REINTERPRETING THE MEANING OF BREAKDOWN*

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ABSTRACT

Alternating bargaining has been extensively used to model two-sided negotiations. The celebrated model of Rubinstein (1982) has provided a formal justification for equitable payoff division. A typical assumption of these models under risk is that the breakdown event means a complete and irrevocable halt in negotiations. We reinterpret the meaning of breakdown as the imposition to finish negotiations immediately. Specifically, after breakdown the last offer becomes definitive. While Rubinstein’s model predicts an immediate agreement with stationary strategies, we show that the same payoff allocation is attainable under non-stationary strategies. Moreover, the payoffs in delayed equilibria are potentially better for the proposer than those in which agreement is immediately reached.

Keywords: breakdown, bargaining, delay.

JEL codes: C72, C78
1 Introduction

Many economic situations can be modelled as two agents with independent interests who may benefit from cooperation. However, no agreement can be imposed without the approval of both individuals. Assuming the agents are rational, it seems natural to predict that cooperation will be carried out, and the question which arises is how the benefits from cooperation are bound to be distributed between them.

In his seminal paper, Rubinstein [16] addressed the above question. He studied a simple bargaining protocol of alternating-offers between two agents who want to share a pie of size 1. The agents alternatively propose a share of the pie until one of the proposals is accepted. No prior deadline is imposed on the number of offers that can be made. The driving force of the negotiation is that agents are impatient. The pie shrinks as time passes by. Binmore, Rubinstein and Wolinsky [3] reformulated the model in a way that the driving force is the possibility of breakdown after a rejection, in which case both agents get nothing. In both models, the only subgame perfect (SP) equilibrium has two main properties: First, it is stationary. Second, the advantage of being the first proposer disappears as the cost of delay (Rubinstein [16]) or the probability of breakdown (Binmore et al. [3]) vanishes\(^1\).

In this paper, we reinterpret the meaning of breakdown. In the classical literature of bargaining, breakdown means a complete and irrevocable halt in negotiations, so that no agreement is possible afterwards. Typically, two reasons are given of why the agents reach such a radical dead-end. First, the agents can get fed up as negotiations get protracted. Second, an unpredictable event or external intervention can eliminate the benefit of cooperation.

In the first case, it is certainly plausible that an agent gets fed up and wants to finish negotiation immediately. In this case, however, we claim that the rational thing to do is to accept the last offer on the table (as long as it is

\(^1\)The resemblance between the results in [16] and [3] has led to the interpretation of the cost of delay as an exogenous probability of breakdown.
better than nothing). In the second case, we must assume that the external intervention can only arise during the negotiation process².

An example can help to clarify this latter point. While two firms bargain over how to divide the returns from the exploitation of a new technology, a third firm may discover a superior technology that makes the previous one obsolete. This situation is described in [13, p. 73] as an example of breakdown due to the intervention of a third party. However, a deeper analysis shows that this is not the case. The superior technology will eliminate the gains from cooperation even after an agreement is reached.

It is then clear that the breakdown should be due to the delay in reaching an agreement. As Binmore et al. point out, while [the agents] are bargaining the opportunity might be snatched by a third party [3, p. 178]. Even then, it may be difficult to justify why, after the external intervention, the agents cannot make a quick agreement. In this case, the most reasonable agreement would be the last one still on the table. Even though there is no room for making a new proposal, or doing so is too costly, there is still possible to accept the last offer.

We model this situation as follows: Two agents bargain over the share of a pie of size 1 by alternating offers. Each time an offer is rejected, there is a (small) probability $1 - \rho$ that the last proposal becomes a ‘take-it-or-leave-it’ offer (breakdown). The responder can then reconsider to accept this last offer. There is a discounting factor $\delta < 1$, so that agents are impatient in the sense that their utility function for a piece of size $u$ at time $t$ is given by $\delta^t u$. If the last offer is rejected after breakdown, the utility is zero for both agents.

Many situations may be interpreted under this assumption. First, there can be an external authority that may force the agents to finish negotiations immediately (for example, a superior authority to which they are delegating). Second, there may exist a small probability that the negotiation proceeds are

²Otherwise, the agents would discount the potential risk of losses in their prior valuation of the pie.
made public and one of the parties cannot change his offer in order to keep its credibility. Third, one of the parties makes a mistake with a small probability (following the ideas presented in [17]) and accepts an offer that could have been improved had he been more insistent.

Hence, after an offer is made, the proposer becomes committed to it in such a way that further renegotiation is not possible. This leads to an endogenous payoff allocation after breakdown, as opposed to the standard alternating-offers model under risk3.

Our model combines both the discounting factor of [16] and the risk of breakdown of [3]. The driving force that induces the agents to reach an agreement is their impatience, i.e. the presence of a discounting factor, and not the risk of breakdown (as opposed to [3], where breakdown has a different meaning).

Our results may be summarized as follows: If the agents are patient (i.e. they do not care when agreement is reached), then their optimal strategy will be to permanently ask for the whole pie. As the agents become more impatient, delay may occur, but an agent eventually presents a reasonable offer and agreement may be reached without breakdown. If agents are impatient enough, we recover Rubinstein’s result of a unique stationary SP equilibrium with immediate agreement.

Our model generalizes [16] in the sense that both models coincide when \( \rho = 1 \). However, this model could easily be adapted to generalize [3] by allowing both kinds of breakdown: the original of [3] with no possibility of agreement, and ours with a last ‘take-it-or-leave-it’ offer.

For arbitrary values of \( \rho \) and \( \delta \), we can distinguish three regions in the set \( \{(\rho, \delta)\}_{\rho, \delta \in [0,1]} \). A first region \( IA \) where “Immediate Agreement” is reached, i.e. the responder agrees on the offer at time 0 (as in [16]). A second region \( DA \) (“Delayed Agreement”) where agreement is reached, but with a

3A recent model with complete information that makes endogenous the level of surplus destruction after the deadline is [12]. In their model, however, the deadline is also endogenously imposed.
possible delay. Finally, a third region $PD$ ("Perpetual Disagreement") where breakdown always occurs. The three regions are represented in Figure 1.

We are interested in studying the agreements that arise when $\rho$ and $\delta$ are close to 1. A value $\delta$ close to 1 can be interpreted as either that agents are patient (see [14, p. 52]) or that the interval between offers and counteroffers is short. A value $\rho$ close to 1 means that the probability of breakdown is small. Under these assumptions, region $PD$ vanishes (Figure 2). Thus, we can consider that the only significant regions are those in which the agents make reasonable offers.

The SP equilibrium payoffs for the proposer as a function of $\delta$ when $\rho = 0.5$ are depicted in Figure 3. The figures for other values of $\rho$ are similar. In $IA$ there exists a unique SP equilibrium payoff allocation $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ which coincides with Rubinstein’s. Moreover, SP equilibria are stationary and efficient (Theorem 3.1). The dotted line in Figure 3 represents the security level payoff $\frac{1-\rho}{1-\rho^2\delta^2}$, which corresponds to the strategy of permanently asking for the whole pie, waiting for breakdown to occur (Remark 2.1).

In $PD$, there does not exist any SP equilibrium with immediate agreement, and thus the dotted line in Figure 3 coincides with the unique SP equilibrium payoff (Proposition 2.1).

In $DA$ there does not exist any stationary SP equilibria, and there exists a continuum of SP equilibrium payoffs (Theorem 4.1). Remarkably, Rubinstein’s allocation $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ is always an attainable outcome in $DA$ under non-stationary strategies for any value of $\rho$ (Corollary 4.1). Hence, even though Rubinstein’s stationary equilibrium is not robust to reinterpretations of the meaning of breakdown, the allocation $\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)$ is. An immediate consequence is that the classical assumption of the meaning of breakdown can be made without loss of generality, as long the final payoff allocation is concerned.

It should be stressed that we characterize all the SP equilibria payoffs, and not only the stationary ones. There do not exist stationary SP equilibria in $DA$, but only non-stationary ones. The implication on the other direction
is also true: if there exist non-stationary SP equilibria, then there do not exist stationary ones. In IA, the only stationary SP equilibrium is efficient (i.e. agreement is reached immediately). In PD, the only stationary SP equilibrium is inefficient (agreement is only reached after breakdown).

In Section 2 we formally present the model and characterize the payoffs in PD. In Section 3 we study the stationary SP equilibria and characterize the payoffs in IA. In Section 4 we study the non-stationary SP equilibria and characterize the payoffs in DA. In Section 5 we briefly discuss the case when the agents are risk averse. In Section 6 we conclude.

2 The model

Two agents, A and B, bargain over the share of a pie of size 1 following a protocol of alternating offers: At time $t = 0, 2, 4, \ldots$, agent A is the proposer. At time $t = 1, 3, 5, \ldots$, agent B is the proposer. At time $t = 0$, A proposes a share $x = (x_A, x_B)$ of the pie, where $x_A + x_B \leq 1$. B should then accept or reject the offer. If he accepts, the game finishes with this share at time $t = 0$. If he rejects, then with probability $\rho$, the process is repeated at time $t = 1$ with B being the new proposer. With probability $1 - \rho$, the proposal made by A becomes final (breakdown), and B should decide if he accepts or rejects it. If he accepts, the game finishes with this share at time $t = 0$. If he rejects, the game finishes in disagreement and each agent receives zero (see Figure 4). Each time an offer is rejected, the value of the pie decreases by a discounting factor of $\delta \in [0, 1]$ for each agent. This means that a share of $x_i$ of the cake at time $t$ yields a utility of $\delta^t x_i$ for agent $i$.

The factor $1 - \rho$ can be considered as some kind of external factor which represents the belief that the proposal on the table will become a ‘take-it-or-leave-it’ offer. On the other hand, the discounting factor $\delta$ is an internal factor that represents the impatience of the agents.

For simplicity, we assume that agents propose admissible shares of the pie (i.e. $x_i \in [0, 1]$ for $i \in \{A, B\}$ besides $x_A + x_B \leq 1$). We say that bargaining
Figure 1: Regions IA, DA and PD.

Figure 2: Regions when $\rho$ and $\delta$ are close to 1.
Figure 3: The SP equilibrium payoffs for the proposer as a function of $\delta$ when $\rho = 0.5$. The dotted line represents the security level payoff.

Figure 4: The first two periods of the bargaining model.
finishes in agreement when the agents follow strategies in which an offer is accepted (without breakdown). Notice that by breakdown we mean that the last offer is made final, and not necessarily that each agent receives zero.

The structure of the game is stationary, and so all the subgames that begin when agent $i$ makes an offer are equivalent. Let $U$ be the set of possible SP equilibrium payoffs for the proposer in these subgames (by symmetry, it is the same for both agents). If $U$ is nonempty, we denote the supremum of $U$ as $M$, and the infimum of $U$ as $m$.

There are two kinds of SP equilibria: SP equilibria in which the first offer of the proposer is accepted and SP equilibria in which the first offer of the proposer is rejected. Let $U^a$ and $U^r$ be their respective sets of possible payoffs for the proposer.

**Remark 2.1** Notice that, given this model, $A$ can assure herself a positive expected final payoff by always offering $(1 - \varepsilon, \varepsilon)$. If her offer becomes final (i.e. breakdown occurs), the perfectness of the equilibrium will imply that $B$ accepts. Since this is true for any $\varepsilon > 0$, we will assume without loss of generality that an agent always accepts a payoff of at least 0 when the proposal is final. Hence, the SP equilibrium payoff for the proposer is at least

$$\sum_{t=0}^{\infty} (\rho\delta)^t (1 - \rho) = \frac{1 - \rho}{1 - \rho^2\delta^2}. $$

This is the final payoff for the proposer when both agents always ask for the whole pie. It represents the security level payoff for the proposer. Hence, if $U \neq \emptyset$, we have

$$\frac{1 - \rho}{1 - \rho^2\delta^2} \leq m. \quad (1)$$

Intuitively, it seems clear that if the agents are sufficiently patient ($\delta$ large with respect to $\rho$) then they will keep asking for the whole pie until a proposal is made final (breakdown). We will see that the set of pairs $(\rho, \delta)$ where this always happens is given by

$$PD := \{ (\rho, \delta) : \delta (1 - \rho) > \rho (1 - \rho\delta^2) \}$$

$^4$From now on, we omit the term ‘expected’ when referring to final payoffs.
where PD stands for “Perpetual Disagreement”.

**Proposition 2.1** If \((\rho, \delta) \in PD\), then there exists a unique SP equilibrium in which the proposer always claims the whole pie, and the responder rejects when this proposal is not final. Moreover, 

\[ U = U^r = \left\{ \frac{1 - \rho}{1 - \rho^2 \delta^2} \right\}. \]

**Proof.** Assume we are in a SP equilibrium and the proposer, say \(A\), offers \((x_A, x_B)\) and \(B\) accepts. By rejecting, \(B\) can ensure himself a final payoff of at least \(\rho \delta m + (1 - \rho) x_B\). Thus, \(\rho \delta m + (1 - \rho) x_B \leq x_B\), i.e. \(\delta m \leq x_B\). Since \(x_A + x_B \leq 1\), we have \(x_A \leq 1 - \delta m\). On the other hand, \(A\) can assure herself a payoff of at least \(m\). Thus, \(m \leq x_A\). Under (1), this implies \(\delta (1 - \rho) \leq \rho \left(1 - \rho \delta^2\right)\) and hence \((\rho, \delta) \notin PD\). This means that in PD there cannot be an accepted proposal in SP equilibrium. From this, it is clear that the only optimal strategy is to ask for the whole pie. The unique SP equilibrium strategy for player \(A\) is to propose the whole pie and accept \((x_A, x_B)\) iff \(x_A > \frac{\delta (1 - \rho)}{1 - \rho^2 \delta^2}\). The strategy for player \(B\) is analogous, and the final payoff for the proposer is \(\frac{1 - \rho}{1 - \rho^2 \delta^2}\). \(\blacksquare\)

The set \(PD\) is stretched in Figure 1. Notice that, even thought the SP equilibria given by Proposition 2.1 are "bad", they only survive when \(\delta\) approaches 1 more quickly than \(\rho\) (Figure 2).

### 3 Stationary SP equilibria

We study the SP equilibria which satisfy the following properties:

**Efficiency** Whenever a proposer makes an offer, this proposal is accepted by the responder. Moreover, the proposals always satisfy \(x_A + x_B = 1\).

**Stationarity** The proposer always makes the same offer.

Given stationarity, we denote the offer made by agent \(i\) in SP equilibrium as \(x_i^*\). Given efficiency, when \(A\) proposes \((x_A^*, 1 - x_A^*)\), \(B\) accepts. By
perfectness, the offer made by $A$ should leave $B$ indifferent to accepting or rejecting. If $B$ rejects and there is no breakdown (probability $\rho$), $B$ will propose $(1 - x^*_B, x^*_B)$ and this proposal is due to be accepted by $A$ (by efficiency). Thus,

$$1 - x^*_A = \rho \delta x^*_B + (1 - \rho) (1 - x^*_A)$$

analogously

$$1 - x^*_B = \rho \delta x^*_A + (1 - \rho) (1 - x^*_B)$$

which yields as unique solution

$$x^*_A = x^*_B = \frac{1}{1 + \delta}.$$ \hspace{1cm} (2)

Thus, any SP equilibrium payoff satisfying efficiency and stationarity should be characterized by this offer. Notice that this payoff coincides with Rubinstein’s [16]. Such a strategy profile is a SP equilibrium in the following region:

$$IA := \{(\rho, \delta) : \delta (1 - \rho) \leq \rho (1 - \delta^2)\}$$

where $IA$ stands for “Immediate Agreement”.

**Theorem 3.1**

a) There exists an efficient and stationary SP equilibrium if and only if $\delta (1 - \rho) \leq \rho (1 - \delta^2)$;

b) if $\rho (1 - \delta^2) > \delta (1 - \rho)$, then the above SP equilibrium is unique; and

c) if $\delta (1 - \rho) = \rho (1 - \delta^2)$, then there exists one inefficient SP equilibrium. In this SP equilibrium, $A$ asks for the whole pie in the first round, $B$ rejects and proposes $\left(\frac{\delta}{1 + \delta}, \frac{1}{1 + \delta}\right)$ in the second round, and $A$ accepts.

Moreover, $\delta (1 - \rho) \leq \rho (1 - \delta^2)$ implies

$$U = U^a = \left\{\frac{1}{1 + \delta}\right\}.$$

**Proof.** a) We have previously proved that any stationary and efficient SP equilibrium should be characterized by (2). The strategy profile "offer $\frac{1}{1 + \delta}$ and accept at least $\frac{\delta}{1 + \delta}$" constitutes a SP equilibrium in $IA$. To see why, assume the proposer $A$ deviates and makes an unacceptable proposal $(x_A, x_B)$.
Thus, $B$ rejects and $A$’s final payoff is not more than $\rho \delta \frac{\delta}{1+\delta} + 1 - \rho$, which is not more than $\frac{1}{1+\delta}$ because $(\rho, \delta) \in IA$.

Assume now $(\rho, \delta) \notin IA$. By deviating and proposing $(1,0)$, $A$ gets $\rho \delta \frac{\delta}{1+\delta} + 1 - \rho$ which is more than $\frac{1}{1+\delta}$ because $(\rho, \delta) \notin IA$. This shows that there does not exist efficient and stationary SP equilibria when $(\rho, \delta) \notin IA$.

We now prove that $U = \{ \frac{1}{1+\delta} \}$. Since the above strategies constitute a SP equilibrium in $IA$, we know that $U \neq \emptyset$ in $IA$. Applying the above reasoning to the arguments presented in [19] (see [13, Section 3.2.2]) we can prove that the following inequalities hold in $IA$: $1 - \delta M \leq m$, $M \leq 1 - \delta m$, $m \leq 1 - \delta M$, and $1 - \delta m \leq M$.

Hence, we deduce that

\[
\begin{align*}
m &= 1 - \delta M \\
M &= 1 - \delta m
\end{align*}
\]

which yield a unique solution $M = m = \frac{1}{1+\delta}$. This proves that $U = \{ \frac{1}{1+\delta} \}$.

b) Assume $\rho \left(1 - \delta^2\right) > \delta \left(1 - \rho\right)$. It is enough to prove that all SP equilibria are efficient. Assume, on the contrary, that there exists a SP equilibrium where $A$ proposes $(x_A, x_B)$ and $B$ rejects. Let $v$ be $A$’s payoff in the subgame which begins when $B$ is the proposer. Thus, $A$’s final payoff is

\[
\rho \delta v + (1 - \rho) x_A \leq \rho \delta \frac{\delta}{1+\delta} + 1 - \rho.
\]

Since we know that $A$’s final payoff is $\frac{1}{1+\delta}$, we deduce that

\[
\frac{1}{1+\delta} \leq \rho \delta \frac{\delta}{1+\delta} + 1 - \rho
\]

which is not possible when $\rho \left(1 - \delta^2\right) > \delta \left(1 - \rho\right)$.

c) Assume $\delta \left(1 - \rho\right) = \rho \left(1 - \delta^2\right)$. Assume $A$ proposes $(x_A, x_B)$ and $B$ rejects. Let $v$ be $A$’s payoff in the subgame which begins when $B$ is the proposer. By an analogous argument as in the proof of part b), we prove that

\[
\rho \delta v + (1 - \rho) x_A = \rho \delta \frac{\delta}{1+\delta} + 1 - \rho
\]
and thus \( v = \frac{\delta}{1+\delta} \) and \( x_A = 1 \). This implies that \( A \) asks for the whole pie in the first round (because \( x_A = 1 \)) and that the second round is efficient (because \( v = \frac{\delta}{1+\delta} \)).

We represent the set \( IA \) in Figure 1. The set \( IA \) includes the case \( \rho = 1 \), where this model coincides with Rubinstein’s [16].

4 Non-stationary SP equilibria

Since we have characterized the SP equilibria in \( IA \) and \( PD \), we focus our attention on the remaining values of \((\rho, \delta)\). We define

\[
DA := \{ (\rho, \delta) : (\rho, \delta) \notin IA \cup PD \}
\]

where \( DA \) stands for “Delayed Agreement”.

Let

\[
\tilde{m} := \max \left\{ \frac{(1 - \delta)(1 + \rho \delta)}{1 - \rho^2 \delta^2}, \frac{1 - \rho}{1 - \rho^2 \delta^2} \right\};
\]

\[
\tilde{M}^r := \rho \delta (1 - \tilde{m}) + 1 - \rho;
\]

and

\[
\tilde{M}^a := 1 - \delta \tilde{m}.
\]

**Theorem 4.1** If \((\rho, \delta) \in DA\), then there exist non-stationary SP equilibria. Furthermore, the set of SP equilibrium payoffs for the proposer is given by

\[
U = U^r = \left[ \tilde{m}, \tilde{M}^r \right]
\]

and

\[
U^a = \left[ \tilde{m}, \tilde{M}^a \right].
\]

These intervals are represented in Figure 3 for \( \rho = 0.5 \). Notice that when \((\rho, \delta) \in DA\), we have \( \tilde{M}^a < \tilde{M}^r \).

The strategy profiles which yield these intervals are described in Table I. We consider the language of automata (see [14, p. 39-40]). Acceptance applies when the offer is not final. If the offer is final, the responder always
accepts (as explained in Remark 2.1). The strategies are the most favorable to \( A \) in \( \text{Favor}_A \); the strategies are the most favorable to \( B \) in \( \text{Favor}_B \); the initial offer is accepted in \( \text{Agreement} \) when \( u < \tilde{M}^a \); and the initial offer is rejected in \( \text{Delay} \).

### TABLE I: SP equilibrium strategies in \( DA \)

<table>
<thead>
<tr>
<th>A offers</th>
<th>Favor ( A )</th>
<th>Favor ( B )</th>
<th>Agreement</th>
<th>Delay</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1, 0))</td>
<td>((\tilde{m}, 1 - \tilde{m}))</td>
<td>((u, 1 - u))</td>
<td>((\frac{u - \rho \delta (1 - \tilde{m})}{1 - \rho}, 0))</td>
<td></td>
</tr>
<tr>
<td>( A ) accepts</td>
<td>( x_A \geq 1 - \tilde{m} )</td>
<td>( x_A &gt; \delta \tilde{m} )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( B ) offers</td>
<td>((1 - \tilde{m}, \tilde{m}))</td>
<td>((0, 1))</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>( B ) accepts</td>
<td>( x_B &gt; \delta \tilde{m} )</td>
<td>( x_B \geq 1 - \tilde{m} )</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Transitions</td>
<td>if ( A ) deviates, go to ( \text{Favor}_B )</td>
<td>if ( B ) deviates, go to ( \text{Favor}_A )</td>
<td>if ( A ) deviates, go to ( \text{Favor}_B )</td>
<td>Otherwise, go to ( \text{Favor}_A ).</td>
</tr>
</tbody>
</table>

The following corollary, whose proof is straightforward, is an important consequence of Theorem 4.1.

**Corollary 4.1** Given \((\rho, \delta) \in DA\),

\[
\frac{1}{1 + \delta} \in U^a.
\]

**Proof of Theorem 4.1.** Assume \((\rho, \delta) \in DA\). Given \( u \in \left[\tilde{m}, \tilde{M}^a\right] \), it is straightforward to check that the strategies in Table I constitute a SP equilibrium when the initial state is \( \text{Agreement} \). Moreover, the first offer is accepted and \( A \)'s final payoff is \( u \). Hence,

\[
\left[\tilde{m}, \tilde{M}^a\right] \subset U^a.
\]  

(3)

Analogously, given \( u \in \left[\tilde{m}, \tilde{M}^r\right] \), the strategies in Table I constitute a SP equilibrium when the initial state is \( \text{Delay} \). Moreover, \( A \)'s first offer is rejected, if there is no breakdown, \( B \)'s first offer is accepted, and \( A \)'s expected final payoff is \( u \). Hence,

\[
\left[\tilde{m}, \tilde{M}^r\right] \subset U^r.
\]  

(4)
from where we conclude that $[\tilde{m}, \tilde{M}] \subset U$.

Let $M^a$ be the supremum of the SP equilibrium payoffs in which the proposer’s initial offer is accepted. Let $M^r$ be the supremum of the SP equilibrium payoffs in which the proposer’s initial offer is rejected. Clearly, $M = \max \{M^a, M^r\}$. Notice that both $M^a$ and $M^r$ are well-defined because $U^a$ and $U^r$ are nonempty.

Since $1 - \delta M \leq m$, under (1) it is straightforward to check that $M \leq \tilde{M}^r$ and $\tilde{m} \leq m$. Hence, $U = [\tilde{m}, \tilde{M}^r]$.

The last step is to prove that $M^a = \tilde{M}^a$ and $m^r = m^a = m$, where $m^r$ (resp. $m^a$) is the minimum of the SP equilibrium payoffs in which the proposer’s initial offer is rejected (resp. accepted).

Under (3), we know that $m^a \leq \tilde{m} = m$. Thus, $m^a = m$.

Under (4), we know that $m^r \leq \tilde{m} = m$. Thus, $m^r = m$.

Under (3), we know that $\tilde{M}^a \leq M^a$. Since $1 - \delta M \leq m$, under (1) it is straightforward to check that $M^a \leq \tilde{M}^a$. Thus, $M^a = \tilde{M}^a$. ■

### 5 Risk aversion

In the previous sections we have assumed that the agents are risk-neutral. Hence, a natural question is how risk aversion would affect our results. In the original Rubinstein’s paper there is no randomness and hence no need to discuss risk attitudes. However, in our model risk aversion is not innocuous because the agents should make comparisons between lotteries.

Without going into formal details, we can still conjecture how risk aversion affects our results. We study the frontier line between $IA$ and $DA$, say

$$\partial A := \{(\rho, \delta) : \delta (1 - \rho) = \rho (1 - \delta^2)\}$$

and between $DA$ and $PD$, say

$$\partial D := \{(\rho, \delta) : \delta (1 - \rho) = \rho (1 - \rho \delta^2)\}.$$
Under Theorem 3.1, when \((\rho, \delta) \in \partial A\) and the agents are risk-neutral, there exist exactly two possibilities for SP equilibria, namely:

- \(A\) offers \(\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)\) and \(B\) accepts.
- \(A\) offers \((1,0)\) and \(B\) rejects. If there is no breakdown, \(B\) offers \(\left(\frac{\delta}{1+\delta}, \frac{1}{1+\delta}\right)\) and \(A\) accepts.

In both cases, \(A\) gets a final (expected) payoff of \(\frac{1}{1+\delta}\). In the first case, \(A\) gets \(\frac{1}{1+\delta}\). In the second case, \(A\) gets either 1 (with probability \(1 - \rho\)) or \(\frac{\delta^2}{1+\delta}\) (with probability \(\rho\)).

However, under risk aversion \(A\) would strictly prefer to get \(\frac{1}{1+\delta}\). Hence, one may expect that the set \(IA\) "advances" whilst the set \(AD\) "retreats". This implies that immediate agreement is more likely under risk aversion.

A similar argument can be followed for \(\partial D\). When the agents are risk-neutral, the strategy of asking for the whole pie (waiting for breakdown to occur) is a possible SP equilibrium in both \(PD\) and \(\partial D\). However, under risk aversion this "security" strategy is less attractive, and hence one may expect that the set \(AD\) "advances" whilst the set \(PD\) "retreats". This implies that agreement is more likely under risk aversion.

6 Concluding remarks

The importance of the alternating-offers model lies on two important features. First, it comprises most of the ideas that one may expect to find in a real process of negotiation (see [8] for a nice discussion). For example, time is valuable, and still the agents perceive that they always have the choice to make a new offer and continue the negotiation under more or less the same circumstances. Second, it answers the question of what is a reasonable payoff division that the agents are bound to agree. For example, Corominas-Bosch [5] avoids the details of the bargaining process between pairs in a network by simply assigning the division \(\left(\frac{1}{1+\delta}, \frac{\delta}{1+\delta}\right)\) in two-sided negotiations.
On the other hand, stationary strategies in general, and immediate agreements in particular, are somehow problematic features. They do not seem to have a counterpart in real-life negotiations. Stationary strategies imply that each agent should not take into account previous offers and rejections. Immediate agreement rules out the possibility of small and harmless delays that seem very willing to happen in real-life negotiations.

This paper maintains the main features of the classical alternating offers model. In particular, it explains delay maintaining the original assumptions of Rubinstein’s, i.e. unbounded rationality, complete information and no outside options, as well as the most tractable concept of subgame perfect equilibrium. See [6] for a recent work explaining delay in a model with incomplete information. Delay due to externalities can be found in [9]. Delay may also arise when the agents can destroy part of the pie ([1, 10]) or are committed not to accept poorer proposals than those already rejected ([7]).

Delay due to outside options can be found in [11] and references therein\(^5\). The impact of an outside option is qualitatively different depending on when an agent can opt out: after having rejected an offer [2], or after his opponent has rejected an offer [18, 15]. In the first case, agreement is reached without delay. In the second case, there may be multiple equilibria and some may include delay. When the outside option is the disagreement payoff, a SP equilibrium that is always present is to ask for the whole pie (proposer) and to accept any offer (responder). The delayed payoffs are obtained by a threat to this strategy (which gives the whole pie to the proposer). The intuition behind this equilibrium is that each proposer believes the responder would behave the same way after roles are changed. The same idea follows the strategy defined as Favor A in Table I. Hence, although a random breakdown is different from an outside option, the way in which they operate is rather similar.

Another remarkable feature in our model is the following. In real-life negotiations, small delays may happen because the agents begin with pro-

\(^5\)Outside options may cancel out the effect of incomplete information, as shown in [4].
posals that are more favorable to them than the ones they actually consider as the most reasonable (cheap talk). The driving force is that there is a small probability that the other party considers the offer as final.

In our model, the payoffs attainable when agreement is immediate (below $M^a$ in Figure 3) are potentially worse for the proposer than those attainable when agreement is delayed (below $M^r$ in Figure 3). The interpretation of this is as follows: The most unfavorable SP equilibria for $A$ are those in which her initial proposal is accepted. Hence, in general it is beneficial to begin with an unacceptable offer. Do bargainers behave like this real-life negotiations? It is obvious that they do. For example, in the Spanish house market, the consultants Re/Max and Look&Find have recently pointed out that sellers put a high price to their houses, planning to decrease it during the bargaining. Claiming high prices gives somehow a favorable status quo in case negotiations should finish immediately.

References


