SKILLED MIGRATION: WHEN SHOULD A GOVERNMENT RESTRICT MIGRATION OF SKILLED WORKERS?*

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Abstract

In the brain drain literature models with heterogeneous agents typically predict that all agents who get tertiary education will try to migrate. Hence, the skill composition of the migration flow is the same as that of the skilled population left behind. This result, however, may not represent the migration pattern of some source countries. In this paper I present and analyze a model of heterogeneous agents where immigrants go through an assimilation process upon arriving to the host country. I start by studying the skill composition of the migration flow of a less advanced country. Then, I characterize conditions that lead a benevolent government to promote migration among the skilled population. I show that the government may promote skilled migration despite the fact that the brain drain decreases per capita income.

Keywords: Assimilation process, brain drain, and migration pattern.

JEL Classification: F22; I28; J24
1 Introduction

The international migration of skilled workers, known as a brain drain, is a significant phenomenon at the world level, with particularly drastic consequences in poor and developing countries. Docquier et al (2005), using data of 2000, find that in Latin America and the Caribbean, Asia and Africa the average migration rate of skilled workers is 7%, while at the world level that rate is 5.3%.\footnote{Moreover, in Latin America, Asia and, Africa the skilled migration is more significant than unskilled migration. For these countries, the average unskilled migration rate is 0.3%.
}

The consequences of the brain drain on the source economy have been studied extensively in the literature. Models with heterogeneous agents, e.g. Miyagiwa (1994), Mountford (1997), Beine et al (2001), and Chau and Stark (1999) show that the possibility of working abroad affects the education process and, hence, alters the level of human capital in the source economy.\footnote{Assuming homogeneity among agents, Vidal (1998), shows that migration prospects may increase the level of education in the source economy.}

Following Beine et al (2001), migration prospects entail two effects: a brain effect and a drain effect. The brain effect captures the rise in the size of the skilled population due to the fact that migration prospects increase the return to human capital. The drain effect measures the fall in the size of the skilled population that follows the migration of skilled workers. In a context of restricted migration, if the brain effect dominates the drain effect the source economy benefits from skilled migration.

Typically, existing literature on brain drain assumes that the possibility of earning higher wages is the main reason of skilled migration. Of course then, all skilled agents have incentives to work abroad and since, migrants are randomly selected from the skilled population, the skill composition of the migration flow is the same as that of the native population.\footnote{Exceptions are Miyagiwa (1994) and Chau and Stark (1999). Miyagiwa finds that the most talented skilled workers always migrate. Therefore, the skilled composition of the migration flow is not the same as that of the native population left behind. Chau and Stark consider the possibility of return to migration. In their model, the skill composition of the migrants that stay abroad differs from that of the skilled workers that return to the source country.}

This result, however, does not represent the migration pattern observed in some source countries. Table 1 provides cross country data supporting this claim. Population with tertiary education is grouped according to the International Standard Classification of Education (ISCE). At tertiary level, ISCE distinguishes between two
types of programmes: type A programmes and type B programmes. The former provide qualification for entry into professions with high skill requirements. The latter permit the acquisition of practical skills. Column 2 shows the skill composition of the adult population with tertiary education in the source country. The third column shows the skill composition of immigrants in the US by country of origin. In countries like Argentina, Mexico and Paraguay the skill composition of the migration flow is not the same as that of the native population. More than 50% of their migration flow is composed of less able skilled agents, i.e. agents with type B programmes, while the percentage of native population with tertiary education and with type A programmes is higher than 50%. On the contrary, in countries like Russia and South Africa more than 50% of the population with tertiary education has type B programmes. However, their migration flows are mainly composed of highly qualified workers.

This is an important observation, as the skill composition of migrants crucially determines not only the production loss caused by migration in the source country, but also e.g. the size of remittances. Different skill compositions of the migration flow can have different economic impacts on the source economy. The following questions immediately arise. How should a government react to this kind of migration? Should a government always restrict skilled migration? Or should the government restrict skilled migration only when all the skilled agents want to migrate? Or, given a skill composition of the migration flow, should a government restrict skilled migration only when the brain drain hurts the source economy while promoting skilled migration when the brain drain benefits the source economy?

These issues cannot be analyzed without studying the factors that determine the composition of the migration flow. In the current paper I characterize different migration patterns. Then, I consider the existence of a benevolent government that seeks to maximize the welfare of those left behind and determine conditions under which the government promotes the migration of skilled workers.

In this paper I make two non-standard assumptions. Both are crucial. The first one is that agents not only migrate because of the possibility of earning a higher wage, but also because of the existence of a lack of employment opportunities for graduates in the source economy. The second assumption is that skilled agents must go through an assimilation
process when they arrive to the host country. These two assumptions allow me to capture some effects on the skill composition of the migration flow that are not detected in the previous models.

The existing literature typically assume full employment in the source economy. As a result, some effects that unemployment has on both education and migration are not detected. For instance, some agents might refrain from becoming skilled since they know that they may fail in getting a skilled job, while others are encouraged to leave their own country. In addition, the assumption of full employment is not in accordance with the reality of some poor and developing countries. According to the World Bank, in some countries there is a lack of employment for graduates because of misguided educational policies that result in a large supply of college graduates for whom no suitable jobs exist.4

Regarding my second assumption, existing models with heterogeneous agents assume that immigrants earn the same wage as natives with equivalent skill levels since they arrive to the host country. Nevertheless, empirical literature shows that immigrants go through an assimilation process when they arrive to the host economy.5 The assimilation process includes the acquisition of information about the host labor market conditions, the improvement of language skills, the validation of the education credential and so on. During this process immigrants are not paid according to their educational attainment. Consequently, their earnings are smaller than those of natives with equivalent skill levels. In the source country, however, firms match the wage they pay with the agents’ educational attainment. Consequently, the opportunity cost of working abroad rises with the educational level. Therefore, the assimilation process may induce the most talented agents to prefer to work in their own country.

I have mentioned above that the brain drain may benefit the source economy. Beine et al (2003) provide empirical support showing that the brain drain benefits those countries that have low levels of human capital and low skilled migration rate. For instance, they obtain that, among other countries, India and Brazil benefit from this type of migration, which is not the case of the Philippines and Mexico.

According to the World Bank there are countries that promote migration of skilled

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workers, e.g. India, Sri Lanka, the Philippines, etc. Of course, it is not surprising that countries that benefit from this type of migration apply such a policy. However, why should a country like the Philippines, which does not benefit from the brain drain, promote skilled migration?

Results that I present in this paper are important, as they offer an explanation to the situations mentioned above. The model shows that when the migration pattern is such that all skilled agents want to migrate, the government promotes skilled migration only if the brain drain benefits the source economy. Interestingly, the model highlights the possibility that in a context where the most talented skilled agents prefer not to migrate, the government promotes skilled migration despite the fact that the brain drain hurts the source economy. It has to do with remittances sent by less able skilled workers are higher than the income they would produce, if they stayed in the source economy. Therefore, the government curtails the fall in per capita income that follows the brain drain by encouraging migration among less able skilled agents.

The rest of the paper is organized as follows: In Section 2 I present the benchmark model where skilled workers are not allowed to migrate. In Section 3 I extend the model to an economy where migration is risky and skilled agents have to decide whether to work abroad or not. Section 4 presents the effects of a selective immigration policy on the composition of the migration flow. Section 5 presents some migration policies applied by a government who wants to maximize per capita income in the source country. Finally, in Section 6 I conclude. Proofs are gathered in an Appendix.

2 The benchmark model without migration

Consider a small open economy in a world where only one good is produced by means of both skilled and unskilled labor. The good is exchanged in competitive markets. There is a continuum of agents. For simplicity I normalize the size of the population to one. Heterogeneity is introduced by assuming that agents possess different abilities to learn, i.e. ability to transform knowledge, acquired by investing in higher education, into productive skills. Every individual has an ability \(a\), which is distributed continuously on a closed interval \([0, A]\) according to some strictly positive density function \(g(a)\), where \(G(a)\)
### Table 1 (a) Distribution of tertiary education among non-migrant population (b) (c) immigrants in the US (b) (d)

<table>
<thead>
<tr>
<th>Country</th>
<th>Type A</th>
<th>Type B</th>
<th>Type A</th>
<th>Type B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Argentina (e)</td>
<td>66</td>
<td>34</td>
<td>42.28</td>
<td>57.72</td>
</tr>
<tr>
<td>Indonesia</td>
<td>55</td>
<td>45</td>
<td>61.94</td>
<td>38.06</td>
</tr>
<tr>
<td>Mexico (e)</td>
<td>88</td>
<td>12</td>
<td>47.73</td>
<td>52.27</td>
</tr>
<tr>
<td>Paraguay</td>
<td>65</td>
<td>35</td>
<td>49.44</td>
<td>50.56</td>
</tr>
<tr>
<td>Peru (e)</td>
<td>51</td>
<td>49</td>
<td>70.32</td>
<td>29.68</td>
</tr>
<tr>
<td>Poland (e)</td>
<td>80.42</td>
<td>19.58</td>
<td>63.96</td>
<td>36.04</td>
</tr>
<tr>
<td>Romania (e)</td>
<td>67.81</td>
<td>32.19</td>
<td>70.37</td>
<td>29.63</td>
</tr>
<tr>
<td>Russia (e)</td>
<td>38</td>
<td>62</td>
<td>84.63</td>
<td>15.37</td>
</tr>
<tr>
<td>South Africa (e)</td>
<td>45</td>
<td>55</td>
<td>73.62</td>
<td>26.38</td>
</tr>
<tr>
<td>Ukraine (e)</td>
<td>64</td>
<td>36</td>
<td>54.41</td>
<td>45.59</td>
</tr>
</tbody>
</table>

(a) Data obtained from: Current Population Survey, LABORSTA of ILO and Education Trends in Perspective of UNESCO-UIS/OECD.

(b) Population grouped according to International Standard Classification of Education:
Type A programmes provide qualifications for entry into professions with high skills requirements.
Type B programmes permit the acquisition of practical skills, e.g. teachers, technicians.

(c) Source countries. Population aged 25 and over and with tertiary education.

(d) Immigrants in the US aged 25 and over and with tertiary education, by country of origin.

(e) Immigrants entered in the US during the period 1996-2001.
is the corresponding cumulative distribution function.\(^6\) For the moment I assume that international migration is not allowed. Consequently, individuals must decide whether to remain unskilled or to invest in higher education. Since agents do not have resources of their own, they must borrow to finance the educational cost \(C\), at a given interest rate \(r\). In the sequel, \(c\) represents the discounted value of the educational cost.

I assume that both skilled and unskilled individuals supply labor services for their remaining working lifetime which is normalized to one, once the education process has finished. Lifetime income is, therefore, denoted in present value discounted at a rate \(r\).

I assume that one unit of unskilled labor produces \(w\) units of the aggregate good. By competition, the wage of unskilled workers is \(w\). Those who work in skilled jobs earn an income according to their productivity. However, investing in higher education is not a sufficient condition for working as skilled worker. More precisely, there is an exogenous probability \(p\), with \(p \in [0, 1]\), with which skilled agent will fail to find a skilled job and, thus, will end up working as an unskilled worker. In that case her lifetime income will be \(w - c\).\(^7\) With probability \(1 - p\) she will get a job as a skilled worker and will earn \(wa - c\). Note that every skilled agent has to repay the educational cost, independently of the job she finally gets.

Individuals are assumed to be risk neutral. Hence, an agent with ability \(a\) invests in higher education if and only if the return to education is higher than \(w\), that is,

\[
 pw + (1 - p) wa - c \geq w.
\]

This defines a threshold ability level \(\hat{a}\) such that, only individuals with abilities above \(\hat{a}\) will get education. The cutoff level is,

\[
\hat{a} = 1 + \frac{c}{(1 - p)w}.
\]

The population is divided into those who remain unskilled and those who invest in higher education. The higher is \(p\), that is, the more difficult is to find a skilled job, the smaller the number of individuals who get higher education. A proportion \(p\) of skilled

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\(^6\) A is assumed to be high enough in order to guarantee that at least the most able agents always find profitable to invest in higher education.

\(^7\) I assume that \(p\) is strictly smaller than 1, since if there were full unemployment of skilled workers nobody would choose to become skilled.
agents ends up working in unskilled jobs, where their knowledge is useless. As a result, per capita income, net of the cost of education ($y$ in the sequel) is,

$$ y = w \left( G(\tilde{a}) + p (1 - G(\tilde{a})) + H c (1 - G(\tilde{a})) \right), $$

where,

$$ H = (1 - p) \int_{\tilde{a}}^{A} ag(a) da. $$

### 3 Equilibrium with migration

In this section I allow skilled individuals to migrate. However, migration need not to be successful. There is an exogenous probability $\pi$ of migration being successful. The reason for this assumption is that most developed countries restrict immigration by giving a limited number of entry visas. I assume that those individuals who are eligible to migrate have to pay a migration cost $M$, where $M \geq 0$. Besides, they go through a period of assimilation when they arrive to the host country. The assimilation process takes up a fraction of time $\theta$, where $\theta \in [0, 1]$. During this process their wage is equal to that of native unskilled workers, $w^F$. Once they are assimilated, they earn the same wage as natives workers possessing an equivalent skill level. Thereby, the income of an immigrant of ability $a$ is $W(a)$, with

$$ W(a) = \theta w^F + (1 - \theta) w^F a - c - M. $$

In order to have some positive migration I assume that $w^F > w + c + M$.

An agent first decides whether to become skilled or to remain unskilled. Unskilled agents earn a wage $w$. Skilled agents decide whether to look for a skilled job in their own country or to look for a skilled job abroad. Consider a skilled agent with ability $a$. She may prefer to look for a skilled job in her own country and try to migrate only if she fails
in getting that job.\(^8\) In this case her expected income is denoted by \(I(a)\) and is given by:

\[
I(a) = (1 - p)(wa - c) + p(\pi W(a) + (1 - \pi)(w - c)). \tag{6}
\]

On the other hand, she may prefer to work abroad, and look for a skilled job in her own country only if she fails in migrating. In this case her expected income is denoted by \(I^F(a)\) and is given by:

\[
I^F(a) = \pi W(a) + (1 - \pi)(pw + (1 - p)wa - c). \tag{7}
\]

Now, I solve the model backwards. I start by determining agents’ migration decisions. Then, I determine agents’ education decisions. Finally, I characterize the equilibrium migration pattern.

### 3.1 Skilled Agents’ Migration Decisions

In this subsection I determine migration decisions of skilled workers. If a skilled agent first looks for a skilled job in the source economy, it reflects that she prefers not to migrate. On the contrary, if she first looks for a skilled job abroad, it reflects that she prefers to migrate. Consider a skilled agent with ability \(a\). She prefers to migrate if and only if:

\[
I^F(a) > I(a) \iff a > a_M,
\]

where:

\[
a_M \equiv 1 - \frac{w^F - w - M}{(1 - \theta) w^F - w}. \tag{8}
\]

The cutoff value \(a_M\) is discontinuous at \(\theta = \hat{\theta}\), where:

\[
\hat{\theta} = 1 - \frac{w}{w^F}. \tag{9}
\]

\(^8\)I do not consider the case where a skilled agent refuses the possibility of trying to migrate despite she has gotten an unskilled job. This decision is not optimal. Assume that this agent exists, her income is \(w - c\). She chooses not to migrate if and only if \(w - c > W(a)\). Since she only wants to work in her own country her ability is as least higher than \(\bar{a}_0\) (see Section (2)). \(W(a)\) is increasing in \(a\), so, I have that:

\[
 w - c > W(\bar{a}_0) \iff w > w^F - c - M,
\]

which is a contradiction.
Therefore, depending on the value of $\theta$, $a_M$ may not exist. In particular, I have three cases: i) $\theta \in \left[0, \frac{M}{w^F}\right]$, ii) $\theta \in \left(\frac{M}{w^F}, \hat{\theta}\right)$ and, iii) $\theta \in \left(\hat{\theta}, 1\right]$. In the first case $\theta$ is very low meaning that assimilation takes a very short time. I find that $a_M > 0$ and skilled agents with abilities below $a_M$ first look for a skilled job in their own country whereas agents with abilities above $a_M$ first try to migrate. In the second case, the length of the assimilation process is such that $a_M$ is negative. This means that all skilled agents prefer to work abroad. They look for a skilled job in their own country only if migration fails. In the following subsection I show that I can discard the first case. Hence, when $\theta$ is smaller than $\hat{\theta}$, the opportunity cost of working abroad is that small that all skilled agents prefer to work abroad. Finally, in the third case the length of the assimilation process is large enough such that the opportunity cost of working abroad, i.e. $\theta w^F$, overweights the wage differential. Therefore, agents with abilities above $a_M$ prefer to look for a skilled job in their own country. They try to migrate, only if they fail in getting that job. On the contrary, agents with abilities below $a_M$ first try to migrate.\footnote{The cutoff level $a_M$ is such that $\lim_{\theta \to \hat{\theta}^-} a_M = -\infty$ and $\lim_{\theta \to \hat{\theta}^+} a_M = \infty$. If I consider the possibility that $a_M > A$, when $a_M > 1$, the analysis is the same than that when $\theta < \hat{\theta}$ and hence $a_M < 1$. Thereby, for the sake of the exposition I assume that $A$ is always higher than $a_M$.}

The following lemma summarizes these results.

**Lemma 1**

(i) If $\theta \in \left[0, \frac{M}{w^F}\right]$, agents with abilities below (above) $a_M$ prefer to work in the source economy (abroad).

(ii) If $\theta \in \left(\frac{M}{w^F}, \hat{\theta}\right)$, all skilled agents first try to migrate.

(iii) If $\theta \in \left(\hat{\theta}, 1\right]$, agents with abilities above (below) $a_M$ prefer to work in the source economy (abroad).

Before leaving this subsection note that the cutoff value $a_M$, when it exists and it is positive, does not depend on the probability of migrating. This stems from the fact that the return to human capital for a skilled agent with productivity $a_M$ is not affected by migration prospects. In her own country, she earns an income equal to that obtained abroad, i.e. $w a_M - c = W(a_M)$. 

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3.2 Agents’ Education Decisions

I call \( \tilde{a}_E \) and \( \hat{a}_E \) the cutoff values satisfying \( I(\tilde{a}_E) = w \) and \( F(\hat{a}_E) = w \), respectively.

\[
\tilde{a}_E = 1 + \frac{c - p \pi (w^F - w - M)}{p \pi (1 - \theta) w^F + (1 - p) w}, \tag{10}
\]
\[
\hat{a}_E = 1 + \frac{c - \pi (w^F - w - M)}{\pi (1 - \theta) w^F + (1 - \pi) (1 - p) w}. \tag{11}
\]

Both threshold levels are negatively related to the probability of migrating. This accounts for how agents’ incentives to invest in higher education are affected by migration opportunities. A rise in \( \pi \) increases the expected return to human capital and, hence, encourages some previously unskilled agents to invest in higher education.

Let \( \tilde{\pi} \) and \( \tilde{\theta} \) be equal to:

\[
\tilde{\pi} = \frac{c}{w^F - w - M}, \tag{12}
\]
and,

\[
\tilde{\theta} = \tilde{\theta} + \frac{1}{\pi} \frac{w}{w^F} (1 - p (1 - \pi)). \tag{13}
\]

**Lemma 2** If \( \pi \in (0, \tilde{\pi}] \) and \( \theta \in \left( \tilde{\theta}, 1 \right] \), the cutoff value \( \tilde{a}_E \) is the education threshold level. If, however, \( \theta < \tilde{\theta} \) the education cutoff value is \( \hat{a}_E \).

**Proof.** See Appendix A. \( \blacksquare \)

Recall that the expected income of an agent who tries to migrate as a last resort is given by \( I(a) \). Keep in mind that I obtained the cutoff value \( \tilde{a}_E \) by equating \( I(a) \) with \( w \). Then, when the probability of migrating is too small and the assimilation process is extremely high, I have that \( \tilde{a}_E < \hat{a}_E \). It means that all agents with abilities above \( \tilde{a}_E \) become skilled with the intention of working in a skilled job in their own country. They try to migrate only if they fail in getting that job. This is not the case when the education threshold level is given by \( \hat{a}_E \), i.e. when \( \hat{a}_E < \tilde{a}_E \).

Now, I am ready to characterize equilibrium migration patterns.
Proposition 1

(I) If \( \theta \in \left[ 0, \hat{\theta} \right) \) all agents with abilities above \( \tilde{a}_E \) become skilled and prefer to work abroad. They work in the source economy only if migration fails.

(II) If \( \theta \in \left( \hat{\theta}, 1 \right) \), agents with abilities above \( \tilde{a}_E \) become skilled. Those skilled agents with abilities below \( a_M \) prefer to work abroad. Skilled agents with abilities above \( a_M \) try to migrate only if they fail in getting a skilled job in the source economy.

(III) If \( \theta \in \left( \hat{\theta}, 1 \right] \) and \( \pi \in (0, \hat{\pi}] \), all agents with abilities above \( \tilde{a}_E \) become skilled and prefer to find a skilled job in their own country. They try to migrate as a last resort.

Proof. See Appendix A.

Figure 1 shows the three types of migration patterns. In the horizontal axes I draw ability and in the vertical axes expected incomes. In Figure 1A all agents with abilities above \( \tilde{a}_E \) get higher education with the intention of migrating. The expected income \( I(a) \) is higher than that of any other alternative. Figure 1B corresponds to Type II migration pattern. In this case agents with abilities in the interval \( [\tilde{a}_E, a_M] \) prefer to work abroad. The most talented agents, however, prefer to try to get a skilled job in their own country. For them the expected income \( I(a) \) is higher than that of any other choice. Figure 1C corresponds to Type III migration pattern. All agents with abilities above \( \tilde{a}_E \) become skilled with the intention of trying to get a skilled job in their own country.

The length of the assimilation process together with the wage differential determine the type of migration pattern. Note that the higher is the wage differential, i.e. the smaller is \( \frac{w}{w^*} \), the less stringent is the condition \( \theta < \hat{\theta} \). Figure 2 illustrates this issue. The bold line corresponds to Equation (9), i.e. \( \tilde{\theta} \), while the curve corresponds to Equation (13), i.e. \( \tilde{\theta} \). In the horizontal axis I draw the wage differential, i.e. \( \frac{w}{w^*} \). The maximum value this ratio can attain is \( \omega = \frac{w^* - c - M}{w^*} \). In the vertical axis I draw values of \( \theta \). The area below the bold line corresponds to the equilibrium where every skilled agent first tries to migrate. This area is denoted by \( I \). On the other hand, the area in between the bold line and the curve corresponds to the equilibrium where the most able agents choose to migrate as a last resort. I denote this area by \( II \). Those pairs of values \( (\frac{w}{w^*}, \theta) \) in region \( II \) represent a situation where the assimilation process takes a relatively large
period of time. This negatively affects the return to human capital abroad, inducing the most talented skilled agents to try to migrate only if they fail in getting a skilled job in their own country. Finally, the area above the function $\tilde{\theta}$ corresponds to the case where all skilled agents prefer to look for a skilled job in the source economy. I denote this area by $\text{III}$. Now, consider two economies, $P$ and $R$, that only differ in their wage rate. In particular, $w^P < w^R$. Observe that in economy $P$, there are more values of $\theta$ that satisfy $\theta < \tilde{\theta}^P$ compared to those values of $\theta$ that satisfy $\theta < \tilde{\theta}^R$ in economy $R$. Therefore, Type $I$ equilibrium is more likely to arise in the poorer economy, i.e. in $P$.

**Corollary 1** The poorer is the source economy, as measured by the wage differential, the more likely is Type I migration pattern.

### 4 Selective Immigration Policy

The significance of the international migration of highly skilled workers may be partly explained by a selective immigration policy that some developed countries apply. Lowell (2001) mentions that countries like Australia and Germany among others have increased their intake of highly skilled agents through programmes that promote immigration of highly skilled agents. On the other hand, Canada applies a point system that favors skilled immigration. Applicants must pass a test which asks about applicants’ age, educational attainment, etc. Applicants who have graduate or professional degree get a given number of points. However, those who are high schools drop outs get zero points. At the end of the test applicants must score a minimum number of points in order to be eligible for an entry visa.\(^{10}\)

In this subsection, I drop the assumption that immigrants are randomly selected from the skilled population in the source economy. On the contrary, I assume that the host country does not restrict the immigration of highly skilled workers. Hence, migration is certain only for those agents with abilities higher than a given threshold level $\overline{a}$. I assume that $\overline{a}$ is in the interval $(x, A)$, where $x = \max \{\widehat{a}_E, \widehat{a}_E, a_M\}$. Without loss of generality I assume that agents with abilities above $\overline{a}$ do not go through the assimilation process. As a result, an agent with ability $a$, with $a > \overline{a}$, earns the following income abroad:

\(^{10}\)See Borjas (1999) for more details on this respect.
\[ \bar{W}(a) = w^F a - c - M. \] (14)

It is easy to check that \( \bar{W}(a) \) is higher than \( I(a) \) for every \( a \) higher than \( \bar{a} \). Consequently, the most talented agents always migrate. On the other hand, expected incomes of less able skilled agents may be \( I(a) \) or \( I^F(a) \), depending on their choices.

Whenever the assimilation process is not large enough, i.e. \( \theta < \bar{\theta} \), the skill composition of the migration flow is the same as in Equilibrium \( I \). That is, the equilibrium where all skilled agents prefer to migrate. If, however, \( \theta \in (\bar{\theta}, \bar{\theta}) \), migrants are mainly drawn from the tails of the skill distribution. In this case I have that \( w \leq I^F(a) \) and \( I(a) < I^F(a) \) for all \( a \in [\bar{a}_E, a_M] \). Therefore, agents with abilities in between \( \tilde{a}_E \) and \( a_M \) get higher education with the intention of migrating. On the other hand, if agents’ abilities are in the interval \( [a_M, \bar{a}] \) their expected incomes are such that \( I^F(a) \leq I(a) \). As a result, these skilled agents try to migrate as a last resort. Finally, those agents with abilities higher than \( \bar{a} \) migrate. Figure 3 illustrates this migration pattern. Figure 4 shows the migration flow that arises when the assimilation process is extremely high, that is \( \theta \geq \bar{\theta} \). In this scenario the lack of employment opportunities turns out to be the main cause of migration for those with abilities below \( \bar{a} \). They get education with the intention of working in the source country and hence, they migrate as a last resort. Still and all, the most talented workers migrate induced by the higher return to human capital they earn in the foreign country.

![Figure 3](image1.png)

![Figure 4](image2.png)
Summarizing, if the host country applies a selective immigration policy it is possible that the most able skilled agents together with the least able skilled agents get education with the intention of working abroad, while agents with intermediate abilities migrate as a last resort.

For the sake of brevity and without loss of generality in the following section I focus on the case where migrants are randomly selected from the skilled population.

5 Government Intervention

Migration prospects entail a brain effect and a drain effect. The former refers to the rise in the size of the skilled population. This is due to the fact that migration prospects increase the return to education. The latter refers to the fall in human capital that follows skilled migration. Depending on the value of the probability $\pi$, the brain drain may benefit the source economy.\footnote{On this respect see Mountford (1997), Vidal (1998), Chau and Stark (1999). Conditions that ensure a rise in per capita income net of education costs are presented in the Appendix.} Therefore, the government can foster the brain effect and ameliorate the drain effect through implementing either a Promoting Migration Policy (PMP) or a Restrictive Migration Policy (RMP). The PMP consists in encouraging agents to become skilled and to try to migrate by subsidizing migration and/or education. On the contrary, the RMP consists in discouraging agents from migrating by levying a tax on migration and/or on education.

I consider a government that seeks to maximize per capita income net of education costs. I assume that successful migrants remit a percentage $\tau$, with $\tau \in [0, 1]$, of their income. Besides, I assume that parameters $\theta, p, w$ and $w^F$ are known and hence, the government is acquainted with the composition of the migration flow that arises in the decentralized economy.\footnote{Superscripts of function refer to the type of migration pattern.} For the sake of clarity, in this section I present the maximization problem associated with Type II equilibrium. Results corresponding to the other two types of equilibria are presented in the Appendix.

Let $P^{II}$ and $y^{II}(\bar{a}, \beta)$ stand for the size of the population left behind and per capita
income, respectively. The government chooses an education cutoff value \( \tilde{a} \) and a migration cutoff value \( \beta \), where \( \beta = \tilde{a} + \varepsilon \), \( \varepsilon \geq 0 \). The optimization problem is:

\[
\max_{\{\tilde{a}, \varepsilon\}} y^I (\tilde{a}, \beta) \\
\text{s.t.: } \beta = \tilde{a} + \varepsilon, \\
\varepsilon \geq 0.
\] (15)

Consider the interior solution, i.e. \( \varepsilon > 0 \). The first order conditions are:

\[
\frac{dy^I (\tilde{a}, \beta)}{d\varepsilon} = \frac{g (\beta)}{PI} \pi^2 (1 - p) (\Psi (\tilde{a}, \beta) - (w\beta - c)) = 0,
\] (16)

\[
\frac{dy^I (\tilde{a}, \beta)}{d\tilde{a}} = \frac{dy^I (\tilde{a}, \beta)}{d\varepsilon} + \frac{g (\tilde{a})}{PI} (w - \Phi^I (\tilde{a}, \beta)) = 0,
\] (17)

where,

\[
\Psi (\tilde{a}, \beta) = y^I (\tilde{a}, \beta) + \tau W (\beta),
\] (18)

and,

\[
\Phi^I (\tilde{a}, \beta) = \pi y^I (\tilde{a}, \beta) + (\pi \tau W (\tilde{a}) + (1 - \pi) (pw + (1 - p) w\tilde{a} - c)).
\] (19)

Affecting skilled agents’ willingness to migrate entails benefits and costs. Consider a skilled agent of ability \( \beta \). She gets a skilled job with probability \( \pi + (1 - \pi) (1 - p) \). Given this probability, Expression (18) is the marginal benefit of inducing her to migrate. The first term captures the rise in per capita income due to a decrease in the size of the population left behind. The second term, \( \tau W (\beta) \), is the migrant agent’s contribution to per capita income. The foregone income associated with this decision is \( w\beta - c \), her contribution to per capita income if she prefers not to migrate. As a result, Condition (16) is satisfied when \( \beta \) is such that the marginal benefit of inducing an additional agent to migrate is equal to the marginal cost.

Affecting agents’ incentives to invest in higher education involves benefits and costs. Expression (19) represents the marginal benefit of inducing an additional agent of ability \( \tilde{a} \) to become skilled. The first term captures the rise in per capita income due to a reduction in the size of the population left behind. Terms in brackets represent skilled agent’s expected contribution to per capita income. The marginal cost associated with this decision is equal to \( w \), i.e. the agent’s contribution to per capita income if she stays unskilled. As a result, the second term of Condition (17) is zero when \( \Phi^I (\tilde{a}, \beta) \) is equal to the marginal cost \( w \).
Expressions (16) and (17) can be rewritten in the following way:

\[ W(\beta) + tm(\tilde{a}, \beta) = w\beta - c, \quad (16') \]
\[ t^{II}(\tilde{a}, \beta) + I^{F}(\tilde{a}) = w, \quad (17') \]

where,

\[ tm(\tilde{a}, \beta) = y^{II}(\tilde{a}, \beta) - (1 - \tau) W(\beta), \quad (20) \]
\[ t^{II}(\tilde{a}, \beta) = \pi(y^{II}(\tilde{a}, \beta) - (1 - \tau) W(\tilde{a})). \quad (21) \]

Consider Equation (17') and let \( \tilde{a} = \tilde{a}_E \), that is the education cutoff value that arises in the decentralized equilibrium. Remember that in the decentralized equilibrium the educational cutoff value is such that \( I^{F}(\tilde{a}_E) = w \). If the government encourages more agents to get higher education the optimal value of \( \tilde{a} \) is smaller than \( \tilde{a}_E \). Keeping in mind that \( I^{F}(a) \) is increasing in \( a \), \( I^{F}(\tilde{a}) < w \) implying that \( t^{II}(\tilde{a}, \beta) \) must be positive.

Applying the same reasoning, I find that a positive (negative) value of \( tm(\tilde{a}, \beta) \) means that the government subsidies (taxes) skilled migration. In such a case, \( \beta \) is higher (smaller) than \( a_M \).

**Proposition 2** In Type II migration pattern, if \( tm(\tilde{a}_E, a_M) \) is positive then a PMP is applied. On the contrary, if \( t^{II}(\tilde{a}_E, a_M) \) is negative then a RMP is applied.

**Proof.** See Appendix D. ■

Remember that in a Type II equilibrium, the least able skilled agents prefer to migrate, whereas, the most talented skilled agents prefer to get a skilled job in the source economy. That is, migration preferences vary among skilled agents. This makes it difficult to find necessary and sufficient conditions for having either a PMP or a RMP. Note that Proposition (2) only states sufficient conditions. When the wage differential is enough high such that an agent with ability \( a_M \) contributes more to per capita income by migrating, i.e. \( \Psi(\tilde{a}_E, a_M) > wa_M - c \), then the government encourages more agents to migrate. The government finds convenient to apply both a subsidy to migration and to education. On the contrary, when the wage differential is small enough such that a skilled agent with ability \( \tilde{a}_E \) is more productive in the source economy, i.e. \( \Phi^{II}(\tilde{a}_E, a_M) < w \), then the government restricts migration. It levies a tax on both education and migration.
Proposition 3

(i) In Type I migration pattern a PMP (RMP) is applied if and only if the Expression below is positive (negative).

\[ t^I (\tilde{a}_E) = y^I (\tilde{a}_E) - (1 - \tau) W (\tilde{a}_E). \] (22)

(ii) In Type III migration pattern a PMP (RMP) is applied if and only if the Expression below is positive (negative).

\[ t^{III} (\tilde{a}_E) = y^{III} (\tilde{a}_E) - (1 - \tau) W (\tilde{a}_E). \] (23)

Proof. See Appendix D. ■

Under Type I migration pattern, a positive value of (22) implies that the marginal benefit of inducing an agent of ability \( \tilde{a}_E \) of becoming skilled is higher than the marginal cost. Therefore, the government fosters the brain effect by subsidizing higher education. The same reasoning applies to Type III migration pattern. In these equilibria migration preferences do not vary among skilled population. In Type I equilibrium all skilled agents first try to migrate, whereas, in Type III migration pattern, all skilled agents first try to get a skilled job in the source economy. Therefore, by subsidizing (taxing) higher education, the government promotes (restricts) skilled migration.

The following corollaries relate the effect of the brain drain on per capita income with the kind of migration policy the government applies.

**Corollary 2** Consider Type I and Type III migration patterns. If the government applies a PMP then in the decentralized equilibrium the brain drain increases per capita income. However, if the brain drain decreases per capita income then a RMP is applied.

Proof. See Appendix D. ■

**Corollary 3** Consider Type II migration pattern. The government may apply a PMP despite the fact that the brain drain decreases per capita income.
Proof. See Appendix D. ■

Consider Type II equilibrium. The fact that migration preferences vary among the skilled population allows for the possibility that the government applies a PMP despite the fact that the brain drain decreases per capita income. Assume that the brain drain reduces per capita income. The government can establish a tax on migration and a subsidy to education such that the size of the migrant population increases. This policy curtails the fall in per capita income that follows skilled migration. The reason is that the tax on migration retains the most able agents, among those who prefer to migrate, and the subsidy to education increases the inflow of remittances by encouraging the least able agents to work abroad.

Since in Type I and in Type III equilibria migration preferences are homogeneous among skilled agents, the situation described above is not feasible. The government cannot mitigate the fall in per capita income by changing migration preferences of the most able skilled agents. In particular, Corollary (2) says that under these type of migration patterns, the government has to restrict skilled migration whenever the brain drain hurts the source economy.

6 Concluding Remarks

In the brain drain literature models with heterogeneous agents typically show that agents get higher education with the intention of working abroad. Interestingly, my model highlights the possibility that some agents get higher education with the intention of working in the source economy. This possibility arises when the length of time that immigrants need to catch up with the wage of natives with equivalent skill levels is relatively large. In other words, when the opportunity cost of working abroad is relatively high for the most talented skilled workers. They first try to get a skilled job in the source economy, and try to migrate only if they fail in getting that job. On the other hand, the model shows that the migration pattern where all skilled agents prefer to migrate is more likely to arise in poor economies. It is in these economies where the wage differential offsets the opportunity cost associated with the assimilation process.

In Section 3 I assumed that the assimilation parameter \( \theta \) is fixed and common to all
skilled immigrants. However, the assimilation process could vary across immigrants even if they are of the same nationality. More talented agents could assimilate faster to the host labor market than less able skilled agents do. Therefore, endogenizing \( \theta \) may detect some migration patterns that do not arise in this model. In Section 4, however, I relaxed the assumption that \( \theta \) is common to all skilled agents. In that section I assumed that the host country applies a selective immigration policy that promotes immigration of highly skilled agents. Under this scenario, the model shows that it is possible that migrants be mainly drawn from the tails of the skill distribution. Skilled agents with intermediate abilities first try to get a skilled job in the source economy. However, the most able skilled agents together with the least able skilled agents invest in higher education with the intention of working abroad.

I analyzed government intervention. I considered a benevolent government that is acquainted with the skill composition of the migration flow. The government weighs the benefits against the costs of encouraging agents to migrate. The model shows that when migration preferences do not differ among the skilled population the government promotes (restricts) skilled migration whenever the brain drain benefits (hurts) the source economy. Interestingly, the model highlights the possibility that the government promotes skilled migration even when the brain drain hurts the source economy. This possibility arises when migration preferences differ among the skilled population. More precisely, when the most talented skilled agents prefer to work in the source economy while the less talented skilled agents prefer to work abroad. The government applies a subsidy to education and a migration tax such that the size of the migrant flow increases. This policy curtails the fall in per capita income. On the one hand, the subsidy to education encourages less able agents to become skilled and migrate, leading to rise in the size of remittances. On the other hand, the migration tax retains the most talented agents among those who prefer to migrate. This result offers an explanation why countries that do not benefit from the brain drain promote this type of migration.

Finally, just a remark to say that this simple model can be generalized by endogenizing the probability that captures unemployment among skilled workers \( p \), assuming that it is positively related to the number of skilled workers in the source economy. However, the equilibria of the model do not depend on that assumption. The only disadvantage of
assuming $p$ exogenous is related with government’s maximization problem. There is a benefit that is not captured in this model. It is the fact that skilled migration relieves skilled job market pressures.

**Appendix A**

**Proof of Lemma 2.**

Let $\delta$ be equal to $w^F - w - M$. The cutoff value $\tilde{a}_E$ is higher than $\hat{a}_E$ if and only if:

$$\delta (\pi + (1 - \pi) (1 - p)) > -c ((1 - \theta) w^F - w) \iff,$$

after rearranging terms I get that,

$$\theta < \tilde{\theta}. \quad (25)$$

If $\pi \in (0, \pi]$, the cutoff $\tilde{\theta}$ may be smaller than 1. As a result, $\tilde{a}_E$ is smaller than $\hat{a}_E$ whenever $\pi \in (0, \bar{\pi}]$ and $\theta \in \left(\bar{\theta}, 1\right]$.

It is possible to show that $\tilde{\theta} > 1$ for all $\pi \in [\bar{\pi}, 1]$. Hence, $\tilde{a}_E < \hat{a}_E$ whenever $\pi \in [\bar{\pi}, 1]$.

Before proving Proposition 1 I state the following Lemmas.

**Lemma 3** If $\tilde{a}_E$ is the education threshold level, then $a_M < \tilde{a}_E$.

**Proof.**

Let $\delta$ be equal to $w^F - w - M$. I have that $a_M < \tilde{a}_E \iff$,

$$-\frac{\delta}{(1 - \theta) w^F - w} < \frac{c - \pi p \delta}{\pi p (1 - \theta) w^F + (1 - p) w}, \quad (26)$$

If $\tilde{a}_E$ is the education threshold level, by Lemma (2), I have that $\theta > \tilde{\theta} > \hat{\theta}$. Then, the denominator of the left hand side is negative. Therefore, after rearranging terms I have:

$$\delta (\pi + (1 - \pi) (1 - p)) > -c ((1 - \theta) w^F - w) \iff,$$

$$\theta > \tilde{\theta}. \quad (28)$$

\[\blacksquare\]
Lemma 4  $\hat{a}_E$ is smaller than $a_M$ whenever $\theta > \hat{\theta}$. On the contrary, $a_M$ is smaller than $\hat{a}_E$ whenever $\theta \in [0, \hat{\theta})$.

Proof. I start with $\theta > \hat{\theta}$. First, consider the case where $\pi \in (0, \bar{\pi}]$ and $\theta \in (\hat{\theta}, \hat{\theta})$. I have that both cutoff values are higher than 1. Then, $\hat{a}_E < a_M$ if and only if:

\begin{align}
  c \left( (1 - \theta) w^F - w \right) &> -\delta \left( \pi + (1 - \pi) (1 - p) \right) \Leftrightarrow, \\
  \theta &< \hat{\theta}.
\end{align}

(29)

By hypothesis the last inequality holds. If $\pi > \bar{\pi}$, I have that $\hat{\theta} > 1$, then the last inequality still holds. In particular, in this case I have: $\hat{a}_E < 1 < a_M$.

Now, I consider the case where $\theta \in [0, \hat{\theta})$. First, consider the case where $\pi \in (0, \bar{\pi}]$. In this case $a_M < 1 < \hat{a}_E$. Finally, assume that $\pi > \bar{\pi}$. Both cutoff values are smaller than 1. Then, $a_M < \hat{a}_E$ if and only if:

\[ -\delta \left( \pi + (1 - \pi) (1 - p) \right) < c \left( (1 - \theta) w^F - w \right) \]

(31)

The left hand side of the last inequality is negative, while the right hand side is positive. Hence, $a_M < \hat{a}_E$. \[ \blacksquare \]

Proof of Proposition 1.

I start with Case (I). By Lemma (2) $\hat{a}_E$ is the educational cutoff value. By Lemma (4), I have that $a_M < \hat{a}_E$. Lemma (1) implies that all skilled agents with abilities above $a_M$ prefer to work abroad. All of this implies that all agents with abilities above $\hat{a}_E$ become skilled and prefer to work abroad.

I turn to Case (II). By Lemma (2) $\hat{a}_E$ is the educational cutoff value. By Lemma (4), I have that $\hat{a}_E < a_M$. Lemma (1) implies that all skilled agents with abilities below $a_M$ prefer to work abroad. All of this implies that agents with abilities in $(\hat{a}_E, a_M)$ become skilled and try to migrate. On the other hand, agents with abilities above $a_M$ become skilled and first try to get a skilled job in the source economy.

Finally, consider Case (III). By Lemma (2) $\hat{a}_E$ is the educational cutoff value. By Lemma (3), I have that $a_M < \hat{a}_E$. Lemma (1) implies that all skilled agents with abilities below $a_M$ prefer to work abroad. All of this implies that all agents with abilities above
become skilled and try to migrate only if they fail to get a skilled job in the source economy.

Appendix B: Government’s intervention.

I start by Type I migration pattern. Let $P^I$ and $y^I$ denote the population left behind and per capita income net of education costs, respectively. The government chooses the cutoff value $\tilde{a}$ above which agents become skilled so as to maximize per capita income. The first order condition (FOC, from now on) is:\(^\text{13}\)

$$\frac{d y^I}{d \tilde{a}} = \frac{g(\tilde{a})}{P^I} \left[ w - \pi y^I - (\pi \tau W(\tilde{a}) + (1 - \pi)(pw + (1 - p)w\tilde{a} - c)) \right] = 0$$

$$= -\frac{g(\tilde{a})}{P^I} \left[ \Phi^I(\tilde{a}) - w \right] = 0,$$

where,

$$\Phi^I(\tilde{a}) = \pi y^I + (\pi \tau W(\tilde{a}) + (1 - \pi)(pw + (1 - p)w\tilde{a} - c)).$$

Expression $\Phi^I(\tilde{a})$ represents marginal benefit of inducing an additional agent of ability $\tilde{a}$ of becoming skilled, while $w$ is the marginal cost. Hence, the FOC is zero if and only if $\Phi^I(\tilde{a}) = w$, or equivalently,

$$t^I(\tilde{a}) + I^F(\tilde{a}) = w,$$

where,

$$t^I(\tilde{a}) = \pi \left( y^I - (1 - \tau)W(\tilde{a}) \right).$$

A positive value of $t^I(\tilde{a})$ means that the government is subsidizing education, and hence, migration. Quite the reverse, a negative value corresponds to the case where the government is restricting migration through a tax on migration (or on education).

I turn to study government’s intervention when the migration pattern is Type III. That is, when all skilled agents prefer to try to get a skilled job in the source economy. The government chooses a cutoff value $\tilde{a}$ above which agents become skilled. The government solves the following problem:

$$\max_{\tilde{a}} \ y^{III}(\tilde{a})$$

$$\text{s.t. : } \tilde{a} > a_M$$

\(^{13}\)Remember that in the decentralized equilibrium $a_M < \tilde{a}_E$. In particular, $a_M < 0$. Therefore, under this configuration of the migration flow, the government only affects the value of $\tilde{a}_E$.  

23
\[
\frac{dy^{III}}{d\tilde{a}} = \frac{g(\tilde{a})}{\Phi^{III}} \left( w - \pi y^{III} - (\pi \tau W(\tilde{a}) + (1 - \pi) I(\tilde{a})) \right) = 0,
\]
\[
= -\frac{g(\tilde{a})}{\Phi^{III}} (\Phi^{III}(\tilde{a}) - w) = 0,
\]
\[
\lambda \tilde{a} = 0,
\]
where,
\[
\Phi^{III}(\tilde{a}) = \pi y^{III} + ((1 - p) w\tilde{a} + p (\pi \tau W(\tilde{a}) + (1 - \pi) (w - c))).
\]
\[\Phi^{III}(\tilde{a})\] represents the marginal benefits of inducing an additional agent of ability \(\tilde{a}\) of getting higher education. The parameter \(w\) corresponds to the marginal cost associated to this decision. Considering the interior solution, i.e. \(a_M < \tilde{a}\), the FOC can also be written in the following way:\footnote{Remember that in type III migration pattern, \(a_M\) is higher than 1. Therefore, in the case of a corner solution, \(\tilde{a} \leq a_M\), the problem of the government is the same as that of the equilibrium II.}
\[
I(\tilde{a}) + t^{III}(\tilde{a}) = w,
\]
where,
\[
t^{III}(\tilde{a}) = \pi p (y^{III} - (1 - \tau) W(\tilde{a})).
\]
A positive (negative) value of \(t(\tilde{a})\) corresponds to a subsidy (tax) to migration (or education).

**Appendix C: Effects of the drain on per capita income**

In this part of the Appendix I compute the effect of the brain drain on per capita income of the source economy. This result will be used in proofs of Propositions (3) and (2) and in Corollaries (2) and (3). The following equations measures the difference between per capita income of equilibrium \(J\) and per capita income of the economy without migration, i.e. \(\Omega_J = y_J - y\).
\[
\Omega^I = \pi \tau \int_{\tilde{a}_E}^A W(a) g(a) da + \pi y (1 - G(\tilde{a}_E)) + \pi e (1 - G(\tilde{a})) + BE - \pi w (G(\tilde{a}) - G(\tilde{a}_E)),
\]
\[
\Omega^{II} = R^{II} + \pi \chi y + \pi c p (1 - G(a_M)) - \pi \int_{\tilde{a}}^a (pw + (1 - p) wa - c) g(a) da
\]
\[
+ BE - \pi w (G(\tilde{a}) - G(\tilde{a}_E) + p (1 - G(a_M))),
\]
\[
\Omega^{III} = \pi \tau \int_{\tilde{a}_E}^A W(a) g(a) da + \pi y (1 - G(\tilde{a}_E)) + \pi c (1 - G(\tilde{a}_E)) +
\]
\[
+ \int_{\tilde{a}_E}^a (pw + (1 - p) wa - c) g(a) da +
\]
\[
- w (G(\tilde{a}) - G(\tilde{a}_E) + \pi p (1 - G(a_1))),
\]

where,
\[
R^{II} = \pi \tau \left( \int_{\tilde{a}_E}^{a_M} W(a) g(a) da + \int_{a_M}^A W(a) g(a) da \right),
\]
\[
BE = (1 - \pi) \int_{\tilde{a}_E}^a ((1 - p) (wa - w) - c) g(a) da,
\]
\[
\chi = \pi (G(a_M) - G(\tilde{a}_E) + p (1 - G(a_M))).
\]

Expressions (43)-(45) must be positive in order to the brain drain increases per capita income in the source economy.

Appendix D

Proof of Proposition 2.

I start by showing that a tax on education implies a tax on migration, that is:
\[
t^{II} (\tilde{a}, \beta) < 0 \Rightarrow tm(\tilde{a}, \beta) < 0, \text{ for all } \tilde{a} \text{ and } \beta,
\]
\[
(49)
\]
since \( t^{II} (\tilde{a}, \beta) < 0 \) I have that:
\[
0 > y^{II} - (1 - \tau) W(\tilde{a}), \text{ and}
\]
\[
y^{II} - (1 - \tau) W(\tilde{a}) > tm(\tilde{a}, \beta).
\]
\[
(50)
\]
Therefore, $t^{II}(\hat{a}_E, a_M) < 0$ implies that $tm(\hat{a}_E, a_M) < 0$. On the contrary, $tm(\hat{a}_E, a_M) > 0$ implies that $t^{II}(\hat{a}_E, a_M) > 0$.

Now, consider an interior solution. Using (17) and (20) I have that $tm(\hat{a}_E, a_M) < 0$ implies that $t^{II}(\hat{a}_E, a_M) < 0$. On the contrary, $tm(\hat{a}_E, a_M) > 0$ implies that $t^{II}(\hat{a}_E, a_M) > 0$.

A positive value of $tm(\hat{a}_E, a_M)$ implies that $t^{II}(\hat{a}_E, a_M) > 0$. This implies that $\hat{a} < \hat{a}_E$ and $\beta > a_M$. Hence, the government enlarges the size of the migrant population when $tm(\hat{a}_E, a_M)$ is positive.

On the other hand, a negative value of $t^{II}(\hat{a}_E, a_M)$ implies $tm(\hat{a}_E, a_M) < 0$. This implies that $\hat{a} > \hat{a}_E$ and $\beta < a_M$. So that, the government restricts the size of the migrant population when $t^{II}(\hat{a}_E, a_M) < 0$. 

Proof of Proposition 3.

Consider Type I migration pattern. Using Expressions (32) and (35) I have that $\Phi^I(\hat{a}_E) > w \Leftrightarrow t^I(\hat{a}_E) > 0$. Then, if $t^I(\hat{a}_E) > 0$, by (32), I have that $\frac{dT^I}{\hat{a}}|_{\hat{a}=\hat{a}_E} < 0 \Rightarrow \hat{a} < \hat{a}_E$, i.e. the government gives a subsidy $t^I(\hat{a})$ to education. On the other hand, using Expressions (16) and (20) I have: $\Psi(\hat{a}_E, a_M > wa_M - c \Leftrightarrow tm(\hat{a}_E, a_M) > 0$. If $tm(\hat{a}_E, a_M) > 0 \Rightarrow \frac{dT^I}{\hat{a}}|_{\hat{a}=a_M} > 0 \Rightarrow \varepsilon > 0$. Thus, the government gives a subsidy $tm(\hat{a}_E, a_M)$ to migration.

Proof of Corollary 2.

Recall Expressions (43) and (45). They measure the effect of the brain drain on per capita income. First, I show that a PMP implies that $J > 0$, with $J = \{I, III\}$. After that, I show that $J < 0$, implies a RMP.

I start by Type I migration pattern. I assume that a PMP is applied when $\Omega^I < 0$. By Proposition (3) a PMP implies that $t^I(\hat{a}_E) > 0$, so that:

$$\pi(\Phi^I(\hat{a}_E) - w) > 0. \quad (51)$$
By integrating Expression (51) over $[\hat{a}_E, \hat{a}]$ I have:

$$\pi y^I \int_{\hat{a}_E}^{\hat{a}} g(a) \, da + \pi \tau \int_{\hat{a}_E}^{\hat{a}} W(a) g(a) \, da + (1 - \pi) \int_{\hat{a}_E}^{\hat{a}} (1 - p) (w a - w) - c) g(a) \, da - \pi w (G(\hat{a}) - G(\hat{a}_E)) > 0. \tag{52}$$

Take the first and second terms of (52), I change the upper limit by $A$ and I add $\pi c(1 - G(\hat{a}))$ to the left hand side of that expression. These transformations do not change the inequality. I denote this new expression by $\tilde{\Omega}^I$, which is equal to:

$$\tilde{\Omega}^I = \pi \tau \int_{\hat{a}_E}^{A} W(a) g(a) \, da + \pi y^I (1 - G(\hat{a}_E)) + \pi c (1 - G(\hat{a})) + BE - \pi w (G(\hat{a}) - G(\hat{a}_E)), \tag{53}$$

$$\tilde{\Omega}^I > 0.$$

The second term of $\tilde{\Omega}^I$ is smaller than that of $\Omega^I$ since I assumed that $\Omega^I < 0$, i.e. $y^I < y$. Thereby, I get a contradiction: $\tilde{\Omega}^I < \Omega^I < 0$.

Now I turn to prove the second part, that is: $\Omega^I < 0 \Rightarrow RMP$. Assume that $\Omega^I < 0$ and the government applies a $PMP$. By the first part, under a $PMP$ I have Expression (51), what implies that $\Omega^I > 0$.

By applying the same reasoning as Type I equilibrium migration pattern, it is possible to show that: (i) a $PMP$ implies that $\Omega^{III} > 0$ and, (ii) $\Omega^{III} < 0 \Rightarrow RMP$. \hfill \blacksquare

**Proof of Corollary 3.**

Recall Expression (44). A positive (negative) value of $\Omega^{II}$ means that the brain drain increases (decreases) per capita income in the source economy. Assume that the government is applying a $PMP$. In particular, assume that the government subsidies both education and migration. Therefore, by Proposition (3) I have that $t^{II}(\hat{a}_E, a_M) > 0$ and $tm(\hat{a}_E, a_M) > 0$. Let \( \varpi^{II} \) be equal to:

$$\varpi^{II}(\hat{a}_E, a_M) - w. \tag{54}$$

A $PMP$ implies that Expression (54) is positive. By integrating it over $[\hat{a}_E, \hat{a}]$, and by applying some transformations that do not change the inequality, I get the following
expression:

\[ \mathcal{II} = \pi y \int _{\hat{a}_E}^{a_I} g(a)\,da + \pi \tau \int _{\hat{a}_E}^{a_I} W(a)\,g(a)\,da + \\
(1 - \pi) \int _{\hat{a}_E}^{a_I} ((1 - p)(wa - w) - c)\,g(a)\,da - \pi w \int _{\hat{a}_E}^{a_I} g(a)\,da, \]

\[ \mathcal{II} > 0. \] (55)

Let \( \lambda \) be equal to:

\[ \lambda = \pi (\Psi(\hat{a}_E, a_M) - (wa_M - c)). \] (56)

By assumption \( \lambda > 0 \). Then the optimal value of \( \beta \) is higher than \( a_M \). I integrate \( \lambda \) on \([a_M, \beta]\).

\[ \pi \rho \tau \int _{a_M}^{\beta} W(a)\,g(a)\,da + \pi pc (G(\beta) - G(a_M)) + \pi py (G(\beta) - G(a_M)) \\
- \pi pw \int _{a_M}^{\beta} ag(a)\,da > 0. \] (57)

After some transformation that do not change the inequality, I get:

\[ \overline{\lambda} = \pi \rho \tau \int _{a_M}^{A} W(a)\,g(a)\,da + \pi pc (1 - G(a_M)) + \pi py (1 - G(a_M)) \\
- \pi pw \int _{a_M}^{\beta} g(a)\,da, \]

\[ \overline{\lambda} > 0. \] (58)

Then by adding \( \mathcal{II} \) and \( \overline{\lambda} \), I have that \( \overline{\lambda} + \mathcal{II} > \Omega^{II} \). So that, \( \overline{\lambda} + \mathcal{II} > 0 \) does not imply that \( \Omega^{II} > 0 \). It means that it is possible that the government applies a PMP even when the brain drain decreases per capita income. ■

7 References


