QUASI –GEOMETRIC CONSUMERS:
PANEL DATA EVIDENCE*

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ABSTRACT

This paper examines the empirical relevance of an intertemporal model of consumption with dynamically inconsistent decision makers. The model has testable implications concerning the relation between the consumers' degrees of short-run patience (self-control) and their consumption-saving decisions. Using Spanish panel data on household expenditure, we estimate the Euler equation derived from the model. We find evidence in favor of consumers' preferences being time inconsistent. Moreover, our results indicate that there are significant differences in the degrees of short-run patience across households.

Keywords: Time-inconsistency, Quasi-geometric discounting, Hyperbolic discounting, Panel data

JEL classification: D91, C23.
1 Introduction

Quasi-geometric (quasi-hyperbolic) consumers are those who have a specific type of time-inconsistent preferences such that the discount factor between the present and the first future periods (the short-run discount factor) is distinct from that between the future period and its successor (the long-run discount factor). If the short-run discount factor is higher (lower) than the long-run discount factor, then consumers are referred to as being short-run patient (impatient). The concept of quasi-geometric discounting was introduced by Strotz (1955-1956) and subsequently developed in Pollack (1968) and Phelps and Pollack (1968). In the last few years, a substantial body of literature has appeared, which studies both analytically and by simulations, the consumption-saving behavior of quasi-geometric consumers, e.g., Laibson (1997), Laibson, Repetto and Tobacman (1998), Hall (1998), Barro (1999), Krusell and Smith (2000), Harris and Laibson (2001). The present paper contributes to the literature by providing evidence on time-inconsistency from household panel data.

A distinctive feature of quasi-geometric consumers is that their actions depend on their capacity to stick to the initial plans (i.e. the degree of self-control). Consequently, such consumers face a constant conflict between their intentions and their actions. Real-life situations that illustrate the importance of a degree of self-control for individual behavior are plentiful. A well-known example is addiction: many smokers plan to quit smoking but only those who have a sufficient degree of self-control will actually drop the habit. A similar tendency is observed with diet, physical exercise, etc. There are also examples of direct relevance to economics. Some people acknowledge that they save too little in relation to what they believe they should; high-interest credit cards are commonly used to finance over-consumption; etc. (see Laibson et al. (1998)).

This paper examines the empirical relevance of a model that links the degree of the consumer’s short-run patience (self-control) to his consumption-saving behavior. Our analysis is carried out within the standard framework

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1These examples point to a general tendency to postpone unpleasant tasks (procrastination). Another related form of time-inconsistency is intoxication (see Asheim (1997) for a discussion).
of intertemporal utility maximization. We assume an infinite horizon and restrict attention to Markov recursive equilibria. Unlike in the standard case of time-consistent preferences, in our model with time-inconsistency, the rate of return on assets in the Euler equation is not equal to the market interest rate, but it is given by a state-contingent function that depends on the individual degree of short-run patience. The more short-run patient the consumer is, the higher his subjective rate of return on assets is, and, therefore, the higher his savings are.

We use Spanish panel data on household expenditure - the Encuesta Continua de Presupuestos Familiares (ECPF) - to estimate the quasi-geometric Euler equation derived from the model. All our findings support the hypothesis of time-inconsistency in preferences. Moreover, our examination suggests that there exist significant differences in the degree of short-run patience across households. To be more precise, when we divide the sample according to house tenure, we find that home-owners are more short-run patient than renters are.

The structure of the paper is as follows: Section 2 formulates the model with quasi-geometric consumers and derives the Euler equation. Section 3 describes the data, discusses the methodology of the empirical study and presents the results. Section 4 concludes.

2 Theoretical framework

Timing is discrete, $t = 0, 1, 2, ..., $ The economy is populated by $I$ infinitely-lived agents, indexed by $i = 1, 2, ..., I$. Agents differ in their degree of quasi-geometric discounting, measured by a discounting parameter $\beta_i > 0$. Agents are endowed with one unit of time, which they supply inelastically in the market. There is an uninsurable stochastic shock to individual labor-productivity. The shock follows a first-order Markov process, which is identical for all agents. The problem, solved by an agent $i$ on each date $t$, is as follows:

$$\max_{\{c_{it}, a_{it+1}\}_{\tau=t}^{\infty}} \left\{ u(c_t) + E_t \sum_{\tau=t}^{\infty} \beta_i \delta^{\tau+1-t} u(c_{i,\tau+1}) \right\}$$

subject to
with $a_{it}$ and $s_{it}$ given. We assume that $a_{it} \in [a_{\min}, a_{\max}] \subset R$ and $s_{it} \in [s_{\min}, s_{\max}] \subset R_+$ for all $\tau$. The variables $c_{it}$, $a_{it}$ and $s_{it}$ are consumption, asset holdings and an idiosyncratic shock to labor productivity, respectively; $r_{\tau} = r + \epsilon_{\tau}$ is the exogenous interest rate where $r > 0$ and $\epsilon_{\tau}$ is iid with $E(\epsilon_{\tau}) = 0; w > 0$ is the exogenous wage per unit of efficiency labor; $\delta \in (0, 1)$ is a discounting parameter, referred to as the long-run discount factor; and $E_t$ denotes the conditional expectation. The momentary utility function $u(c)$ is continuously differentiable, strictly increasing and strictly concave.

The assumption of quasi-geometric discounting leads to time-inconsistency in preferences. The intuition is as follows: From the perspectives of agent $i$ in period $t$, the expected marginal rate of substitution between utilities in two subsequent periods $t + \tau$ and $t + \tau + 1$ is equal to the inverse of the long-run discount factor $\delta$ for all $\tau > 0$

$$- E_t \left[ \frac{du(c_{it+\tau+1})}{du(c_{it+\tau})} \right] = \frac{1}{\delta}. $$

However, as period $t + \tau$ arrives, the agent’s preferences change: the expected marginal rate of substitution between utilities in periods $t + \tau$ and $t + \tau + 1$ is now equal to the inverse of the short-run discount factor $\beta_i \delta$

$$- E_{t+\tau} \left[ \frac{du(c_{it+\tau+1})}{du(c_{it+\tau})} \right] = \frac{1}{\beta_i \delta}. $$

The time-inconsistency affects the consumption-saving decisions. If $\beta_i < 1$, an agent systematically saves less (consumes more) in period $t + \tau$ relative to his previous saving (consumption) plans for this period. On the contrary, if $\beta_i > 1$, the agent always saves more at time $t + \tau$ than he thought he would. As defined in the introduction, if $\beta_i < 1$ ($\beta_i > 1$), then the agent is short-run impatient (short-run patient). If $\beta_i = 1$, then the preferences are time-consistent: the agent is equally patient in the short-run and in the long-run and always fulfills his original plans.

Due to time-inconsistency, the agent’s lifetime preferences cannot be expressed in terms of just one value function. The problem (1), (2) however can
be written recursively by introducing two distinct value functions reflecting the agent’s short-run and long-run utility valuations (see Harris and Laibson (2001) for a discussion). A recursive formulation is provided in the Appendix. We restrict our attention exclusively to such solutions to the problem (1), (2) which are interior, recursive and Markov in the current state. We assume that in all of the periods, the agent decides on consumption according to the same decision rule, called the consumption function, $c_{it} = C^i(a_{it}, s_{it})$.\footnote{The interest rate $r_t$ is not included in the decision rules because $\epsilon_t$ is assumed to be \textit{iid}.} Furthermore, we assume that $C^i(a_{it}, s_{it})$ is single-valued, continuous and differentiable on the whole domain.

As shown in the Appendix, with an interior solution, the optimal individual choice must satisfy the quasi-geometric Euler equation

$$u'(c_{it}) = \delta E_t \left\{ u'(c_{it+1}) \left[ 1 + r_{t+1} - (1 - \beta_i) \cdot C^i_1(a_{it+1}, s_{it+1}) \right] \right\},$$

where $C^i_1(a_{it+1}, s_{it+1})$ is the first-order partial derivative of the consumption function with respect to the first argument, i.e. the future Marginal Propensity to Consume (MPC) out of assets.

Consequently, in the presence of quasi-geometric discounting, it appears as if the agent faces an interest rate that depends on his discounting parameter $\beta_i$ and on the future MPC, i.e.,

$$u'(c_{it}) = \delta E_t \left\{ u'(c_{it+1}) \left[ 1 + r_{it+1} \right] \right\},$$

where an endogenous subjective interest rate $r_{it+1}$ is given by

$$r_{it+1} = r_{t+1} - (1 - \beta_i) \cdot C^i_1(a_{it+1}, s_{it+1}).$$

The relation between the degree of quasi-geometric discounting and the subjective interest rate can be understood by looking at formula (5). If preferences are time-consistent, $\beta_i = 1$, then the subjective interest rate is equal to the market interest rate, $r_{it+1} = r_{t+1}$. Provided that the MPC is positive, a decrease (an increase) in the discounting parameter $\beta_i$ makes the subjective interest rate lower (higher) than the market interest rate, $r_{it+1} < r_{t+1}$ ($r_{it+1} > r_{t+1}$). Consequently, a short-run impatient (short-run patient) agent
faces a lower (higher) subjective interest rate and saves less (more) than an agent who is equally short- and long-run patient. In the remainder of the paper, we present evidence on time-inconsistency in preferences from Euler equation estimates.

3 Empirical analysis

This section is dedicated to the empirical analysis. We first describe the data and the sample selection; we then derive the empirical model and discuss the estimation procedure; and, finally, we present the results.

3.1 The data

We use household data from the Spanish Family Expenditure Survey (Encuesta Continua de Presupuestos Familiares, ECPF). The ECPF is a rotating panel conducted by the Spanish National Statistics Office. The survey was carried out every quarter, from 1985 to 1996. The sample size of each wave is about 3,000 families. In each period, 1/8 of the households are replaced by a new sample, so that households stay in the sample for, at most, eight consecutive quarters. The ECPF provides very detailed information on expenditures, income and family characteristics (see Collado (1998) and Browning and Collado (2001) for a detailed description of the data).

The ECPF has important advantages over other data sets that have information on expenditures. Firstly, in the Spanish survey, households are interviewed for eight quarters, while the consumer surveys most widely used do not have this panel structure. For example, the British Family Expenditure Survey consists of independent cross-sections; the American Consumer Expenditure Survey follows households for four quarters, and, therefore, annual changes are not observed. Secondly, compared to the PSID, the ECPF gives detailed information on household consumption, while the PSID only reports on food expenditure.

In this paper, we use a subsample of the ECPF. We drop households that do not provide full information for at least six consecutive quarters. We select couples with the husband in full-time employment and the wife out of the labor force. We drop agricultural workers and the self-employed. We also delete households who change their dwellings during the sample period. Our final sample consists of 1,549 households that are observed for at least

6
six consecutive quarters. The total number of observations is 11,419.

3.2 The empirical model and econometric issues

The empirical model is based on the Euler equation (3). In terms of actual values, we can re-write the Euler equation as follows:

\[
\delta u'\left(c_{it+1}\right) \left[ 1 + r_{t+1} - (1 - \beta_i) C_i\left(a_{it+1}, s_{it+1}\right) \right] \left( u'(c_{it}) \right)^{-1} = 1 + \varepsilon_{it+1}, \quad E_t(\varepsilon_{it+1}) = 0.
\] (6)

The ECPF reports on individual consumption and earnings. However, it provides no information about individual asset-holdings. Therefore, to proceed, we eliminate the variable \(a_{it+1}\) from the Euler equation. Specifically, by combining the consumption function, \(C_i\left(a_{it}, s_{it}\right)\), and the individual budget constraint (2), we define the asset function, \(a_{it+1} = A_i\left(a_{it}, s_{it}\right)\). Assuming that consumption and asset functions are invertible with respect to the first argument, we construct the new asset function \(a_{it+1} = h A_i\left(c_{it}, s_{it}\right)\). By using such a function, we then re-write the MPC in terms of consumption and shocks, \(\tilde{C}_i\left(c_{it}, s_{it}, s_{it+1}\right) \equiv C_i\left(A_i\left(c_{it}, s_{it}\right), s_{it+1}\right)\).

We assume a constant relative risk aversion utility function

\[
u(c_{it}) = \frac{c_{it}^{1-\theta} - 1}{1 - \theta},
\]

where \(\theta > 0\) is the coefficient of relative risk aversion.

We consider the first-order approximation to the Euler equation. Taking the logarithms in (6) and computing the first-order Taylor expansion yields

\[
\Delta \ln(c_{it+1}) = \frac{1}{\theta} \ln \delta + \frac{1}{\theta} r_{t+1} - \frac{1}{\theta} (1 - \beta_i) \tilde{C}_i\left(c_{it}, s_{it}, s_{it+1}\right) + \frac{1}{\theta} \varepsilon_{it+1},
\] (7)

where \(\Delta\) is the first differences operator. For the MPC, we use a log-linear approximation

\[
(1 - \beta_i) \tilde{C}_i\left(c_{it}, s_{it}, s_{it+1}\right) = \mu_{0it+1} + \mu_{1i} \ln c_{it} + \mu_{2i} \ln s_{it} + \mu_{3i} \ln s_{it+1},
\] (8)

where \(\mu_{0it+1}\) depends on demographics and seasonal dummies. By substituting (8) in (7), we get the empirical Euler equation

\[
\Delta \ln(c_{it+1}) = \alpha_{0it+1} + \alpha_{1i} \ln c_{it} + \alpha_{2i} \ln s_{it} + \alpha_{3i} \ln s_{it+1} + \alpha_{4i} r_{t+1} + \nu_{it+1},
\]

\[
E_t(\nu_{it+1}) = 0,
\] (9)
where the coefficients are defined by

\[
\begin{align*}
\alpha_{0i} \equiv & \frac{\ln (\delta) - \mu_{0i}}{\theta}, \\
\alpha_{1i} \equiv & -\frac{\mu_{1i}}{\theta}, \\
\alpha_{2i} \equiv & -\frac{\mu_{2i}}{\theta}, \\
\alpha_{3i} \equiv & -\frac{\mu_{3i}}{\theta}, \\
\alpha_{4} \equiv & \frac{1}{\theta}.
\end{align*}
\]

(10)

We will estimate the Euler equation in (9) by the Generalized Method of Moments (GMM). To allow for unobserved persistent differences in household behavior, we assume that the error term in (9) has the following structure:

\[
\nu_{it+1} = \eta_i + u_{it+1}, \quad E_t(u_{it+1}) = 0,
\]

where \(\eta_i\) is a household-specific effect and \(u_{it+1}\) is a pure idiosyncratic expectational error. The undesirable consequence of this assumption is that the coefficients in the Euler equation in "levels" (equation 9) are, in general, not identified. In principle, it is possible to estimate the coefficients from the first-difference version of the Euler equation. In such a case, however, lagged values of the endogenous variables would be typically poor instruments.

Arellano and Bover (1995) show that one can identify the coefficients in the Euler equation in "levels", if the endogenous variables have a constant correlation with the household specific effects. In terms of our model, this would imply that, for all \(t, t'\), consumption and wages satisfy

\[
E(\eta_i \ln c_{it}) = E(\eta_i \ln c_{it'}), \quad E(\eta_i \ln s_{it}) = E(\eta_i \ln s_{it'}).
\]

(11)

The estimation procedure suggested by Arellano and Bover (1995) makes use of two sets of instruments: lagged levels of the endogenous variables for the equation in first differences, and lagged first differences of the endogenous variables for the equation in "levels". In this paper, we shall adopt Arellano and Bover’s (1995) approach.

As instruments, we shall use lags of consumption, wages and the interest rate. We allow for the possibility of serially uncorrelated measurement errors in consumption and wages. As a result, the first lag of consumption and wage will not be valid instruments for the equation in levels. Similarly, the second lag of consumption and wages cannot be used as instruments for the equation in first differences. According to all the above considerations, the set of instruments will be as follows:

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• For the equation in “levels”, we use first differences of consumption and wage lagged two and three periods, the interest rate lagged one period, and the exogenous variables.

• For the equation in first differences, we use consumption and wage lagged three and four periods, the interest rate lagged two periods, and the exogenous variables.

We shall empirically test the validity of the instrument set used. This will allow us to verify the validity of the model’s underlying assumptions.

Regarding the hypothesis of quasi-geometric discounting, two main tests will be carried out. Firstly, we will assess whether the preferences of the typical household are time consistent by testing the null hypothesis that the coefficients in equation (9) are equal to zero. Secondly, we will attempt to determine whether there are significant differences in the degrees of quasi-geometric discounting across households by testing the null hypothesis that the coefficients in (9) are identical for all households.

Ideally, we would also like to know whether consumers are short-run patient or short-run impatient. This however is not possible in the context of our model. The problem we face is two-fold. Firstly, we cannot distinguish between the unknown function $C_i^1$ and the term $(1 - \beta_i)$ in the approximation (8). Secondly, we cannot distinguish between the short-run and long-run discounting parameters. To be more precise, given the $\alpha$’s, defined in (3.2), we can recover all the $\mu$’s except $\mu_{0it+1}$ (to identify the latter coefficient, we would need to fix some value for $\delta$). However, we can compute the following function relating the short-run and long-run discounting parameters

$$\gamma_i (c_{it}, s_{it}, s_{it+1}) \equiv -\ln (\delta) + (1 - \beta_i) \tilde{C}_i^1 (c_{it}, s_{it}, s_{it+1}).$$

Using the estimated values for $\gamma_i$, we shall attempt to make some inferences regarding the underlying parameters of the model. For instance, for a fixed value of $\delta$, it is possible to recover the term $(1 - \beta_i) \tilde{C}_i^1$ and to use its sign for deducing whether a consumer is short-run patient or short-run impatient.

### 3.3 Results

We present the results in Table 1. The demographic variables used are the number of children (nch), the number of adults (nad), the age (hage) and the age squared (hage$^2$) of the household head. The interest rate is one on
deposits, the wage variable is labor income, and the consumption measure is total real expenditure. As is usually the case, the data on consumption has a very strong seasonal pattern (see Browning and Collado (2001) for details on the seasonal pattern of consumption in this data set). To account for seasonality, we include a set of dummies in the model that indicate the week of the year in which the survey was carried out.

We first estimate the model under the assumption that the short-run discounting parameter is identical for all consumers, $\beta_i \equiv \beta$ for all $i$, (column (1)). In this case, the coefficients $\alpha_{1i}$, $\alpha_{2i}$ and $\alpha_{3i}$ in equation (9) are constant across households, i.e. $\alpha_{1i} = \alpha_1$, $\alpha_{2i} = \alpha_2$ and $\alpha_{3i} = \alpha_3$, for all $i$. The Sargan test does not reject the validity of the set of instruments used, and this provides evidence in favor of the assumption of a constant correlation between the endogenous variables and the household-specific effects (11). To assess whether the households’ preferences are time consistent ($\beta = 1$), we test the null hypothesis that the coefficients $\alpha_1$, $\alpha_2$ and $\alpha_3$ are jointly equal to zero. The Wald test unambiguously rejects the null hypothesis ($p-value = 0.000$). This finding provides evidence that supports the hypothesis of time-inconsistency. Furthermore, by using the estimated parameters, we compute the value of $e_{\gamma_i}$ for each household in the sample. The median is positive and equal to 0.0143. Direct calculations show that, under the above value of $e_{\gamma_i}$, the median consumer is short-run impatient (patient) if the annual long-run discount factor is higher (lower) than 0.9444.\(^3\)

We then allow for heterogeneity in the dimension of the short-run discount factor. According to the theoretical model, we would expect to observe a positive relation between the degree of short-run patience and wealth. In order to test this implication of the model, we need some proxy for wealth (as we have said, wealth is not reported in the ECPF). Given that, for the typical household, the value of housing constitutes a large fraction of total wealth, we use housing tenure (reported in the ECPF) as a proxy for wealth. We proceed by dividing the sample into two groups: home-owners and renters. If owners and renters, indeed, discount the short-run future differently, all of the coefficients except the one for the interest rate in equation (9), will differ for the two groups distinguished. The results of the estimation for owners and renters are presented in columns (2) and (3), respectively.

\(^3\)The value of the annual long-run discount factor, which is typically used in macroeconomic literature, is 0.96, e.g., Aiyagari (1994), Krusell and Smith (1998). This value would imply that the median consumer in our sample is short-run impatient. However, our results are clearly not conclusive in this respect.
The first thing that we should notice here, is that the Wald test for the null hypothesis that the coefficients of lagged consumption, current and lagged wages are jointly equal to zero, rejects the null hypothesis at a 1% level, for both home-owners and renters. This result leads us to the conclusion that the preferences of both owners and renters are time inconsistent. Our next step therefore is to explore whether there are any differences between the degrees of short-run patience of owners and renters. We test the null hypothesis that the coefficients of lagged consumption, current and lagged wages are equal for both owners and renters. We find that the null hypothesis can be rejected at a 1% level. We then evaluate the function γ_i for the households in the two subsamples by using the estimated parameters. The medians for owners and renters are 0.0146 and 0.0165, respectively. Given our assumption that the long-run discount factor is the same for all households, this finding suggests that home-owners are more short-run patient than renters are.\footnote{We should point out that taking the life-cycle effect into account would only re-enforce our results. Indeed, home-owners are typically older than renters (in our sample, the difference between the average age of these two groups is about three years). If we assume that the (long-run) discount factor decreases with age, as estimates of the life-cycle models suggest (see Samwick (1998)), then the difference between the short-run patience of home-owners and renters, implied by the definition of γ_i, would increase.}

In principle, the difference observed between the median of γ_i for owners and renters could be attributed not only to differing degrees of short-run patience, but also to other existing differences between the two groups, for example, their level of consumption, their wages, demographic characteristics, etc. We therefore proceed by separating the effect associated with the different degrees of short-run patience of owners and renters, from all the other effects. Specifically, we compute the values of γ_i for the entire sample by first using the estimated coefficients for the home-owners and then using the estimated coefficients for the renters. We then perform a test of equality of medians for pairwise samples, and we reject the null hypothesis at a 1% level. This result confirms our previous conjecture that there are significant differences in the degrees of short-run patience between owners and renters, and that the short-run discount factor is higher for owners than for renters.
4 Conclusion

This paper has studied the empirical implications of an intertemporal model of consumption in which the consumers’ preferences are time inconsistent due to quasi-geometric discounting. A distinctive feature of such a model is that the rate of return on assets in the Euler equation is in general not equal to the market interest rate, but it is given by a state-contingent agent-specific function. It appears as if short-run patient (impatient) agents face a rate of return on assets which is higher (lower) than the market interest rate.

We have estimated the quasi-geometric Euler equation by using Spanish panel data on household expenditures. Our two main results are as follows: First, we have found evidence supporting the hypothesis that households’ preferences are time inconsistent. Second, after splitting the sample in two groups according to housing tenure, we have found that home-owners are typically more short-run patient than renters. We interpret the latter finding as an indication that there exists a link between the consumers’ degrees of short-run patience and their amount of wealth.

The main short-coming of our approach is that it does not allow us to identify the degrees of the consumers’ short-run patience. The following two-step procedure could help us to resolve the identification problem. First, estimate the consumption function from household data and calculate the corresponding MPC out of assets. Second, substitute the obtained MPC in the Euler equation and estimate the structural parameters of the model including the consumers’ short-run discount factors. The theoretical foundations of this approach are yet to be developed. Also, its practical implementation would require data on wealth which are not available in our data set. The outlined extension is left for future research.

References


5 Appendix

In this section, we describe the recursive formulation of the problem (1), (2). The recursive formulation of the problem faced by an agent with current state \( (a_{it}, s_{it}) \) is as follows:

\[
W^i (a_{it}, s_{it}) = \max_{c_{it}} \left\{ u(c_{it}) + \beta_i \delta E \left[ V^i (a_{it+1}, s_{it+1}) \mid s_{it} \right] \right\}, \tag{12}
\]

where given \( a_{it}, s_{it} \), the value function \( V^i (a_{it+1}, s_{it+1}) \) solves the functional equation

\[
V^i (a_{it+1}, s_{it+1}) = u \left[ C^i (a_{it+1}, s_{it+1}) \right]
+ \delta E \left\{ V^i \left[ w s_{it+1} + (1 + r_{t+1}) a_{it+1} - C^i (a_{it+1}, s_{it+1}), s_{it+2} \right] \mid s_{it+1} \right\} \tag{13}
\]

subject to

\[
a_{it+1} = w s_{it} + (1 + r_{t+1}) a_{it} - c_{it}. \tag{14}
\]

Here, one and two primes on variables indicate their values one and two periods from now, respectively.

Under the assumption of an interior solution, the optimal allocation satisfies

\[
u' (c_{it}) = \beta_i \delta EV^i (a_{it+1}, s_{it+1}), \tag{15}\]

where \( V^i \) denotes the first-order partial derivative of the value function \( V^i \) with respect to the first argument, \( a_{it+1} \). The value function \( V^i \) satisfies

\[
V^i_1 (a_{it+1}, s_{it+1}) = u' (c_{it+1}) C^i_1 (a_{it+1}, s_{it+1})
+ (1 + r_{t+1} - C^i_1 (a_{it+1}, s_{it+1})) \cdot \delta EV^i_1 (a_{it+2}, s_{it+2}), \tag{16}
\]

where \( C^i_1 \) is the first-order partial derivative of the consumption function \( C^i \) with respect to \( a_{it+1} \). Substituting \( \delta EV^i_1 (a_{it+2}, s_{it+2}) \) from the updated version of (15) into equation (16), we get

\[
\beta V^i_1 (a_{it+1}, s_{it+1}) = u' (c_{it+1}) \left[ 1 + r_{t+1} - (1 - \beta_i) \cdot C^i_1 (a_{it+1}, s_{it+1}) \right].
\]

By premultiplying both sides of the last condition by \( \delta \) and by substituting the resulting condition into (15), we obtain

\[
u' (c_{it}) = \delta E \left\{ u' (c_{it+1}) \left[ 1 + r_{t+1} - (1 - \beta_i) \cdot C^i_1 (a_{it+1}, s_{it+1}) \right] \right\}. \tag{17}
\]

This is the quasi-geometric Euler equation (3) in the main text.
Table 1
Total Expenditures

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<th>(2)</th>
<th>(3)</th>
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<td>-0.3534</td>
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<td>$r_{t+1}$</td>
<td>4.4805</td>
<td>12.0274</td>
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<tr>
<td></td>
<td>(6.6495)</td>
<td>7.7670</td>
<td></td>
</tr>
<tr>
<td>nch</td>
<td>0.0239</td>
<td>0.0296</td>
<td>0.0097</td>
</tr>
<tr>
<td></td>
<td>(0.0094)</td>
<td>(0.0135)</td>
<td>(0.0145)</td>
</tr>
<tr>
<td>nad</td>
<td>0.0887</td>
<td>0.1062</td>
<td>0.0546</td>
</tr>
<tr>
<td></td>
<td>(0.0122)</td>
<td>(0.0173)</td>
<td>(0.0237)</td>
</tr>
<tr>
<td>hage</td>
<td>0.0050</td>
<td>0.0100</td>
<td>0.0042</td>
</tr>
<tr>
<td></td>
<td>(0.0105)</td>
<td>(0.0195)</td>
<td>(0.0148)</td>
</tr>
<tr>
<td>hage$^2$</td>
<td>0.0000</td>
<td>0.0001</td>
<td>0.0001</td>
</tr>
<tr>
<td></td>
<td>(0.0001)</td>
<td>(0.0002)</td>
<td>(0.0002)</td>
</tr>
</tbody>
</table>

Sargan Test (df) 136.48 (120) 96.38 (81)

$p$ - value 0.144 0.117

$\chi^2$ Test$^a$ (df) 124.96 (3) 84.27 (3) 15.57 (3)

$p$ - value 0.000 0.000 0.001

$\chi^2$ Test$^b$ (df) 14.26 (3)

$p$ - value 0.003

$^a$ The null hypothesis is that the coefficients of lagged consumption, current and lagged wages are jointly equal to zero.

$^b$ The null hypothesis is that the coefficients of lagged consumption, current and lagged wages are equal for owners and renters.