ELECTION ON RETIREMENT AGE*

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ABSTRACT

We analyze the relationship between wage distribution, degree of redistribution of the Social Security and effective retirement age. We develop a two-staged political economy model. In the first stage government chooses the redistribution level of the Social Security Program, according to three different criteria. In second stage the retirement age is elected through a majority voting process by agents with different wages, knowing exactly the redistribution level and voting accordingly. We analyze the different elected retirement age under each government criterion.

Key words: Wage distribution; retirement age; level of redistribution; median voter.
"I know we make our own reality and we always have a choice, but how much is pre-ordained?" John Lennon.

1. Introduction

Most of the OECD countries are preparing reforms in the public pensions schemes in order to solve the financing problem of the Social Security System. The main reform that is being proposed is the delay in the retirement age. Due to this, the retirement age has recently become an important issue.

In the present paper, as in our previous one, Lacomba and Lagos [2000]; we seek to analyze the importance of the retirement age by considering this retirement age the issue to be collectively elected. In consequence, we consider retirement age as a compulsory age that will be the variable to be chosen by the individuals through a majority voting process. In this manner we can deduce not only the popular support lying behind each retirement proposal, but also the economic effects derived from changes in this elected retirement age.

One of the more significant variables that affects individual retirement decisions is the wage, therefore the wage distribution of the population will play an important role in the determination of the implemented retirement age. For this reason we have studied the behaviour of countries with a different wage distribution with regard to the retirement age in order to test the relationship between retirement age and wage distribution.

In our empirical tests, we have observed a positive partial correlation between wage distribution and effective retirement age. That is, the higher the wage inequality is, the higher the retirement age will be. See figures 1 and 2.

The simple bivariate regressions that we test are not enough to conclude the positive effect of inequality on the retirement age. But from this intuition arises the model developed in this article.

As Casamatta et al.[2000]; we develop a two-staged political economy model. In the rst stage government chooses the redistribution level of the Social Security Program, according to three different criteria. At this constitutional stage the choice is made under the knowledge that the retirement age will be chosen by majority voting in the second stage.

\[1\] See appendix.
In the second stage, the majority voting election of retirement age is made by individuals knowing exactly the redistribution level and voting accordingly.

In societies where the pretax median voter’s wage is below the pretax mean wage (the usual assumption), the retirement age being chosen through majority voting system allows us to observe the following: the less wage dispersion a society has, that is, the less the difference between the mean and the median wage is, the later the socially elected retirement age will be. So, if Social Security System increases its level of redistribution, reducing the difference between post-tax median and mean income, then the elected retirement age will be delayed. All that will lead to positive effects on total production from redistribution.

Also, our model implies that not only the poor people but also the richest one would prefer some level of redistribution, because of the elected retirement age would be closer to the rich people’s optimal ones.

The organization of this paper is as follows: the next section presents the theoretical model; the third one shows the majority voting process on retirement age; the fourth section presents the government decision on redistribution; in the fifth one we show the concluding remarks and the paper winds up with the appendix showing the empirical tests.

2. The model

In this model we have a continuous distribution of agents on wage that will be located between a minimum and a maximum wage level, \([w_p; w_r]\); belonging to the same generation and differentiated only in wage. There is no uncertainty on the length of life and each individual lives exactly \(T\) years. On the first \(R\) years the individual will be a full time worker whereas on the following \(T-R\) ones the individuals will be retired.

As in our previous model, Lacomba and Lagos [2000]; these individuals have a stationary and temporally independent utility function, which is separable and strictly increasing in consumption and leisure. Leisure yields utility to the individual only when this individual is retired. So the only way utility coming from leisure can be modified is by changing the retirement age. The pension benefits are received only after they leave the labor force. The instantaneous utility function is, then, as follows

\[
U \left( c_t; \mu_t \right) = u \left( c_t \right) + v \left( \mu_t \right)
\]  

\(2.1\)
where $c_t^i$ is the consumption at period $t$ of agent $i$. The utility of consumption is twice differentiable with $u_0^C > 0$, $u_{00}^C < 0$. Let $\mu_t$ be the leisure at period $t$, being the utility of leisure $v(\mu_t) = 0$; in their working years and $v'(\mu_t) = v$; in their retirement years. Besides, we assume that the elasticity of marginal utility of consumption with respect to consumption $\frac{1}{2} = \frac{u''(c)}{u'(c)}$ is non-increasing and smaller than one.\(^2\)

Workers plan the consumption, savings and retirement in order to maximize the discounted value of utility subject to their lifetime budget constraint. We assume that individuals earn an annual wage, $w_i$, until they retire. Later, they receive an annual pension, $p_i$, from a Social Security Program $\xi; \zeta$ where $\xi$ is a constant Social Security contribution tax rate and $\zeta$ a variable that measures the redistribution level of the system. The labor supply is constant in order to avoid incentives problems.

Social Security Programs may be financed through two different systems. On the one hand, a "pay as you go" system (PAYG) where the pensions of retired people are paid by working people through taxes. On the other hand, a "fully funded" system where the pensions of retired people are financed through the return of the taxes that they paid during their working life. In the PAYG the return will depend on the population growth rate, while in a fully funded system will depend on the interest rate. When the population growth rate is identical to the interest rate, if the rest of parameters of Social Security Program are equals, the pension that retired people receives will be the same in both financing systems.

In the present model the population growth rate and the interest rate are constant and equal to zero. Therefore the pension will be the same independently of the system. We assume a fully funded system due to that there are no retired people during the working years of the first generation.

Let $\pm$, $r$ be the subjective rate of time preference and the market rate of interest. Then the lifetime utility can be written as

\[
\begin{align*}
\sum_{t=0}^{T} u_i c_t^i \mu_t^\xi dt = & \sum_{t=0}^{T} u_i c_t^i e^{\pm t} dt + \sum_{t=R}^{T} u_i c_t^i e^{\pm t} + \sum_{t=R}^{T} v_i \mu_t \phi_t e^{\pm t} dt \\
0 & 0 \quad R
\end{align*}
\] (2.2)

\(^2\)This elasticity is the well-known coefficient of relative risk aversion.
\[
Z^r \quad Z^k \quad Z^r
\]
\[s:t: \quad c_i e^{rt} dt = w_i (1 - \xi) e^{rt} dt + p_i e^{rt} dt \quad (2.3)\]

We assume that saving earns no interest and that individuals do not discount their future income, \((\pm = r = 0)\). There is a perfect capital market, so people may borrow at zero interest rate. These assumptions together with the separability and concavity of utility function imply that each individual consumes the same in every period, \(c_t^i = c_i\) for any \(t\).

Thus the utility function of an individual over his life-cycle can be reduced to

\[U = U(c_i) + (T - R)v\quad (2.4)\]

where

\[c_i = \frac{1}{T} \left( R w_i (1 - \xi) + (T - R) p_i \right)\quad (2.5)\]

We assume that the pension, \(p_i\), is given by

\[p_i = \frac{R}{T} \frac{R - \xi}{R} W_i\quad (2.6)\]

being \(R = (T - R)\) the ratio between working and retirement years and \(W_i = [(1 - \xi) \mu + \xi w_i]\) a linear combination of the mean wage \(\mu\), and the individual \(i\)'s wage, \(w_i\), where \(\xi \in [0; 1]\); with \(\xi = 0\) meaning full redistribution, and \(\xi = 1\), actuarially fairness.\(^3\)

The budget is annually balanced, that is, total tax contributions are equal to total pension benefits.

3. Majority Voting Process

In this section, the agents have to choose the compulsory retirement age through a majority voting system. The others parameters are exogenously given. According to this, we can start studying the optimal retirement age for each individual at birth. Once we substitute \((2.5)\) and \((2.6)\) for \((2.4)\), and after some simplifications, the optimization problem that the individual faces is as follows

\(^3\)The case in which \(\xi = 1\) is equivalent to private system, where the consumption is \(c_i = \frac{R}{T} w_i\).
\[
\max_R \quad \mathcal{U} \left[ \frac{1}{\mu} \left[ w_i (1 \cap z) + \zeta \left( (1 \cap \theta) \psi + \theta w_i \right) \right] + (T \cap R) \nu \right] \quad (3.1)
\]

which yields the following first and second order conditions

\[
\frac{\partial \mathcal{U}}{\partial R_i} = u^0(c_i) [w_i (1 \cap z) + \zeta \left( (1 \cap \theta) \psi + \theta w_i \right)] \quad \nu = 0 \quad (3.2)
\]

\[
\frac{\partial^2 \mathcal{U}}{\partial R_i^2} = H = [w_i (1 \cap z) + \zeta ((1 \cap \theta) \psi + \theta w_i)]^2 \frac{1}{T} u^{\theta}(c_i) < 0 \quad (3.3)
\]

In this manner we obtain the optimal retirement for each individual. Since (3.3) is negative for all \( R \), the utility function is concave with respect to retirement age. That is, preferences are single peaked with respect to \( R \); therefore the median voter theorem may be also applied.\(^4\) Let \( R_i^\mu \) be the optimal retirement age for agent \( i \); that is, agent with wage \( w_i \). Then if \( R_i^\mu \) is a monotonic function of \( w_i \) the voting equilibrium can be easily obtained since the median voter will be the individual with median wage. In order to determine if this occurs we use the following expression that is derived from implicitly differentiating (3.2)

\[
\frac{\partial R_i^\mu}{\partial w_i} = \frac{[1 \cap \zeta ((1 \cap \theta) \psi)] [u^0(c_i) (1 \cap \frac{\nu}{\mu})]}{H} \quad (3.4)
\]

This expression is positive, \( \frac{\partial R_i^\mu}{\partial w_i} > 0 \); since \( (1 \cap \frac{\nu}{\mu}) > 0 \) (recall that we assume a relative risk coefficient less than one). If the coefficient of relative risk aversion is larger than one, \( \frac{\nu}{\mu} > 1 \), the sign of (3.4) would be the opposite, \( \frac{\partial R_i^\mu}{\partial w_i} < 0 \); which would yield opposite results.

Remark 1. The optimal retirement age is increasing with the wage.\(^5\) The median wage voter is the median voter.

Therefore the higher the wage is, the later an agent wishes to stop working, i.e., richer people desire to work longer than poorer people (see Muñoz [1995]).

\(^4\) We suppose voters have no strategic behaviour. They vote for the closest age to their own optimal retirement age.

\(^5\) It is important to note that if an agent \( i \) increases his wage, then the mean wage (\( \$ \)) will be affected too. But here we are focusing on another problem, how the optimal retirement age would change if the individual is in different levels of wage. In this case the mean wage does not change, thus we assume \( \frac{\partial \mathcal{U}}{\partial w_i} = 0 \).
Higher compulsory retirement age yields higher total production, since delaying retirement age implies that there will be more workers in the society.\(^6\)

Recall that we consider that the higher the amount of people close to the mean wage the more egalitarian the society is. This way, it can be pointed out that if we consider two societies, both with the same mean wage, when median wage is lower than mean wage, which is the usual case, the more egalitarian one will have the highest production and the highest total consumption. Since the median voter in the egalitarian economy will have a larger wage and, hence, he will choose a higher retirement age.

**Proposition 3.1.** When median wage is lower than mean wage, given a Social Security Program, the closer the median wage to the mean one is,\(^7\) the higher the retirement age would be. More egalitarian societies yield higher level of total production.\(^8\)

The proof follows directly from (3.4).

Another issue to observe is how the result of the voting changes when the redistribution level of the Social Security Program is modified. In order to come to this point, we derive the following expression from (3.2)

\[
\frac{\partial R}{\partial \delta} = \frac{\partial}{\partial \delta} \left( w_i - \$ \right) u^0(c_i) \left( 1 - \frac{1}{2} \right)
\]

The sign of this expression depends on the wage level of each individual.

> From (3.5) and the comparative-static response three results are obtained, but we focus just on the most realistic one, namely, when median wage is below

\(^6\)Labor supply could depend on \(R\) but as we assume a fixed labor supply there will not be incentives problem and this effect is avoided. We also consider that labor demand is able to absorb any increase in the working period.

\(^7\)Assuming that mean wage is fixed.

\(^8\)This result goes apparently in opposite direction with the partial correlation referred in the introduction, that is, the higher the wage inequality is, the higher the retirement age will be. But it is necessary to point out that in the model there are no retired people. However, in the empirical contrast we have used real data where, obviously, retired people are taken into account. It is logical to assume that these retired people would support the highest retirement age, since this would affect positively the dependency ratio and it would therefore improve the pension benefits. Due to this retired people's weight, the median voter's wage would be higher than median wage and it could be higher than mean wage. If this would happen the theoretical results would coincide with the empirical contrast.
mean wage.\textsuperscript{9} Let $R_m^w$ be the optimal retirement age for median voter, that is, the agent with median wage $w_m$:

**Proposition 3.2.** Let $w_m < \$\$ then $@R_m^w=\$\$ < 0$. An increase in the degree of redistribution delay the elected retirement age.

The proof follows directly from (3.5). The median voter will increase his income due to the increase in the redistribution degree, so he will have a later optimal retirement age that will be implemented through the voting process. All that will lead to an increase in the working population and therefore to a rise in the total production of the society.

Thus given a Social Security Program with parameters $h; \iota$, increasing progressivity of $\$\$ would produce a rise in total production\textsuperscript{10}.

Since in this system retirement age is chosen by the population in a voting process, it may be better for the richer people some degree of redistribution. This way, by increasing this degree, the elected retirement age would be therefore closer to their optimal ones. Moreover, it may happen that in high wage dispersion societies, richest people would be better with a full redistribution program ($\$\$ = 0) rather than with an individual’s earning one ($\$\$ = 1), given that implemented retirement age will be chosen by the median voter on a voting process.

This occurs since the median voter would receive much more money on his pension in the r st case than in the second one. This would lead him to have a higher optimal retirement age. So the elected retirement age would be closer to the one preferred by the richest people.

4. Government Decision

We now analyze how the degree of redistribution is determined. Social Security Program is defined by two parameters, $h; \iota$; where the tax level is exogenously given and the redistribution level of Social Security is determined by the government.

\textsuperscript{9}When median wage is equal to mean wage, $w_m = \$\$; $@R_m^w=\$\$ = 0$. That is, changes in the redistribution level do not alter the elected retirement age. When median wage is above mean wage, $w_m > \$\$, $@R_m^w=\$\$ > 0$. More redistribution leads to decreases in the total production since retirement age is put forward.

\textsuperscript{10}The same results can be obtained if, instead of $@R_m^w=\$\$, we would have calculated $@R_m^\iota=\$\$: That is, for $w_m < \$\$, increases in the tax level would delay the elected retirement age.
We define three social welfare criteria. A government with a Downsian criterion that cares only about the median citizen. A government with a right-wing criterion that cares only about the richest people. A government with a left-wing criterion that cares only about the poorest people.

If the political parties do not have policy preferences and the policy space is one dimensional then the only possible government criterion in equilibrium is the Downsian one. But if parties are ideological ones and they are uncertain about preferences of the voters then they may have different criteria in equilibrium. Therefore, as Lee [1999]; we simply assume that the three criteria are possible in a majority voting context and analyze the results under each one.

The government chooses the level of redistribution taking into account that this one will affect the voting process on $R$. Once $R$ is determined, retirement age will be chosen democratically, as described in the previous section, i.e., it will be the median voter’s optimal one.

Given that $w_m$ is median voter’s wage level, $R_m^a$ will be the implemented retirement age. We define an individual indirect utility function, $V_i$; on the median voter’s retirement age and redistribution degree, $R_m^a$ and $\alpha$

$$V(R_m^a(\alpha); \alpha) + (T_i \cdot R_m^a(\alpha)) v$$

Thus, the government solves the following problem

$$\max_{\alpha} V_i (R_m^a(\alpha); \alpha) \Rightarrow \max_{\alpha} T u(c_i(\alpha)) + (T_i \cdot R_m^a(\alpha)) v$$

for $i = p; m; r$ (4.1)

being $V_i$ the indirect utility function of the poorest individual, $V_p$, the median, $V_m$, and the richest one, $V_r$, depending on the Government’s criterion. By differentiating $V(R_m^a(\alpha); \alpha)$ with respect to $\alpha$ we get

$$\frac{dV_i}{d\alpha} = \frac{\partial V_i}{\partial \alpha} + \frac{\partial V_i}{\partial R_m^a} \frac{\partial R_m^a}{\partial \alpha}$$

where the first term, $\frac{\partial V_i}{\partial \alpha}$, gives us the direct impact on the individuals’ utility of a change in the redistribution system, $\alpha$. The second term, $\frac{\partial V_i}{\partial R_m^a} \frac{\partial R_m^a}{\partial \alpha}$; shows the indirect impact of an increase in the redistribution level on their welfare, as a consequence of the changes on the retirement chosen by the median voter.
4.1. Downsian Criterium
In this model, the median citizen is the individual with the median wage, so we consider that under Downsian criterium, government cares about median worker’s wage. Therefore, here the government maximizes the utility of the median wage’s worker, $V_m$; with respect to the redistribution degree, $\alpha$.

Thus, the government solves the following problem

$$\max_{\alpha} V_m (R_m^\alpha (\alpha); \alpha) \quad \max_{\alpha} T u (c_m (\alpha)) + (T_i - R_m^\alpha (\alpha)) v$$  \hspace{1cm} (4.3)

Let $\alpha^m$ be the solution to (4.3).

Proposition 4.1. Let $w_m < $ (w_m > $) then $\alpha^m = 0$ ($\alpha^m = 1$):

Proof. By differentiating (4.3) with respect to $\alpha$ we get

$$\frac{dV_m}{d\alpha} = \frac{\partial V_m}{\partial \alpha} + \frac{\partial V_m}{\partial R} \frac{\partial R_m^\alpha}{\partial \alpha}$$  \hspace{1cm} (4.4)

If $w_m < $ (w_m > $) the direct impact will be always negative (positive)

$$\frac{\partial V_m}{\partial \alpha} = u^0 (c_i) R_m^\alpha (w_m < $) < 0$$  \hspace{1cm} (4.5)

The less redistributive the program is, the lower the utility of the median worker would be. With respect to the sign of the indirect impact

$$\frac{\partial V_m (R_m^\alpha (\alpha); \alpha)}{\partial R}$$  \hspace{1cm} (4.6)

and

$$\frac{\partial R_m^\alpha}{\partial \alpha}$$  \hspace{1cm} (4.7)

Since the utility function is evaluated at $R_m^\alpha$, (4.6) is always equal to zero. Hence, the indirect impact will be zero.

Therefore, if $w_m < $ (w_m > $) then $dV_m/d\alpha < 0$ ($dV_m/d\alpha > 0$) what implies $\alpha^m = 0$ ($\alpha^m = 1$): Q.E.D.

Although changes in the redistribution level lead to changes on the retirement age chosen by the median voter, which affect indirectly the agent’s utility, for the
median worker this indirect effect will be null, since the elected retirement age will be his own optimal one.

Therefore, in this case the redistribution degree chosen by the government will depend only on the relation between median and mean wage. When median voters' wage is lower than mean wage, i.e., \( w_m < \$ \), the government under Downsian criterium will implement a Social Security System with maximal redistribution:

Consequently, from Downsian criterium arises that when \( w_m < \$ \), the Social Security System will be fully redistributive. But, in this model, we do not consider retired people, who could lead median voter's wage to be higher than median and mean wages.\(^{11}\) In that case, we could find societies in which more than 50% of the working population have wages lower than mean wage and, however, under Downsian criterium, there would not be redistribution. Because when median voters' wage is higher than mean wage, i.e., \( w_m > \$ \); the government under Downsian criterium will implement a Social Security System with no redistribution, that is, \( \alpha^m = 1 \):

4.2. Right-wing Criterium

Here the government maximizes the utility of the highest wage (\( w_r \)) individuals, \( V_r \), with respect to the redistribution degree, \( \alpha \).

Thus, the government solves the following problem

\[
\max_{\alpha} V_r \left( R_m^\alpha ; \alpha \right) \quad \text{max} \quad T \left( c_r \left( \alpha \right) \right) + \left( T - R_m^\alpha \left( \alpha \right) \right) v
\]

(4.8)

Let \( \alpha^* \) be the solution to (4.8).

Proposition 4.2. Let \( w_m < \$ \). A right-wing criterium does not imply no redistribution:

Proof. By differentiating (4.8) with respect to \( \alpha \) we get

\[
\frac{dV_r}{d\alpha} = \frac{\partial V_r}{\partial \alpha} + \frac{\partial V_r}{\partial R_m^\alpha} \left( \frac{\partial R_m^\alpha}{\partial \alpha} \right)
\]

(4.9)

The direct impact will be always positive. The less redistributive the program is, the higher the utility of the richer worker would be.

\(^{11}\)Given that retired people would support the highest retirement age, since there would be more working population, and their pension benefits depend positively on them. See Lacomba and Lagos [2000].
\[
\frac{dV_r}{\partial \delta} = u^0(c_i) R_m^\mu \cdot (w_r ; \$) > 0
\]

Again, we need to know the sign of the indirect impact

\[
\frac{dV_r (R_m^\mu (\delta); \delta)}{\partial \delta}
\]

and

\[
\frac{dR_m^\mu}{\partial \delta}
\]

Since optimal retirement age of the richest individual, \(R_r^\mu\), is higher than the optimal one of the median voter, \(R_m^\mu\), and taking into account the single peakness of the preferences, when the retirement age is increased from \(R_m^\mu\), the difference between the optimal retirement age of the low wage individuals and the compulsory one will be shorter. This will affect his utility positively. In other words, the richest group is currently working less than its optimum. This way, increases in the working years lead to increases in the utility for the individuals. For this reason (4.11) is always positive. On the other hand, the sign of (4.12) depends on whether the median voter's wage is above or below the mean wage (see (3.5)). Hence, the indirect impact may be positive or negative.

When median voters' wage is lower than the mean wage, i.e., \(w_m < \$\); (4.12) is negative. Hence, the indirect impact is negative.

As we have that the direct effect, (4.10), is always positive, by these opposite effects, we can find a level of redistribution \(\delta < 1\) where \(dV_r = d \delta \geq 0\): Q.E.D.

Then it may be deduced that the richest people would be better with some level of redistribution, i.e., \(\delta < 1\):

On the other hand, although \(w_m > \$\) is an unrealistic case, it is interesting to include this result since allow us to compare it with the realistic case, \(w_m < \$\): As it would be expected, if \(w_m > \$\) and the government would have a right-wing criterium, there would be no redistribution in the Social Security system.\(^{12}\)

\(^{12}\)In this case, both direct and indirect effects have the same positive sign. This implies \(\delta = 1\):
4.2.1. Numerical example

Here we show an example where the rich people are better with some degree of redistribution than with no redistribution. In this exercise we calculate the utility of the rich but evaluated at $R_m^\alpha$; that is, the retirement age given by the voting process. The optimization problem that an individual faces is as follows

$$
\max_{R} U(R) = \mu R + (T - R) v
$$

(4.13)

where $W_i = w_i(1 - \xi) + \xi ((1 - \beta) i + \beta w_i)$ with $W_i = w_i$ if $\beta = 1$. In our example the utility function that we use is

$$
U(c) = k^D c
$$

with $k > 0$

We have to determine $R_m^\alpha$; i.e., the optimal retirement age of the median voter. According to this, we obtain the first order conditions

$$
\frac{\partial U}{\partial R_m^\alpha} = \frac{kW_m}{2} \mu v = 0, \quad \frac{kW_m}{2} \frac{\partial R_m^\alpha}{W_i} = v
$$

Then the optimal retirement age of the median voter is given by

$$
R_m^\alpha = \frac{K}{4v^2} W_m T
$$

Substituting $R_m^\alpha$ in the utility function of the rich we obtain

$$
U_r = \mu R_m^\alpha + (T - R_m^\alpha) v
$$

Equivalently

$$
U_r = K^D W_m \frac{\mu}{4v^2} W_r + \frac{\mu}{4v^2} \frac{K^D W_m T}{4v^2} v
$$

We want to test if the utility of the rich with some redistribution is higher than with no redistribution.

$$
U_r(\beta < 1) > U_r(\beta = 1)
$$

Equivalently
As \( w_r = aw_m \) and \( $ = bw_m \) with \( a > b > 1 \) what we have to obtain is

\[
2^p \left( (1 \tilde{\zeta} + \tilde{\zeta}((1 \tilde{\zeta} \oplus b + \oplus)(a(1 \tilde{\zeta}) + \tilde{\zeta}((1 \tilde{\zeta} \oplus b + \oplus)\right) \\
> 2^p \frac{a}{\tilde{\zeta} + 1}
\]

This inequality will be true when the highest and mean wages are much bigger than the median wage. It is not difficult to find values where this inequality holds, for instance: \( a = 4; b = 3; \tilde{\zeta} = 0.3; \oplus = 0 \) In this particular case, the rich agents prefer not only redistribution but full redistribution.

The intuition here is that although it is true that lowering \( \oplus \) decreases the richest people's wealth, this redistribution also would increase the median voter's income, (recall that \( w_m < $ \) ) and this would delay his optimal retirement age. Consequently, the compulsory age would be increased having positive consequences for the richest group, since the implemented retirement age will be closer to their optimal one.

### 4.3. Left-wing criterion

Here the government maximizes the utility of the lowest wage \( (w_p) \) individuals, \( V_p \), with respect to the redistribution degree, \( \oplus \).

Thus, the government solves the following problem

\[
\max_{\oplus} V_p \left( R_m^n (\oplus); \oplus \right) + (T \tilde{\zeta} R_m^n (\oplus)) \nu \quad (4.14)
\]

Let \( \oplus^p \) be the solution to (4.14).

Under the left-wing criterion the results are equal to those derived from right-wing criterion but in the opposite way.

**Proposition 4.3.** Let \( w_m < $ \): A left-wing criterium does not imply maximal redistribution:
Proof. By differentiating (4.14) with respect to \( \bar{\sigma} \) we get

\[
\frac{dV_p}{d\bar{\sigma}} = \frac{\partial V_p}{\partial \bar{\sigma}} + \frac{\partial V_p}{\partial R} \frac{\partial R_m^*}{\partial \bar{\sigma}} \tag{4.15}
\]

The direct impact will be always negative

\[
\frac{\partial V_p}{\partial \bar{\sigma}} = u^0(c_i) R_m^* (w_p - \$) < 0 \tag{4.16}
\]

The less redistributive the program is, the lower the utility of the poor one would be. On the other hand, we need to know the sign of the indirect impact

\[
\frac{\partial V_p (R_m^* (\bar{\sigma}) ; \bar{\sigma})}{\partial R} \tag{4.17}
\]

and

\[
\frac{\partial R_m^*}{\partial \bar{\sigma}} \tag{4.18}
\]

Since optimal retirement age of the poorest individual, \( R_m^* \), is lower than the optimal one of the median voter, \( R_m^* \); and taking into account the single peakness of the preferences, when the retirement age is increased from \( R_m^* \), the difference between the optimal retirement age of the low wage individuals and the compulsory one will be larger. This will have a negative effect on his utility. In other words, the poorest group is currently working more than its optimum. This way, increases in the working years lead to decreases in the utility for the individuals. For this reason (4.17) is always negative.

On the other hand, the sign of (4.18) depends on whether the median voter’s wage is above or below the mean wage (see (3.5)). Hence, the indirect impact may be positive or negative.

When median voters’ wage is lower than mean wage, i.e., \( w_m < \$ \); (4.18) is negative. Hence, the indirect impact is positive, and since the direct effect is always negative, by these opposite effects, we can find a level of redistribution \( \bar{\sigma} = 0 \) where \( dV_p = d\bar{\sigma} \geq 0 \).

Q.E.D.

Hence, when \( w_m < \$ \); it could be possible that a left-wing criterium does not imply maximal redistribution.

An intuitive explanation can be given to understand the case in which \( \bar{\sigma} = 0 \). It is easy to check that if the government could implement both parameters \( (R; \bar{\sigma}) \),
in the left-wing criterium, the poorest individual’s optimal redistribution degree would be full redistribution. But since we are considering the second-best option for government, that is, it chooses a parameter ($R$) and people choose the other one ($R'$), the implemented retirement age will be the median voter’s one. Then, it may be inferred that the poorest individual would not prefer the maximal degree of redistribution because they will have to work more than their optimum.

On the other hand, with $w_m > \$ $ and a left-wing government, there would be maximal redistribution in the Social Security system.\footnote{In this case, both direct and indirect effects have the same negative sign. This implies $\beta = 0$.}

In summary, we have analyzed three different criteria in order to determine the level of redistribution of the Social Security system. Under Downsian criterium since there is no indirect effect, the final level of redistribution depends only on the relation between median and mean wage. This explains why are so different the results under Downsian and left-wing criterium.

Under Downsian criterium, if $w_m < \$ $, given that the median wage worker always works until his optimal retirement age, there is maximal redistribution. Under left-wing criterium, if $w_m < \$ $, given that there is a decrease in the utility of the lowest wage worker derived from an increase in the compulsory retirement age due to a higher redistribution, it is possible to find a Social Security with no maximal redistribution.

Under right-wing criterium, when $w_m < \$ $, without altruism, it is possible to find some positive level of redistribution, in spite of a right-wing government.

5. Conclusions

We have tried to analyze the relation between wage distribution and effective retirement age. According to this, we have developed a voting process on compulsory retirement age, once the level of redistribution of pension benefits is determined by the government. This way, we can study the popular support lying behind each retirement proposal.

Although considering the retirement age as compulsory could seem "excessive", most of the countries have a Social Security Program without flexibility in pensionable age (Disney, [1996]):
Moreover, empirical studies (Gruber and Wise, [1997]) have shown that there is a strong relationship between the age at which benefits are available and departure from the labor force. That is, social provisions themselves provide enormous incentive to leave the labor market. Therefore, to assume $R$ as the age at which people have to leave the labor force is not so excessive.

The results obtained are rather intuitive. The optimal retirement age increases with the wage level. If the median voter’s wage is lower than mean wage, the median voter’s income is increasing with the degree of redistribution. Then if the degree of redistribution is raised, the elected retirement age will be larger and, consequently, the people will work longer. Therefore, the degree of redistribution might be positively correlated to the total production through retirement age.

In our model, median voter’s wage being higher than the mean wage implies that less egalitarian countries will have higher retirement ages. This case would explain the positive partial correlation that we have found between inequality and retirement ages.

Median voter’s wage being higher than the mean one in the regressions could be possible although median wage would be lower than mean wage, due to the presence of retired people in the voting process. Since the retired people would support the highest retirement age, this would increase the median retirement age.

This result shows the crucial role of the retired people in the different voting processes, as in this case, in the election on retirement age.

With respect to the redistribution degree, we have analyzed three different criteria. Under Downsian criterium, the final level of redistribution depends on the relation between median and mean wage. Under right-wing criterium, it is worth to note that it is possible, without altruism, to find some positive level of redistribution. At the same time, under left-wing criterium we highlight that it can be found a Social Security system with no full redistribution.

For future research, a new empirical insight could arise. Nowadays, almost every industrialized countries face unfavorable demographic trends. A decrease in the birth rate and an increase in the life expectancy affect negatively on the viability of the Social Security system. Moreover, this pressure is enforced by another trend, employees are leaving the labor force at younger and younger age. Therefore, we consider that it would be very interesting, to introduce in our
model the aging of the population to study if the effects of postponing retirement age allow to guarantee the social benefits without increasing contribution rates.
6. Appendix

6.1. Empirical Results

Most of the countries have a compulsory retirement age. Besides, this compulsory age is very similar in almost all the countries, around sixty+ve years. Due to this, it would be very di$cult to show that there exists a relation between retirement age and wage distribution using these compulsory retirement ages.

For this reason, we decide to use a di$erent data base, from U.S. Department of Health and Human Service, the e$ective retirement age. Although every country has a compulsory retirement age, most of their Social Security Programs allow for some $exibility. Workers may leave their jobs in an age di$erent from the compulsory one, but in these cases they will have to su$er some kind of punishment in terms of reduction of their pension bene$ts.

In contrast with the compulsory retirement age, signi#cant di$erences can be observed among countries when these e$ective retirement ages are compared. This allows us to verify if there is any relation between retirement age and wage distribution.

We show two di$erent regressions. We estimate by a OLS regression, where the explanatory variable is the Gini coe$cient and the explained one is the e$ective retirement age. We try to test the signi$cance of the Gini coe$cient as a variable that explains the e$ective retirement age of the countries.

\[ R_i = \hat{\beta}_1 + \hat{\beta}_2 \text{Gini}_i + U_i \quad \text{with } i = 1; \ldots; 13 \]

where \( R_i \) is the e$ective retirement age and the error terms of the model \( U_i \) are normally distributed. The results obtained in regression 1 are

<table>
<thead>
<tr>
<th>OLS estimation</th>
<th>t-statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\beta}_1 = 54.3 )</td>
<td>( t = 3.98 )</td>
</tr>
<tr>
<td>( \hat{\beta}_2 = 0.25 )</td>
<td>( R^2 = 0.22 )</td>
</tr>
</tbody>
</table>

\(^{14}\text{We use Gini coe$cient as a proxy measure of the wage inequality, since this coe$cient provide us information about the distance between median and mean wage. This data-base is described in Deininger and Squire [1996] and Atkinson et al [1995].}\)
Once the estimation is made, we obtain that the Gini coefficient is significant and, therefore, explains the effective retirement age. Also, the Gini coefficient and the effective retirement age are positively correlated since $\hat{\gamma}_2 > 0$. The determinacy coefficient $R^2$ is small, but we consider only an explanatory variable and very few data.

In regression 1 we observe a small number of countries (table 1). In spite of this, this table is useful since the data is very homogeneous and it allows us to show a positive correlation between Gini coefficient and effective retirement age.

From regression 1 it can be derived that the higher the Gini coefficient, i.e., the less egalitarian a country is, the higher the effective retirement age is. See figure 1.

With this first table the problem is that we do not have enough countries to test it correctly. For that reason, we make another OLS estimation and we use a second table (table 2) which is formed by a larger number of countries, all the OECD countries (except Iceland). The results obtained are

\begin{tabular}{|c|c|}
\hline
OLS estimation & t-statistic \\
\hline
$\hat{\gamma}_1 = 54.7$ & \\
$\hat{\gamma}_2 = 0.21$ & $t = 5.26$ \\
$R^2 = 0.23$ & \\
\hline
\end{tabular}

where similar conclusions can be interpreted from this OLS estimation. Again, we observe the positive correlation between the Gini coefficient and effective retirement age (see figure 2). That is, the higher the wage inequality is, the higher the retirement age will be.

Besides, given that the data-bases used in the empirical test are post-tax wage, it can be inferred that higher inequality leads to a lower level of redistribution ($\beta$) in the Social Security program of our model. In this way, the empirical contrast would show a negative relation between level of redistribution and retirement age. In our model this would happen when the median voter’s wage be higher than the mean wage.

\[\text{We test } H_0 : \hat{\gamma}_2 = 0 \text{ with a level of signification of } 5\%. \text{ The statistic of contrast } t = 3.98 \text{ is higher than the critical value } t_{12:0.975} = 2.179, \text{ therefore we reject the null hypothesis.}\]
<table>
<thead>
<tr>
<th>Country, Year</th>
<th>Gini levels</th>
<th>Effective Retirement Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finland, 1987</td>
<td>20.7</td>
<td>59.0</td>
</tr>
<tr>
<td>Sweden, 1987</td>
<td>22.0</td>
<td>63.3</td>
</tr>
<tr>
<td>Norway, 1986</td>
<td>23.4</td>
<td>63.8</td>
</tr>
<tr>
<td>Belgium, 1988</td>
<td>23.5</td>
<td>57.6</td>
</tr>
<tr>
<td>Luxembourg, 1985</td>
<td>23.8</td>
<td>58.4</td>
</tr>
<tr>
<td>Germany, 1984</td>
<td>25.0</td>
<td>60.5</td>
</tr>
<tr>
<td>Netherlands, 1987</td>
<td>26.8</td>
<td>58.8</td>
</tr>
<tr>
<td>France, 1984</td>
<td>29.6</td>
<td>59.2</td>
</tr>
<tr>
<td>United Kingdom, 1986</td>
<td>30.4</td>
<td>62.7</td>
</tr>
<tr>
<td>Italy, 1986</td>
<td>31.0</td>
<td>60.6</td>
</tr>
<tr>
<td>Switzerland, 1982</td>
<td>32.3</td>
<td>64.6</td>
</tr>
<tr>
<td>Ireland, 1987</td>
<td>33.0</td>
<td>63.4</td>
</tr>
<tr>
<td>United States, 1986</td>
<td>34.1</td>
<td>63.6</td>
</tr>
</tbody>
</table>

17 Year: 1995
Table 2

<table>
<thead>
<tr>
<th>Country</th>
<th>Gini levels¹⁸</th>
<th>Effective Retirement Age¹⁹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>41.72</td>
<td>61.8</td>
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<tr>
<td>Austria</td>
<td>31.60</td>
<td>58.6</td>
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<tr>
<td>Belgium</td>
<td>26.92</td>
<td>57.6</td>
</tr>
<tr>
<td>Canada</td>
<td>27.65</td>
<td>62.3</td>
</tr>
<tr>
<td>Denmark</td>
<td>33.20</td>
<td>62.7</td>
</tr>
<tr>
<td>Finland</td>
<td>26.11</td>
<td>59.0</td>
</tr>
<tr>
<td>France</td>
<td>34.91</td>
<td>59.2</td>
</tr>
<tr>
<td>Germany</td>
<td>25.20</td>
<td>60.5</td>
</tr>
<tr>
<td>Greece</td>
<td>37.67</td>
<td>62.3</td>
</tr>
<tr>
<td>Ireland</td>
<td>34.60</td>
<td>63.4</td>
</tr>
<tr>
<td>Italy</td>
<td>32.19</td>
<td>60.6</td>
</tr>
<tr>
<td>Japan</td>
<td>35.00</td>
<td>66.5</td>
</tr>
<tr>
<td>Luxembourg</td>
<td>27.13</td>
<td>58.4</td>
</tr>
<tr>
<td>Netherlands</td>
<td>29.38</td>
<td>58.8</td>
</tr>
<tr>
<td>New Zealand</td>
<td>40.21</td>
<td>62.0</td>
</tr>
<tr>
<td>Norway</td>
<td>33.31</td>
<td>63.8</td>
</tr>
<tr>
<td>Portugal</td>
<td>35.63</td>
<td>63.6</td>
</tr>
<tr>
<td>Spain</td>
<td>25.91</td>
<td>61.4</td>
</tr>
<tr>
<td>Sweden</td>
<td>32.44</td>
<td>63.3</td>
</tr>
<tr>
<td>Switzerland</td>
<td>32.90</td>
<td>64.6</td>
</tr>
<tr>
<td>United Kingdom</td>
<td>27.80</td>
<td>62.7</td>
</tr>
<tr>
<td>United States</td>
<td>37.94</td>
<td>63.6</td>
</tr>
</tbody>
</table>

¹⁸Data-base described in Deininger and Squire (1996).
¹⁹Year: 1995
7. References


