $y$ is the quantity of the numeraire commodity consumed and $Q_i, p_i, i = 1, 2$, are the quantity of the brand produced by the manufacturer $i$ and its market price, respectively. The function $U$ is assumed to be separable, linear in the numeraire
commodity and quadratic and strictly concave in the differentiated good:
\[ U = y + a_1 Q_1 + a_2 Q_2 - [b(Q_1^2 + Q_2^2) + 2dQ_1Q_2]/2, \]
where \( a_i, i = 1, 2, \) \( b \) and \( d \) are positive, \( b^2 > d^2 \) and \( a_i b - a_j d > 0 \) for \( i \neq j \). This utility function gives rise to a linear demand schedule, where inverse demands are given by,

\[
\begin{align*}
p_1 &= a_1 - bQ_1 - dQ_2 \\
p_2 &= a_2 - bQ_2 - dQ_1
\end{align*}
\]

We assume, without loss of generality, that \( a_1 > a_2 \) meaning that the highest price (when quantities are set to zero) consumers are willing to pay for the good produced by \( M_1 \) is greater than for the one produced by \( M_2 \). Also, since \( d > 0 \) and \( b > d \), own effects on prices are greater than cross effects. Note that the distance between \( b \) and \( d \) is measuring the degree of inter-brand rivalry, that is, how similar the brands are perceived by consumers. Then, brands 1 and 2 are imperfect substitutes and when \( d \) approaches \( b \) brands become closer substitutes, this meaning that inter-brand rivalry increases. Intra-brand rivalry, that is how similar the retailers’ services are perceived by consumers to be when selling the same brand, is maximal. They are perfect substitutes: retailers are not differentiated.

We begin by computing the Nash equilibrium quantities for each possible distribution scheme inherited from the second stage. Then, using the Nash equilibrium quantities, we compute the retailers’ equilibrium distribution choice (either Nash or in dominant strategies) given the transfer prices. Finally, we find the Nash equilibrium choice of transfer prices by manufacturers.

2.1 The retailers’ decisions on quantities

Given the symmetry between retailers, 10 different distribution schemes out of 16 need to be analyzed. Computations are relegated to Appendix A. In this subsection, we present the equilibrium quantities and gross profits (leaving aside any fixed costs) for each of the distribution schemes.\(^{10} \) Denote by

\(^{10}\)Remark that in the Table in the Appendix whenever two distribution schemes are reported, the equilibrium quantities and gross profits follow by a simple exchange of subindices.
and $q_k(s_1, s_2)$, $k = 1, 2$ the retailer $k$’s equilibrium gross profits and total quantity, respectively, when $R_1$ has chosen the action $s_1 \in S_1$ and $R_2$ has chosen the action $s_2 \in S_2$.

The equilibrium quantities are a function of earlier choices, $w_1$ and $w_2$, and the distribution schemes. The quantities corresponding to some distribution systems may become negative depending on the size of $(a_1 - w_1)/(a_2 - w_2)$, the relative per unit profitability of brands for retailers, and then the corner solutions have to be taken. It is therefore convenient to distinguish the different intervals displayed in figure 1.

Thus, whenever the ratio belongs to interval $I_1 \equiv \left[ \frac{3bd}{2b^2 + d^2}, \frac{3b^2 + d^2}{3bd} \right]$ all the quantities corresponding to every distribution system are strictly positive. The table in Appendix A shows them along with the corresponding pay-offs. As the ratio decreases (either $w_1$ goes up or $w_2$ goes down) brand 1 becomes relatively more expensive than brand 2, and this implies that its distribution does not pay retailers for some distribution systems. In the interval $I_2 \equiv \left[ \frac{d}{b}, \frac{3bd}{2b^2 + d^2} \right]$ quantity $q_{11}(12, 1)$ and $q_{12}(1, 12)$ are set to zero in the subgames $(12, 1)$ and $(1, 12)$, respectively. Consequently, the second stage retailers’ choice of $(12, 1)$ and $(1, 12)$ is payoff equivalent to choosing $(2, 1)$ and $(1, 2)$, respectively. For $I_3 \equiv \left[ \frac{d}{b}, \frac{d}{b} \right]$ the following quantities are zero: $q_{11}(12, 1)$, $q_{12}(1, 12)$, $q_{11}(12, 2)$, $q_{12}(2, 12)$, $q_{11}(12, 12)$, $q_{12}(12, 12)$, $q_{11}(12, 0)$ and $q_{12}(0, 12)$. In words, brand 1 is never jointly distributed with brand 2 by the same retailer. Consequently, the choice of strategy 12 is payoff equivalent to the choice of strategy 2 in stage two. Finally, $I_4 \equiv \left[ 0, \frac{d}{b} \right]$ supposes that, in addition to the above quantities, $q_{11}(1, 2)$ and $q_{12}(2, 1)$ are set to zero. Hence, for every distribution system where brand 1 competes with brand 2 the latter remains as the only brand in the market.

Alternatively, when the ratio $(a_1 - w_1)/(a_2 - w_2)$ increases brand 1 becomes relatively cheaper than brand 2, and this implies that the distribution of brand 2 does not pay retailers for some distribution systems. A similar reasoning as above can be applied by exchanging 1 by 2. This gives rise to intervals $I_5 \equiv \left( \frac{2b^2 + d^2}{3bd - d^2}, \frac{b}{d} \right]$, $I_6 \equiv \left( \frac{d}{b}, \frac{b}{d} \right]$ and $I_7$ for $\frac{a_1 - w_1}{a_2 - w_2} > \frac{2b}{d}$. The equilibrium quantities for intervals $I_5$ to $I_7$ follow by applying the above reasoning to those reported in the Table in Appendix A.
2.2 The retailers’ choice of distribution schemes

This is the central stage of the game and it requires the introduction of some useful terminology. In particular, the term "distribution" has to do with the (number of) channels employed by manufacturers, whereas the word "purchasing" is used to refer to the (number of) brands that retailers wish to sell. In the second stage of the game, each retailer \( k \) decides simultaneously and independently the action \( s_k \) that maximizes his payoffs taking as given the transfer prices. The possible combination of actions gives rise to the following distribution schemes:

- **Non-exclusive distribution and exclusive purchasing** (Figure 2a): both retailers distribute one of the manufacturer’s branded product whereas the rival manufacturer is not present in the market. It refers to the above mentioned schemes \((1,1)\) and \((2,2)\). This distribution scheme is equivalent to a homogeneous product duopoly and then, only intra-brand rivalry appears in the market. Besides, this situation can be understood as one with vertical foreclosure of one of the manufacturers.

- **Duopoly exclusive distribution and purchasing** (Figure 2b): each retailer purchases only one brand and each manufacturer uses just one retailer. It refers to the distribution schemes \((1,2)\) and \((2,1)\). This distribution scheme is equivalent to a differentiated duopoly and therefore there is only inter-brand rivalry, and it has been usually called exclusive dealing by papers in the literature. Further note that it is related
to those papers invoking the no-intra-brand-competition in retailing (NICR) assumption.

• **Non-exclusive distribution and purchasing** (Figure 2c): the two retailers purchase both brands. It relates to the distribution scheme (12, 12). Therefore, both retailers are multi-product dealers. Furthermore, each retailer faces for each brand inter-brand rivalry by both the other brand he is selling and the one sold by the competing retailer. Also, he faces intra-brand rivalry by the same brand sold by his competitor dealer.

• **Exclusive distribution and non-exclusive purchasing** (Figure 3a): a single retailer distributes both manufacturers’ brands, as in the distribution schemes (12, 0) and (0, 12). The retailer behaves as a multi-product monopoly and thus there is only inter-brand rivalry. This distribution scheme has been usually referred to as common agency or common distribution system by related papers in the literature.

• **Monopoly exclusive distribution and purchasing** (Figure 3b): it corresponds with schemes (1, 0), (0, 1), (2, 0) and (0, 2) where only one retailer and one manufacturer are active in the market. Note that this distribution scheme is equivalent to what has been termed as a successive monopoly, either in brand 1 or in brand 2.

• **Mixed schemes** (Figure 3c): one retailer distributes both brands whereas the other sells only one of the brands. Therefore, one of the retailers is a non-exclusive dealer while the other is an exclusive dealer. It embodies the (1, 12), (12, 1), (2, 12) and (12, 2) distribution schemes.

As noted in the introduction, the interaction between retailers leads to the comparison of more distribution schemes relative to other papers in the literature. The assumption that more bargaining power is on the retailers’ side will precisely change the resulting equilibrium structures, as shall shortly be seen. [As noted above, each retailer $k$ chooses simultaneously and independently one element, $s_k$, from the set $S_k = \{0, 1, 2, 12\} \ k = 1, 2$, where

\[11\]This is assumed in Lin (1990), O’Brien and Shaffer (1993) and Gabrielsen (1997). This separated structure is the equilibrium outcome in Bonanno and Vickers (1988) who study, in a product differentiation setting, whether manufacturers wish to delegate sales to independent retailers (separation) or not (integration).
$s_k = 0$ denotes not to deal, $s_k = 1$ denotes that the retailer will deal with $M_1$, likewise for $s_k = 2$, and finally $s_k = 12$ means that the retailer will deal with both manufacturers.] Then, sixteen different distribution schemes may result from the retailers’ strategic choice of brands. The corresponding payoff matrix depends on the pair $(w_1, w_2)$ which in turn implies that the ratio $(a_1 - w_1)/(a_2 - w_2)$ will belong to either of the intervals mentioned. We will end up with a different payoff matrix for each of these intervals. Table 1 displays that of $I_1$.

**Proposition 1** The equilibrium of the second stage in dominant strategies is:
- a) the non-exclusive distribution and purchasing scheme, $(12, 12)$, if and only if the relative per unit profitability of brands belongs to the interval $I_1 \cup I_2 \cup I_5$;
- b) the non-exclusive distribution and exclusive purchasing of brand 2 scheme, $(2, 2)$, if and only if the relative per unit profitability of brands belongs to the interval $I_3 \cup I_4$; and
- c) the non-exclusive distribution and exclusive purchasing of brand 1 scheme, $(1, 1)$, if and only if the relative per unit profitability of brands belongs to the interval $I_6 \cup I_7$.

**Proof.** See Appendix B.

The second stage equilibrium is obtained by iterative deletion of dominated strategies in the corresponding payoff matrix for each of the aforementioned intervals. The second stage equilibrium may be of three types. The equilibrium distribution configuration for $I_1 \cup I_2 \cup I_5$ implies that both inter and intra-brand rivalry are present when the retailers decide strategically on the distribution schemes. For each retailer $k$ being a multi-product dealer is preferred to being a single-product dealer of whichever brand irrespective of the rival’s decision. Hence, both retailers choose $s_k = 12$ for $k = 1, 2$ as the equilibrium in dominant strategies. The deletion of dominated strategies is as follows. Note first that, given the assumptions in the model, strategy $s_k = 0$ for $k = 1, 2$, is always strictly dominated. We show in Appendix B that $s_k = 12$ strictly dominates $s_k = 1$ and $s_k = 2$ for interval $I_1$, while for intervals $I_2$ strategy $s_k = 1$ is strictly dominated by $s_k = 2$ which in turn is strictly dominated by $s_k = 12$. Similarly, for interval $I_5$, $s_k = 2$ is strictly dominated by $s_k = 1$ which in turn is strictly dominated by $s_k = 12$. 

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Table 2: The Second Stage Payoff Matrix for $I_3$

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Table 3: The Second Stage Payoff Matrix for $I_6$

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<td>$R_1(1,2)$</td>
<td>$R_2(2,1)$</td>
<td>$R_2(2,2)$</td>
<td>$R_2(1,1)$</td>
<td>$R_2(1,2)$</td>
<td>$R_1(2,0)$</td>
<td>$R_2(2,0)$</td>
</tr>
<tr>
<td>0</td>
<td>$R_1(0,1)$</td>
<td>$R_1(0,2)$</td>
<td>$R_1(0,1)$</td>
<td>$R_1(0,2)$</td>
<td>$R_2(0,1)$</td>
<td>$R_2(0,2)$</td>
<td>$R_2(0,1)$</td>
<td>$R_2(0,2)$</td>
<td>$R_1(0,0)$</td>
<td>$R_2(0,0)$</td>
</tr>
</tbody>
</table>

14
Suppose now that the ratio \((a_1 - w_1)/(a_2 - w_2)\) belongs to the interval \(I_3\). Table 2 shows the corresponding payoff matrix. It can be seen that strategies \(s_k = 2\) and \(s_k = 12\) are equivalent for both retailers since none of them involves the supply of brand 1 from manufacturer 1 at the third stage equilibrium. Then, by deleting any of them we are left with a two by two matrix. Let us delete \(s_k = 12\). It is easy to see that to choose \(s_k = 2\) for \(k = 1, 2\), is a dominant strategy and manufacturer one would be foreclosed from the market. The equilibrium distribution structure would involve the existence of only intra-brand competition. A similar reasoning can be applied for \((a_1 - w_1)/(a_2 - w_2)\) belonging to \(I_6\). The relevant payoff matrix is given in Table 3.

### 2.3 The manufacturers’ decision on transfer prices

At this stage, each manufacturer decides simultaneously and independently the transfer price \(w_i\) that maximizes his payoffs. We look for the subgame perfect equilibrium of the full game. Let \(\bar{w} = \frac{a_1 - w_1}{a_2 - w_2}\). For a given \(w_2\), manufacturer 1 maximizes

\[
M_1(w_1, w_2) = \begin{cases} 
0 & \text{for } w_1 \text{ such that } \bar{w} \in I_3 \cup I_4 \\
\frac{2(w_1 - c)(b(a_1 - w_1) - d(a_2 - w_2))}{3(b^2 - d^2)} & \text{for } w_1 \text{ such that } \bar{w} \in I_1 \cup I_2 \cup I_5 \\
\frac{2}{3b}(w_1 - c)(a_1 - w_1) & \text{for } w_1 \text{ such that } \bar{w} \in I_6 \cup I_7 \text{ and } w_1 > c
\end{cases}
\]

where \(c\) is the common per unit cost of production. For a given \(w_1\), manufacturer 2 maximizes

\[
M_2(w_1, w_2) = \begin{cases} 
0 & \text{for } w_2 \text{ such that } \bar{w} \in I_6 \cup I_7 \\
\frac{2(w_2 - c)(b(a_2 - w_2) - d(a_1 - w_1))}{3(b^2 - d^2)} & \text{for } w_2 \text{ such that } \bar{w} \in I_1 \cup I_2 \cup I_5 \\
\frac{2}{3b}(w_2 - c)(a_2 - w_2) & \text{for } w_2 \text{ such that } \bar{w} \in I_5 \cup I_4 \text{ and } w_2 > c
\end{cases}
\]

Let \((w_1^*, w_2^*)\) denote the equilibrium pair of transfer prices. An inspection of the profit functions above shows that, for a given transfer price of the rival, there exist two local maxima for each manufacturer. The global maximum depends on the ratio \(C = \frac{a_1 - c}{a_2 - c}\) which can be interpreted as the relative per unit profitability of brands for the manufacturers and it is greater than or equal to one. When \(C \in [1, \frac{2b^2 - d^2}{db}]\) the equilibrium pair \((w_1^*, w_2^*) = (\bar{w}_1, \bar{w}_2)\).
implies that $\bar{w} \in I_1 \cup I_2 \cup I_5$. As $C$ increases manufacturer one finds it profitable to deviate from the above equilibrium. The new equilibrium will imply either manufacturer one setting a limit transfer price $\bar{w}_1(w_2)$ or the monopoly transfer price $w_{1m}^\ast$. Both these cases involve that $\bar{w} \in I_6 \cup I_7$. Notice that there is no analogous reasoning for manufacturer two given brand asymmetry. The next proposition summarizes the above discussion.

**Proposition 2** The equilibrium of the full game is a pair of transfer prices equal to either:

a) $w_i^\ast = \hat{w}_i = \frac{(2b^2-d^2)a_i-bda_j+b(2b+d)c}{db^2-d^2} i, j = 1, 2$ when $C \in [1, \frac{2b^2-d^2}{db}]$,

or b) $w_1^\ast = \hat{w}_1(c) = a_1 - \frac{b(a_2-c)}{d}, w_2^\ast = c$ when $C \in \left[\frac{2b^2-d^2}{db}, \frac{2b}{d}\right]$,

or c) $w_1^\ast = w_{1m}^\ast = a_1 + \frac{c}{2}, w_2^\ast = c$ when $C > \frac{2b}{d}$.

**Proof.** See Appendix B.

**Corollary 1** The subgame perfect distribution equilibrium systems are $(12, 12)$ if $C \in [1, \frac{2b^2-d^2}{db}]$ or $(1, 1)$ if $C > \frac{2b^2-d^2}{db}$.

The result above claims that, for low brand asymmetry (a low $C$), it is sufficient that retailers hold the power to choose the number of products they wish to distribute to obtain endogenously both inter and intra-brand competition; both retailers become multi-product sellers. However, as the profitability of brands diverges sufficiently, only the most profitable brand will be distributed by both retailers thus only arising intra-brand competition at equilibrium. Note that the greater the degree of product differentiation, $\frac{b}{d}$, the greater the asymmetry required to end up with only intra-brand competition in the market. It is worth remarking that, regardless of the degree of product differentiation, the non-exclusive distribution and purchasing distribution system is the retailers’ equilibrium choice when brands are equally profitable, $a_1 = a_2$. Finally, when inter-brand competition is maximal, $b = d$, the retailers will only select to be supplied by the manufacturer with the greatest consumers’ valuation, the greatest $a$.

As mentioned in the Introduction, our analysis considers a larger number of possible channel structures compared with the existing literature. Our findings suggest that this assumption, derived from more retailer power, is not irrelevant provided that the well-known exclusive and common distribution systems are among the choice set and do not arise at equilibrium. A
natural question to ask is whether the \((12, 12)\) and \((1, 1)\) distribution systems would be selected when it is manufacturers who choose the number of retailers they wish to employ. Suppose that manufacturers choose simultaneously and independently whether to employ none, retailer one, retailer two, or both. In the second stage, they decide upon transfer prices and finally retailers compete à la Cournot. It can be shown that, for \(C \in [1, \frac{2b^2 - d^2}{db}]\), \((12, 12)\) is not always an equilibrium since manufacturer \(M_2\) has an incentive to deviate from 12 when \(C\) belongs to a subset included in \([1, \frac{2b^2 - d^2}{db}]\), which implies sufficient brand asymmetry.

The literature has typically analyzed the exclusive and the common distribution system; the systems \((1, 2)\), \((2, 1)\) and \((12, 0)\), \((0, 12)\) in our terminology, respectively. Where one system prevails upon the other depending on the degree of product differentiation and on whether competition is in either prices or quantities. Of course, these are not equilibria with more retailer power. What our analysis highlights is that there is a conflict between manufacturers’ and retailers’ choices and it does matter who are the agents with more power in the trading relationship.

### 3 Conclusions

The scarcity of shelf space coupled with the rise in retailer concentration has shifted the balance of power away from manufacturers and endowed retailers with a better bargaining position. We have considered a three-stage non-cooperative game in which, despite the fact that manufacturers play first in the choice of transfer prices, retailers play a relevant role in the shaping of the distribution system. The literature on distribution systems has typically considered exclusive versus common distribution systems finding that manufacturers prefer an exclusive distribution system. This is the equilibrium choice unless we resort to either a mutual manufacturer-retailer agreement (as in Dobson and Waterson, 1997), or to a cooperative approach (as in Gabrielsen, 1996), or to a dynamic game (as in Gabrielsen, 1997), or to the possibility of foreclosure under both systems (as in Gabrielsen and Sørgard, 1999b). We have found that exclusive dealing systems are not observed in a setting with more retailer decision power. Furthermore, vertical foreclosure of the less profitable brand will show up the lower the degree of product differentiation and the higher the asymmetry between brands.
A Appendix: the third stage equilibrium outcomes.

We proceed to present the equilibrium outputs, prices and gross profits for each distribution scheme.

a) The \((1, 0), (0, 1), (2, 0), \) and \((0, 2)\) distribution schemes.

In any of these cases there is only one retailer selling in the market. The distribution scheme \((i, 0)\) means that \(R_1\) is distributing brand \(i\), \(i = 1, 2\), while \(R_2\) is not distributing any brands; similarly for \((0, i)\). Suppose that \(R_1\) is the only active retailer. He maximizes his profits defined as follows by choosing \(q_i\),

\[
R_1(q_i) = (a_i - bq_i - w_i)q_i \quad i = 1, 2
\]

and we find that the equilibrium outcomes are

\[
q_i(i, 0) \equiv q_1(i, 0) = \frac{a_i - w_i}{2b}
\]

\[
p_i(i, 0) = \frac{a_i + w_i}{2}
\]

\[
R_1(i, 0) = \frac{(a_i - w_i)^2}{4b}
\]

for \(i = 1, 2\) while for \(R_2\) it is obvious that it distributes zero and gets zero profits. The restriction on the parameter space to get positive output equilibria is that \((a_i - w_i) > 0\) for \(i = 1, 2\).

b) The \((12, 0)\) and \((0, 12)\) distribution schemes.

Now, we have that only one retailer is active in the market as above, but he distributes both manufacturers’ brands. Therefore, there is one multi-product retailer. Take for example the case \((12, 0)\), \(R_1\) maximizes the following

\[
R_1(q_{11}, q_{21}) = (a_1 - bq_{11} - dq_{21} - w_i)q_{11} + (a_2 - bq_{21} - dq_{11} - w_2)q_{21}
\]

which results in the following equilibrium outcomes

\[
q_{1}(12, 0) \equiv q_{11}(12, 0) + q_{21}(12, 0)
\]

\[
= \frac{b(a_1 - w_1) - d(a_2 - w_2)}{2(b^2 - d^2)} + \frac{b(a_2 - w_2) - d(a_1 - w_1)}{2(b^2 - d^2)}
\]
\[ p_1(12, 0) = \frac{a_1 + w_1}{2} \]
\[ p_2(12, 0) = \frac{a_2 + w_2}{2} \]
\[ R_1(12, 0) = \frac{(a_1 - w_1)[b(a_1 - w_1) - d(a_2 - w_2)]}{4(b^2 - d^2)} \]
\[ \quad + \frac{(a_2 - w_2)[b(a_2 - w_2) - d(a_1 - w_1)]}{4(b^2 - d^2)} \]
\[ \quad = \frac{b(a_1 - w_1)^2 + b(a_2 - w_2)^2 - 2d(a_1 - w_1)(a_2 - w_2)}{4(b^2 - d^2)} \]

In this case, to get positive equilibrium outputs, we need to restrict the parameters to the next interval
\[ \frac{d}{b} < \frac{a_1 - w_1}{a_2 - w_2} < \frac{b}{d} \]

c) The (1, 1) and (2, 2) distribution schemes.
In both cases we have a homogenous duopoly. Both retailers distribute the same brand while the other manufacturer’s brand is not sold in the market. Take as an example the case of (1, 1). Each retailer maximizes his profits choosing quantities.

\[ R_1(q_{11}, q_{12}) = (a_1 - b(q_{11} + q_{12}) - w_1)q_{11} \]
\[ R_2(q_{11}, q_{12}) = (a_1 - b(q_{12} + q_{11}) - w_1)q_{12} \]

The equilibrium quantities are obtained by solving the two-equation system of first order conditions for \( q_{11} \) and \( q_{12} \). These are:

\[ q_1(1, 1) \equiv q_{11}(1, 1) = \frac{a_1 - w_1}{3b} \]
\[ q_2(1, 1) \equiv q_{12}(1, 1) = \frac{a_1 - w_1}{3b} \]
\[ p_1(1, 1) = \frac{a_1 + 2w_1}{3} \]
\[ R_1(1, 1) = R_2(1, 1) = \frac{(a_1 - w_1)^2}{9b} \]
with the same restriction on the parameters as in case presented in the first place.

d) The (1, 2) and (2, 1) distribution schemes.

Here, we have that each retailer is distributing one and only one manufacturer’s brand. Therefore, there is a differentiated duopoly. Consider the case (1, 2). Each retailer maximizes his profits choosing quantities.

\[
R_1(q_{11}, q_{22}) = (a_1 - bq_{11} - dq_{22} - w_1)q_{11}
\]

\[
R_2(q_{11}, q_{22}) = (a_2 - bq_{22} - dq_{11} - w_2)q_{22}
\]

We obtain the following equilibrium outcomes

\[
q_{1}(1, 2) = q_{11}(1, 2) = \frac{2b(a_1 - w_1) - d(a_2 - w_2)}{4b^2 - d^2}
\]

\[
q_{2}(1, 2) = q_{22}(1, 2) = \frac{2b(a_2 - w_2) - d(a_1 - w_1)}{4b^2 - d^2}
\]

\[
p_1(1, 2) = \frac{2b^2a_1 + (2b^2 - d^2)w_1 - bd(a_2 - w_2)}{4b^2 - d^2}
\]

\[
p_2(1, 2) = \frac{2b^2a_2 + (2b^2 - d^2)w_2 - bd(a_1 - w_1)}{4b^2 - d^2}
\]

\[
R_1(1, 2) = \frac{b[2b(a_1 - w_1) - d(a_2 - w_2)]^2}{(4b^2 - d^2)^2}
\]

\[
R_2(1, 2) = \frac{b[2b(a_2 - w_2) - d(a_1 - w_1)]^2}{(4b^2 - d^2)^2}
\]

where the restriction on the parameter space in order to get positive equilibrium outputs becomes:

\[
d \frac{2b}{2b} \frac{a_1 - w_1}{a_2 - w_2} < \frac{2b}{d}
\]

e) The (12, 12) distribution scheme.

In this case, both retailers distribute the brands of both manufacturers. Then, we have a multi-product duopoly. Each retailer takes two decisions on outputs to maximize his profits,

\[
\max_{q_{11}, q_{21}} R_1(q_{11}, q_{12}, q_{21}, q_{22}) = (a_1 - b(q_{11} + q_{12}) - d(q_{21} + q_{22}) - w_1)q_{11}
\]

\[
+ (a_2 - b(q_{21} + q_{22}) - d(q_{11} + q_{12}) - w_2)q_{21}
\]

\[
\max_{q_{12}, q_{22}} R_2(q_{11}, q_{12}, q_{21}, q_{22}) = (a_1 - b(q_{11} + q_{12}) - d(q_{21} + q_{22}) - w_1)q_{12}
\]

\[
+ (a_2 - b(q_{21} + q_{22}) - d(q_{11} + q_{12}) - w_2)q_{22}
\]
The equilibrium outputs come as the solution to the four-equation system of first order conditions for \(q_{11}, q_{12}, q_{21}\), and \(q_{22}\). We obtain

\[
q_{1}(12, 12) = \frac{b(a_1 - w_1) - d(a_2 - w_2)}{3(b^2 - d^2)} + \frac{b(a_2 - w_2) - d(a_1 - w_1)}{3(b^2 - d^2)}
= \frac{a_1 - w_1 + a_2 - w_2}{3(b + d)}
\]

\[
p_1(12, 12) = \frac{a_1 + 2w_1}{3}
\]

\[
p_2(12, 12) = \frac{a_2 + 2w_2}{3}
\]

\[
R_1(12, 12) = R_2(12, 12) = \frac{(a_1 - w_1)[b(a_1 - w_1) - d(a_2 - w_2)]}{9(b^2 - d^2)}
+ \frac{(a_2 - w_2)[b(a_2 - w_2) - d(a_1 - w_1)]}{9(b^2 - d^2)}
= \frac{b(a_1 - w_1)^2 + b(a_2 - w_2)^2 - 2d(a_1 - w_1)(a_2 - w_2)}{9(b^2 - d^2)}
\]

with the same restriction on the parameters as in the case presented in the second place.

e) **The (12, 1), (1, 12) and (12, 2), (2, 12) distribution schemes.**

This is an asymmetric case where one of the retailers distributes both manufacturers’ brands while the other only distributes one brand. Then, we have a multi-product retailer facing a single-product one. Take as an example the distribution scheme (12, 1). Each retailer maximizes his profits,

\[
\max_{q_{11}, q_{21}} R_1(q_{11}, q_{12}, q_{21}) = (a_1 - b(q_{11} + q_{12}) - dq_{21} - w_1)q_{11}
+ (a_2 - bq_{21} - d(q_{11} + q_{12}) - w_2)q_{21}
\]

\[
\max_{q_{12}} R_2(q_{11}, q_{12}, q_{21}) = (a_1 - b(q_{11} + q_{12}) - dq_{21} - w_1)q_{12}
\]

The equilibrium outputs come as the solution to the three-equation system of first order conditions for \(q_{11}, q_{12}\), and \(q_{21}\). We obtain

\[
q_{1}(12, 1) = q_{11}(12, 1) + q_{21}(12, 1) =
\]

\[
\frac{(2b^2 + d^2)(a_1 - w_1) - 3bd(a_2 - w_2)}{6(b^2 - d^2)} + \frac{b(a_2 - w_2) - d(a_1 - w_1)}{2(b^2 - d^2)}
\]
\[
q_2(12, 1) \equiv q_{12}(12, 1) = \frac{a_1 - w_1}{3b}
\]
\[
p_1(12, 1) = \frac{a_1 + 2w_1}{3}
\]
\[
p_2(12, 1) = \frac{3b(a_2 + w_2) - d(a_1 - w_1)}{3}
\]
\[
R_1(12, 1) = \frac{(a_1 - w_1)(2b^2 + d^2)(a_1 - w_1) - 3bd(a_2 - w_2)}{18b(b^2 - d^2)}
\]
\[
+ \frac{3b(a_2 - w_2) - d(a_1 - w_1)[b(a_2 - w_2) - d(a_1 - w_1)]}{12b(b^2 - d^2)}
\]
\[
= \frac{(4b^2 + 5d^2)(a_1 - w_1)^2 + 9b^2(a_2 - w_2)^2 - 18bd(a_1 - w_1)(a_2 - w_2)}{36b(b^2 - d^2)}
\]
\[
R_2(12, 1) = \frac{(a_1 - w_1)^2}{9b}
\]
where the restrictions on the parameters to ensure that \(q_{11}(12, 1) = q_{12}(11, 12)\) and \(q_{21}(12, 2) = q_{22}(2, 12)\) are positive are,
\[
\frac{3bd}{2b^2 + d^2} < \frac{a_1 - w_1}{a_2 - w_2} < \frac{2b^2 + d^2}{3bd}
\]
The restrictions displayed at the end of each of the above distribution schemes gives rise to the parameter interval in Figure 1 in the text.
B Appendix: proofs.

Proof of Proposition 1.

In this proof we apply iterated deletion of dominated strategies for each of the relevant intervals in which the $\bar{w} = \frac{a_1 - w_1}{a_2 - w_2}$ space is divided to characterize the second stage equilibrium. Since retailers’ payoffs are symmetric, we just prove the result for $R_1$. Also, it is easy to see that action $s_1 = 0$ is dominated by the other three for each of the possible intervals.

Part a) of Proposition 1 states that $(12,12)$ is the second stage equilibrium for $\frac{a_1 - w_1}{a_2 - w_2} \in I_1 \cup I_2 \cup I_3$.

We prove that $(12,12)$ is the equilibrium for each of the referred intervals:

**I₁-Interval**, $\frac{3bd}{2b^2 + d^2} \leq \bar{w} \leq \frac{3bd}{32b^4 + 10b^2d^2 + 5d^4}$.

We show that for retailer $1$ action $s_1 = 12$ dominates the other three actions, i.e. $R_1(12, s_2) > R_1(s_1, s_2), \forall s_2 \in \{1,2,12\}, \forall s_1 \in \{1,2\}$. Firstly, we check that action $s_1 = 12$ dominates action $s_1 = 2$. This is equivalent to checking when the following inequalities are satisfied: a) $R_1(12,1) > R_1(2,1), b) R_1(12,2) > R_1(2,2), c) R_1(12,12) > R_1(2,12)$.

First, a) $R_1(12,1) > R_1(2,1)$ if $[64b^6 + 12b^4d^2 + 5d^6][a_1 - w_1]^2 + 9b^2d^2(8b^2 + d^2)(a_2 - w_2)^2 - 18bd(8b^4 + d^4)(a_1 - w_1)(a_2 - w_2) = [(2b^2 - d^2)(a_1 - w_1) - 3bd(a_2 - w_2)][(32b^4 + 10b^2d^2 + 5d^4)(a_1 - w_1) - (24b^3d + 3bd^3)(a_2 - w_2)] > 0$.

Therefore, the inequality is positive when either both terms are positive or both are negative. Both are positive when

$$\bar{w} > \frac{3bd}{2b^2 + d^2} > \frac{3bd(8b^2 + d^2)}{32b^4 + 10b^2d^2 + 5d^4}$$

which is always satisfied for $\bar{w} \in I_1$.

Second, $R_1(12,2) > R_1(2,2)$ and $R_1(12,12) > R_1(2,12)$ iff $[b(a_1 - w_1) - d(a_2 - w_2)]^2 > 0$, which is always satisfied. Therefore, we conclude that action $s_k = 12$ dominates action $s_k = 2$ for $k = 1, 2$.

Next, we check when action $s_1 = 12$ dominates the action $s_1 = 1$. This is equivalent to checking when the following inequalities are satisfied: a) $R_1(12,1) > R_1(1,1), b) R_1(12,2) > R_1(1,2), c) R_1(12,12) > R_1(1,12)$.

First, $R_1(12,1) > R_1(1,1)$ and $R_1(12,12) > R_1(1,12)$ iff $[b(a_2 - w_2) - d(a_1 - w_1)]^2 > 0$, which is always satisfied.
Second, b) \( R_1(12, 2) > R_1(1, 2) \) if \[ [(2b^2 - d^2)(a_2 - w_2) - 3bd(a_1 - w_1)][(32b^4 + 10b^2d^2 + 5d^4)(a_1 - w_2) - (24b^3d + 3bd^3)(a_1 - w_1)] > 0. \] This inequality is satisfied when either both terms are positive or when both are negative. Both are positive when

\[
\bar{w} < \frac{2b^2 + d^2}{3bd} < \frac{32b^4 + 10b^2d^2 + 5d^4}{3bd(8b^2 + d^2)}
\]

which is always satisfied for \( \bar{w} \in I_1 \).

Therefore, we conclude that action \( s_k = 12 \) dominates action \( s_k = 1 \) for \( k = 1, 2 \).

Hence for \( \bar{w} \in I_1 \), we have shown that \( s_1 = s_2 = 12 \) is the equilibrium in dominant strategies, i.e. the \textit{non-exclusive distribution and dealing scheme} is the equilibrium distribution scheme in dominant strategies.

\textbf{I}_2-\textbf{Interval, } \frac{d}{b} \leq \bar{w} < \frac{3bd}{2b^2 + d^2}.

When the pair \((w_1, w_2)\) is such that \( \bar{w} \in I_2 \) the third stage equilibrium quantities \( q_{11}(12, 1) \) and \( q_{12}(1, 12) \) are set to zero in the subgames \((12, 1)\) and \((1, 12)\), respectively. Consequently, the second stage retailers’ choice of \((12, 1)\) and \((1, 12)\) is payoff equivalent to choosing \((2, 1)\) and \((1, 2)\), respectively. We first show that action \( s_1 = 2 \) dominates action \( s_1 = 1 \), which is equivalent to showing that a) \( R_1(2, 1) > R_1(1, 1) \) and b) \( R_1(2, 2) > R_1(1, 2) \), and c) \( R_1(2, 12) > R_1(1, 12) \).

First, a) \( R_1(2, 1) > R_1(1, 1) \) if \( (16b^4 - 17b^2d^2 + d^4)(a_1 - w_1)^2 + 36b^3d(a_1 - w_1)(a_2 - w_2) - 36b^4(a_2 - w_2)^2 < 0 \), or equivalently if

\[
0 < \bar{w} < \frac{6b^2}{(4b - d)(b + d)}
\]

but \( \frac{6b^2}{(4b - d)(b + d)} > 1 \) and therefore, \( R_1(2, 1) > R_1(1, 1) \) for \( \bar{w} \in I_2 \).

Second, note that since \( R_1(2, 12) \) is always equal to \( R_1(2, 2) \) and that \( R_1(1, 12) = R_1(1, 2) \) for \( \bar{w} \in I_2 \), inequalities b) and c) are the same. Therefore, we find when it is true that \( R_1(2, 2) > R_1(1, 2) \), or equivalently, when

\[
(16b^4 - 17b^2d^2 + d^4)(a_2 - w_2)^2 < 0
\]

which in terms of \( \bar{w} \) amounts to

\[
0 < \bar{w} < \frac{(4b - d)(b + d)}{6b^2}
\]
but \( \frac{(4b-d)\bar{d}+d}{6b^2} > \frac{3bd}{2b^2+2} \) and therefore, \( R_1(2,2) > R_1(1,2) \) for \( \bar{w} \in I_2 \).

Next, we show that after deleting actions \( s_1 = 0 \) and \( s_1 = 1 \), action \( s_1 = 12 \) dominates \( s_1 = 2 \). This is equivalent to showing that \( R_1(12,2) > R_1(2,2) \) and \( R_1(12,12) > R_1(2,12) \), but we know that they are satisfied iff \( [b(a_1 - w_1) - d(a_2 - w_2)]^2 > 0 \). Therefore, we conclude that action \( s_k = 12 \) dominates action \( s_k = 2 \) for \( k = 1,2 \).

Hence for \( \bar{w} \in I_2 \), we have shown that \( s_1 = s_2 = 12 \) is the equilibrium in dominant strategies.

**I_3-Interval, \( \frac{2b^2 + d^2}{3bd} < \bar{w} \leq \frac{b}{\bar{a}} \).**

The third stage equilibrium quantities \( q_{21}(12,2) \) and \( q_{22}(2,12) \) are set to zero in the subgames \((12,2)\) and \((2,12)\), respectively. Consequently, the second stage retailers’ choice of \((12,2)\) and \((2,12)\) is payoff equivalent to choosing \((1,2)\) and \((2,1)\), respectively. We first show that action \( s_1 = 1 \) dominates action \( s_1 = 2 \), that is equivalent to showing that \( a) \ R_1(1,1) > R_1(2,1) \), and \( b) \ R_1(1,2) > R_1(2,2) \), and \( c) \ R_1(1,12) > R_1(2,12) \).

First, note that since \( R_1(1,12) = R_1(2,1) \) for \( \bar{w} \in I_3 \), inequalities \( a) \) and \( c) \) are the same. Therefore we find when it is true that \( R_1(1,1) > R_1(2,1) \). That is, \( R_1(1,1) > R_1(2,1) \), if

\[
\frac{6b^2}{(4b-d)(b+d)} < \bar{w}
\]

but as \( \frac{6b^2}{(4b-d)(b+d)} < \frac{2b^2 + d^2}{3bd} \) then \( R_1(1,1) > R_1(2,1) \) for \( \bar{w} \in I_3 \)

Second, \( R_1(1,2) > R_1(2,2) \) if

\[
\frac{(4b-d)(b+d)}{6b^2} < \bar{w}
\]

but since \( \frac{(4b-d)(b+d)}{6b^2} < 1 \) we have that \( R_1(1,2) > R_1(2,2) \) for \( \bar{w} \in I_3 \).

Next, we show that after deleting actions \( s_1 = 0 \) and \( s_1 = 2 \), action \( s_1 = 12 \) dominates \( s_1 = 1 \). This is equivalent to showing that \( R_1(12,1) > R_1(1,1) \), and \( R_1(12,12) > R_1(1,12) \), but we know that they are satisfied iff \( [b(a_1 - w_1) - d(a_2 - w_2)]^2 > 0 \). Therefore, we conclude that action \( s_k = 12 \) dominates action \( s_k = 1 \) for \( k = 1,2 \).

Hence for \( \bar{w} \in I_3 \), we have shown that \( s_1 = s_2 = 12 \) is the equilibrium in dominant strategies. Finally, we conclude that for \( \bar{w} \in I_1 \cup I_2 \cup I_3 \), the pair of actions \( s_1 = s_2 = 12 \) is the second stage equilibrium in dominant strategies.
Part b) of Proposition 1 states that (2,2) is the second stage equilibrium for \( \hat{w} \in I_3 \cup I_4 \)

\[ I_3\text{-Interval, } \frac{d}{2b} \leq \frac{a_1 - w_1}{a_2 - w_2} < \frac{d}{b} \]

The third stage equilibrium quantities \( q_{11}(12,1), q_{12}(1,12), q_{11}(12,2), q_{12}(2,12), q_{11}(12,12), q_{12}(12,12), q_{11}(12,0) \) and \( q_{12}(0,12) \) are zero. In words, brand 1 is never jointly distributed with brand 2 by the same retailer. Consequently, the choice of action \( s_k = 12 \) is payoff equivalent to the choice of action \( s_k = 2 \) in stage two. After the deletion of actions 0 and 12 we prove that action \( s_k = 2 \) dominates action \( s_k = 1 \). This amounts to proving that \( R_{11}(2,1) > R_{11}(1,1) \) and \( R_{11}(2,2) > R_{11}(1,2) \). However we saw above that both are satisfied iff

\[
0 < \hat{w} < \frac{(4b - d)(b + d)}{6b^2} < \frac{6b^2}{(4b - d)(b + d)}
\]

which is the case since \( I_3 \subset (0, \frac{(4b - d)(b + d)}{6b^2}) \). Therefore, the second stage equilibrium for \( \hat{w} \in I_3 \), is \( s_1 = s_2 = 12 \) in dominant strategies.

\[ I_4\text{-Interval, } 0 < \hat{w} < \frac{d}{2b} \]

In addition to the quantities that become zero for the \( I_3\text{-Interval} \) case, the quantities \( q_{12}(2,1) \) and \( q_{11}(1,2) \) are also set to zero. In words, the only way in which brand 1 can be distributed is when it is not competing with brand 2, that is, distribution systems \((1,1), (1,0)\) and \((0,1)\). Therefore the payoff matrix after deletion of dominated strategies becomes,

\[
\begin{array}{c|cc}
 & 1 & 2 \\
\hline
1 & R_{11}(1,1) & 0 \\
2 & R_{11}(2,0) & R_{11}(2,2) \\
\end{array}
\]

where it is easily proven that \( s_k = 2 \) dominates \( s_k = 1 \) provided that \( R_{11}(2,0) > R_{11}(1,1) \), that is when \( \hat{w} < \frac{3}{2} \), which is the case since \( I_4 \subset (0, \frac{3}{2}) \).

Therefore, the second stage equilibrium for \( \hat{w} \in I_4 \), is \( s_1 = s_2 = 2 \) in dominant strategies. Then it is concluded that for \( \hat{w} \in I_3 \cup I_4 \), the pair of actions \( s_1 = s_2 = 2 \) is the second stage equilibrium in dominant strategies.

A parallel reasoning as the one above is employed to prove Part c) of Proposition 1: where \((1,1)\) is the second stage equilibrium for \( \hat{w} \in I_6 \cup I_7 \).
Proof of Proposition 2.

The strategy of the proof is to find whether and when manufacturer $M_1$ has incentives to deviate from the equilibrium in which both manufacturers supply both retailers. Fix $w_2$ equal to $\hat{w}_2 = \frac{\left(2b^2-d^2\right)a_2-bda_1+b(2b+d)c}{4b^2-d^2}$. Given that, we may write the corresponding profit function of $M_1$ as $\Pi_1(w_1, w_2^*)$. Let $\Pi(w_1, \hat{w}_2) = \frac{2(\Pi(w_1, w_2^*))}{\Pi(\hat{w}_2)}$ and let $\Pi'(w_1, \hat{w}_2) = \frac{\partial}{\partial w_1} \Pi(w_1, \hat{w}_2)$. The unconstrained equilibrium transfer price $w_1^*$ for $\Pi'(w_1, \hat{w}_2)$ is $w_1^* = \frac{(2b^2-d^2)a_1-bda_2+b(2b+d)c}{4b^2-d^2}$; the unconstrained equilibrium transfer price $w_1^*$ for $\Pi''(w_1, \hat{w}_2)$ is the monopoly transfer price $w_1^{m} = \frac{a_1+c}{2}$; the limit transfer price is obtained from the intersection of $\Pi'(w_1, \hat{w}_2)$ and $\Pi''(w_1, \hat{w}_2)$, $\tilde{w}_1(\hat{w}_2) = a_1 - \frac{b(a_2-\hat{w}_2)}{d}$. These equilibrium transfer prices must be ranked in order to find any profitable deviation.

It turns out that: a) for $\frac{a_1-c}{a_2-a_2}$ belonging to $[1, \frac{2b^2-d^2}{bd}]$, $w_1^m > \hat{w}_1(\hat{w}_2)$; b) for $\frac{a_1-c}{a_2-a_2}$ belonging to $[\frac{2b^2-d^2}{bd}, \frac{2b}{d}]$, $w_1^m > \hat{w}_1(\hat{w}_2) > \hat{w}_1$ and c) for $\frac{a_1-c}{a_2-a_2}$ greater than $\frac{2b}{d}$, $\hat{w}_1(\hat{w}_2) > w_1^m > \hat{w}_1$. Note that when manufacturer $M_1$ sets a transfer prices smaller than or equal to the limit transfer price, the rival manufacturer is foreclosed from the market, manufacturer $M_1$ relevant profit branch is the lower one. Thus, in case a), the maximum of $\Pi''(w_1, \hat{w}_2)$ subject to $w_1 \in [c, \hat{w}_1(\hat{w}_2)]$ is $\tilde{w}_1(\hat{w}_2)$, while the maximum of $\Pi'(w_1, \hat{w}_2)$ subject to $w_1 > \hat{w}_1(\hat{w}_2)$ is $\tilde{w}_1(\hat{w}_2)$. And noting that $\Pi'(\tilde{w}_1(\hat{w}_2), \hat{w}_2) = \Pi''(\tilde{w}_1(\hat{w}_2), \hat{w}_2)$ we conclude that the global maximum is $\tilde{w}_1$ and manufacturer $M_1$ has no incentive to deviate.

In case b), the maximum of $\Pi''(w_1, \hat{w}_2)$ subject to $w_1 \in [c, \hat{w}_1(\hat{w}_2)]$ is $\tilde{w}_1(\hat{w}_2)$, while the maximum of $\Pi'(w_1, \hat{w}_2)$ subject to $w_1 > \hat{w}_1(\hat{w}_2)$ is $\tilde{w}_1(\hat{w}_2)$.

We conclude that the global maximum is $\tilde{w}_1(\hat{w}_2)$ and manufacturer $M_1$ has an incentive to deviate.

Finally, in case c), the maximum of $\Pi''(w_1, \hat{w}_2)$ subject to $w_1 \in [c, \hat{w}_1(\hat{w}_2)]$ is $w_1^m$, while the maximum of $\Pi'(w_1, \hat{w}_2)$ subject to $w_1 > \hat{w}_1(\hat{w}_2)$ is $\tilde{w}_1(\hat{w}_2)$.

We conclude that the global maximum is $\tilde{w}_1(\hat{w}_2)$, and noting that $\Pi'(\tilde{w}_1(\hat{w}_2), \hat{w}_2) = \Pi''(\tilde{w}_1(\hat{w}_2), \hat{w}_2)$ we conclude that the global maximum is $w_1^m$; manufacturer $M_1$ has an incentive to deviate.

Whenever manufacturer $M_1$ has an incentive to deviate, manufacturer $M_2$ may lower the transfer price $w_2$ in order to remain in the market. The lowest $w_2$ it can fix is $w_2 = c$. Substituting in cases b) and c) above $\hat{w}_2$ for $w_2 = c$, lead to the intervals for $C = \frac{a_1-c}{a_2-c}$ stated in the proposition.
<table>
<thead>
<tr>
<th>Distribution Scheme</th>
<th>Quantities</th>
<th>Gross Profits</th>
</tr>
</thead>
</table>
| (1,0) and (0,1)     | $q_1(1,0) = \frac{a_1-w_1}{2b}$  
$q_2(1,0) = 0$ | $R_1(1,0) = \frac{(a_1-w_1)^2}{4b}$  
$R_2(1,0) = 0$ |
| (2,0) and (0,2)     | $q_1(2,0) = \frac{a_2-w_2}{2b}$  
$q_2(2,0) = 0$ | $R_1(2,0) = \frac{(a_2-w_2)^2}{4b}$  
$R_2(2,0) = 0$ |
| (12,0) and (0,12)   | $q_1(12,0) = \frac{(a_1-w_1+a_2-w_2)}{2(b+d)}$  
$q_2(12,0) = 0$ | $R_1(12,0) = \frac{2(a_1-w_1)^2 + (a_2-w_2)^2 - 2d(a_1-w_1)(a_2-w_2)}{4(b^2-d^2)}$  
$R_2(12,0) = 0$ |
| (1,1)               | $q_1(1,1) = q_2(1,1) = \frac{a_1-w_1}{2b}$ | $R_1(1,1) = R_2(1,1) = \frac{(a_1-w_1)^2}{4b}$ |
| (2,2)               | $q_1(2,2) = q_2(2,2) = \frac{a_2-w_2}{2b}$ | $R_1(2,2) = R_2(2,2) = \frac{(a_2-w_2)^2}{4b}$ |
| (12,12)             | $q_1(12,12) = q_2(12,12) = \frac{(a_1-w_1+a_2-w_2)}{2(b+d)}$ | $R_1(12,12) = R_2(12,12) = \frac{b(a_1-w_1)^2 + (a_2-w_2)^2 - 2d(a_1-w_1)(a_2-w_2)}{4(b^2-d^2)}$ |
| (1,2) and (2,1)     | $q_1(1,2) = \frac{2b(a_1-w_1)-d(a_2-w_2)}{4b^2-d^2}$  
$q_2(1,2) = \frac{2b(a_2-w_2)+(a_1-w_1)}{4b^2-d^2}$ | $R_1(1,2) = \frac{b(2b(a_1-w_1)-d(a_2-w_2))^2}{(4b^2-d^2)^2}$  
$R_2(1,2) = \frac{b(2b(a_2-w_2)-(a_1-w_1))^2}{(4b^2-d^2)^2}$ |
| (12,1) and (1,12)   | $q_1(12,1) = \frac{(2b-d)(a_1-w_1)+3d(a_2-w_2)}{6b(b+d)}$  
$q_2(12,1) = \frac{(a_1-w_1)}{3b}$ | $R_1(12,1) = \frac{((4b^2+5d^2)(a_1-w_1)^2 + 3b^2(a_2-w_2)^2 - 18bd(a_1-w_1)(a_2-w_2))}{36b(b^2-d^2)}$  
$R_2(12,1) = \frac{(a_1-w_1)^2}{4b}$ |
| (12,2) and (2,12)   | $q_1(12,2) = \frac{(2b-d)(a_2-w_2)+3d(a_1-w_1)}{6b(b+d)}$  
$q_2(12,2) = \frac{(a_2-w_2)}{3b}$ | $R_1(12,2) = \frac{((4b^2+5d^2)(a_1-w_1)^2 + 3b^2(a_2-w_2)^2 - 18bd(a_1-w_1)(a_2-w_2))}{36b(b^2-d^2)}$  
$R_2(12,2) = \frac{(a_2-w_2)^2}{4b}$ |
| (0,0)               | $q_1(0,0) = q_2(0,0) = 0$ | $R_1(0,0) = R_2(0,0) = 0$ |
Figure 1: The Relevant Intervals for the Second Stage of the Game.
Figure 2a: Non-Exclusive Distribution and Exclusive Dealing

Figure 2b: Duopoly Exclusive Distribution and Dealing

Figure 2c: Non-Exclusive Distribution and Dealing
Figure 3a: Exclusive Distribution and Non-Exclusive Dealing

Figure 3b: Monopoly Exclusive Distribution and Dealing

Figure 3c: Mixed Schemes
References


