ECONOMIC INTEGRATION AND MONETARY UNION IN EUROPE
OR THE IMPORTANCE OF BEING EARNEST:
A TARGET-ZONE APPROACH*

Enrique Alberola**

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** European University Institute Firenze, Universitat de València.
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ABSTRACT

This decade contemplates the completion of the process which will lead to the
Economic and Monetary Union by the end of the century. This paper suggests that target-
zone models could be an appropriate framework for the study of this process. Upon a basic
model, two central elements in the integration process, capital controls and credibility are
built in and their relationships studied with the help of a little self-made computer
programme. The conclusion stresses that, after the process of financial integration has been
fulfilled, only convergence can guarantee the success of EMU.

RESUMEN

La terminación del proceso que llevará a la Unión Monetaria y Europea tendrá lugar
a finales de este siglo. En este trabajo se propone utilizar los modelos de zonas objetivo
como el marco apropiado para el estudio de este proceso. Se incorporan al modelo básico dos
elementos centrales del proceso de integración, controles de capital y credibilidad, y se
estudian sus relaciones con la ayuda de un programa informático. La conclusión es que,
después de que se haya cumplido el proceso de integración financiera, sólo la convergencia
podrá garantizar el éxito de la UME.
1.- INTRODUCTION.

The European Community is heading towards Economic and Monetary Union (EMU) after the Maastricht Treaty signed in December 1991. A first essential step -regarding the Economic Union- will be taken by January 1st, 1993, when the single market enters into force, while by the end of the century the single currency, at least for some countries, will probably be a reality, though some doubts have emerged recently.

The eighties have contemplated a push forward in the economic integration ideal of the Rome Treaty: from an institutional point of view the adoption of the Single Act in 1986 opened the way to the Single Market we are about to experience; from the economic perspective the European Monetary System (EMS) has strengthened and become quite credible. It has also helped the main economic indicators to converge up to a certain extent\(^1\), without losing sight of its underlying goal of achieving the desired exchange rate stability within the EC.

The improvement in the behaviour of the EMS is a necessary condition for the success of the Single Market and, once it has been achieved, it will play even a more central role in its success, since the Single Market implies, among other facts, the free movement of capital and its restrictions have been important in the consolidation of the EMS\(^2\).

In the light of this, it is no surprise that, from the moment of the reshuffle of the EC (mid-eighties) to the Maastricht summit there has been a mounting literature on the evaluation, consequences and future of the EMS, proposals for what kind of monetary union to carry out, etc. Furthermore, from a more academic point of view, there has been a new area in International Economics derived directly from the EMS experience and from the proposals for global exchange rate management\(^3\), namely, the target zone approach to exchange rate determination.

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\(^1\) For an updating of the integration process and a descriptive analysis of the convergence see Ungerer et al.(1990), c.IV and III, respectively and on a brief summary of the Maastricht Accords see The Economist (1991).

\(^2\) Actually, the impulse in the integration had its counterpart in the EMS with the Basel-Nyborg Agreement in September 1987, aimed at the reinforcement of the ERM and attempting to accommodate the instability effects of the last stage of the liberalization of the capital markets. For details, see European Economy (1988), introduction and Gros & Thygesen (1988).

In a previous paper (Alberola (1992)) I have reviewed the literature on target zones and pointed out some of the points I consider interesting. The aim of this essay is to provide a proper framework to convey the process of financial integration and to explore the possible consequences of the future EMU.

Incorporating these issues in a target-zone setup requires us to modify the traditional models (*first generation models*) provided by the literature which I present in section 2; the modifications consist of adding two elements that I consider essential: credibility and capital controls. Capital controls have not been considered explicitly in the model by the literature up to now, so it constitutes an original contribution of this paper (section 3). In order to incorporate the credibility issue -section 4- I have chosen one of the alternatives that the literature offers, which is, in my opinion, the one which best adapts to the framework. The final result is a new target-zone model which accounts for both elements. Then, in section 5 I will explore the relationship between both factors; this link, in the form of *trade-off* is graphically derived from the final model. I have developed a simple computer routine in MATLAB -to which I will refer as TRAZOS- in which the target zone models I present in this paper can be simulated and which facilitates the derivation of mentioned interactions. Finally, I claim that these factors and their connection have been important to the success of the EMS experience in the last years and from that point of view the model I present can be a promising starting point for the subsequent study of the final stage of the process of economic integration e.g: the EMU.

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4 This programme is available on request.
2.- GENERAL SETUP.

The Exchange Rate Mechanism (ERM) in the EMS is a system of fixed exchange rates (around a band) which can be adjusted, preferably as infrequently as possible. The national central banks which belong to the ERM try to keep the currency within the predetermined bands and in that objective are sustained by a framework of financing facilities and obliged by a set of intervention rules. For details on its functioning and origins, see for instance Giavazzi & Giovannini (1989), ch. 1 and 2. Thus the EMS in conjunction with the ERM constitutes the most significant example of target zone.

As I have mentioned above, literature on EMS and target zones has been growing during the last years. Regarding target zones models, the original paper by Krugman (1989) was followed by a series of papers which a) developed and extended the basic model and b) adapted the mathematical framework target zones models, since the techniques were borrowed from financial economics. Examples of the first group are Svensson (1989), Froot & Obstfeld (1989), Miller & Weller (1989). Some of the most significant articles are collected in Krugman & Miller (1991); the main contribution of the second group will be given in the appendix.

After that “first generation” followed the empirical evidence that failed to test the target zones hypothesis (see Alberola (1992) s.3) and which led to the exploration of new issues, such as reserves (Krugman & Rothemberg (1990), Flood & Garber (1991)), devaluation risks and credibility (Svensson (1990,1991), Bertola & Svensson (1990), Frankel & Phillips (1991)). The literature has become so wide in so little time that several surveys have already been published, trying to make attainable the mathematical tools and collecting the latest developments (Bertola(1991), Pesenti(1991)). This paper, intended to emphasize the role of capital controls in the model and relate it to the credibility issues, could be included in this “second generation” of models.

As a starting point, the simplest and most classical model of target zones is introduced. Then an outline of how target zones can be sustained is presented to advance the necessary modifications of the basic model and to facilitate the understanding of the following steps. In order to be more easily followed, the text includes the least possible mathematical technicalities. These are to be found in the appendix, where a derivation of the models in this and the following sections and their solution is presented.
2.1.- The basic model.

In this subsection I will follow mainly Svensson (1989). Let us make use of a simple monetary model of exchange rate determination. This model contains evident weaknesses and there are other approaches which assume more elaborated specifications, such as the Dornbusch model used by Miller & Weller (1989); however, most of the literature is developed on this too simple underlying model. The demand for real balances (m-p) depends on the level of income (y) and the interest rate (i); e.g. a basic LM curve in which \( \alpha \) is the semielasticity of money demand to interest rates; the Purchasing-Power-Parity (PPP) holds by assumption, so the (log) exchange rate (s) equals the difference between (log) domestic and foreign prices (p-p*); the uncovered interest parity (UCP) does not necessarily hold: the interest rate differential equals the expectation of depreciation of the currency, plus a risk premium (r). Finally, we consider the money market equilibrium in the foreign country (LM*) and, for simplicity, we assume that the parameters are the same than in the domestic market:

\[
LM \rightarrow \quad m_i - p_i = \psi y_i - \alpha i_i
\]

\[
PPP \rightarrow \quad s_i = p_i - p^* \quad ,
\]

\[
UCP \rightarrow \quad (i_i - i^*_i) = E_t \frac{ds_i}{dt} + r_i
\]

\[
LM^* \rightarrow \quad m^*_i - p^*_i = \psi y^*_i - \alpha i^*_i
\]

(1)

Now we can solve this system to get an expression of the exchange rate. Solving for prices (p, p*) in the LM functions, substituting the expression of its difference in the PPP equation and substituting the interest rate differential in the UCP for the expectation, we can derive the following expression for the exchange rate which constitutes the basic equation of the target zones models:

\[
s_i = f_i^* \alpha E_t \frac{ds_i}{dt}
\]

(2)

It states that the current exchange rate (s) depends on some economic forces, the fundamentals (f) and of the expectations on the future exchange rate. Let us have a look at
the fundamentals; it is convenient to split them into two components: the (log) money supply (m) and the velocity (v):

\[ f_i = m_i + v_i \]  \hspace{1cm} (3)

\[ v_i = \psi(y_i - y_*) - m_i + \alpha r_i \]  \hspace{1cm} (4)

\[ m_i = \ln(D + R) \]  \hspace{1cm} (5)

Velocity in this very simple formulation depends on the difference between incomes - it would increase if the foreign country grew at higher rates than our country-, the foreign money supply and the risk premium which we will assume at the moment to be zero. The domestic money supply can be displayed in terms of a very simplified central bank balance sheet: the (high-powered) money equals the sum of domestic credit (D) and reserves (R).

It can be seen then that, a too relaxed monetary policy (m - m' > 0) depreciates the currency. The angular stone of this specification is the expectation component. The stochastic calculus techniques are intended to eliminate the expectation term and to achieve a formulation which makes the exchange rate depend only on the fundamentals (s = s(f)).

The appendix shows the mathematics. Here, it is enough to point out that the expected exchange rate depends on the future behaviour of fundamentals, and then from (2), it can be seen that the current exchange rate depends on the current and expected behaviour of fundamentals:

\[ s_t = \frac{1}{\alpha} E_t \int_{t}^{\infty} e^{\alpha(\tau - t)} f_\tau d\tau \]  \hspace{1cm} (6)

Under the stochastic specifications of the process driving the fundamentals given in the appendix, when the exchange regime is not constrained by any arrangement or intervention, that is, under a free float regime, there is a linear relationship between the current levels of exchange rates and fundamentals:

\[ s_t = k + f_t \]  \hspace{1cm} (7)
where \( k \) is a constant term. This implies that when the process is driftless (see appendix) the appreciation expectation is zero.

**Target zones**

The linear relationship in (7) breaks down in a target-zones regime because the expectations on the exchange are affected by the interventions of the central banks to defend the bands \((s_-, s_+)_1\). In the EMS case, the ERM arrangements compel the central banks to defend the band: when the currency hits the band the authorities are supposed to intervene: they will sell (buy) reserves when \( s \) reaches the upper (lower) band, reducing the quantity of money and the level of fundamentals to alleviate the pressure on the exchange rate. The agents have to take into account this feature when constructing their expectations. The probability of the authorities effectively implementing the interventions— in other words, the credibility of the system— also influences those expectations. Under a completely credible target zone, the agents know that the future exchange rate will not surpass its lower or upper limits \((s_- \text{ and } s_+, \text{ respectively})\), and that implies that the future path of the fundamentals will fluctuate between a lower \((f_-)\) and an upper limit \((f_+)\). The mathematical solution given in the appendix shows the following relationship:

\[
s = f - \frac{\sinh(\lambda f)}{\lambda \cosh(\lambda f_+)}
\]

This equation has been obtained by applying the “value matching” conditions, derived from the required continuity of \( s(f) \) in order to rule out arbitrage opportunities at the edges of the band. These conditions imply that \( s(f_+) = 0 \) \((s_-(f) = 0)\), that is, the relationship between exchange rates and fundamentals becomes 0 at the edges and is positive between the bands. Figure 1 plots the known S-shaped curve which arises from this relationship, for a concrete example (the diagonal line is the linear relationship in a free float exchange regime).

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5 The intervention can be carried out before the exchange rate hits the band (inframarginal interventions) and in fact a large proportion of them are inframarginal; the standard solution of the model only considers marginal intervention, but it can be easily extended to the inframarginal case (see Flood and Garber (1989) and Pesenti (1991)).

6 This simple specification involves complete credibility; in section 4 we will add incomplete credibility to the model.

7 The example has been carried out with TRAZOS. The values are an exchange band of 6% around the central parity, a semielasticity of demand of money \((\alpha)\) equal to two and a standard deviation for the process in the fundamentals, \( \sigma \) which equals 0.1—see appendix. Then the fundamentals are allowed to vary a 15.05% and the maximum interest rate differential is 4.53%.
What is the intuition behind this figure? As it has been said the causes of the curvature arise from the changes in the expectation which comes from a completely credible target-zones regime. The certainty of an intervention at the edges of the bands determine the expectations, since the future path of the fundamentals -the expectation’s determinants are known to be bounded by the constraints that the band imposes. In the upper part of the band there is a widening wedge between the expectation of depreciation under the different rules. Comparing (2),(7) and (8) we can see that the depreciation expectation is no longer zero but negative (expected appreciation): under a target zone regime, the closer the fundamentals get to its upper limit $f_+$, the greater the appreciation expectation, since the room for the (future) fundamentals to increase becomes smaller and the room to decrease larger biasing downwards the expectations. At the edge of the band the fundamentals can only go down and at that point the appreciation expectation is maximum, such as to make the exchange rate insensitive to further increases in the fundamentals.
The difficulties in the definition and measurement of the fundamentals make convenient the use of the interest rate differentials ($\delta = i - i'$) which are directly observable. The relationship between them is easily derived in the appendix and plotted in Figure 2, for the two bands in operation in the EMS$^8$ (2.25% and 6%):

$$\delta(f) = \frac{(s'(f) - f)}{\alpha}$$

(9)

**FIGURE 2**

RELATION BETWEEN EXCHANGE RATES, FUNDAMENTALS AND INTEREST RATES

At the upper (lower) part of the band, the interest differential is negative (positive) and equivalent smooth pasting conditions hold as the lower left part of the figure displays: in general terms, a strict monetary policy which keeps fundamentals at negative levels means

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$^8$ It is evident that the narrower the band, the smaller is the room for the fundamentals to move.
a positive interest rate differential which attracts capital and strengthens the currency, keeping it in the lower part of the band.

2.2.- Determinants of the performance of a target-zone.

The structural characteristics of the economy (conveyed in our model by the parameters of the monetary model and the process which drives the fundamentals) and the institutional setup of the band (fluctuation bands, intervention rules) determine the features of the basic target zone model such as displayed in Figures one and two. The basic model assumes perfect capital mobility and complete credibility; both assumptions will be relaxed later in the specification of the model. Here, I intend to explain their importance in the sustainability of the target zone. Figure 3 displays a scheme with the determinants of a sustainable target zone, in our case the EMS. The terms in brackets refer to the target-zone nomenclature I am using.

FIGURE 3
SUSTAINABILITY OF THE TARGET ZONE
The first factor is discipline. By that I mean the commitment of the authorities to keep the range of variation of the fundamentals within the bounds of the target zone \( f < f < f_+ \). This is the most straightforward way to preserve the target zone and it is the key feature of the success of EMS in the years ahead as I will try to explain.

Though the term is rather vague it embeds several essential concepts in the integration agenda. On the one hand *policy coordination* contributes to homogenize the national (monetary) policies, reducing their differences and, from (3)-(4), forcing the fundamentals to approach; on the other hand the very same membership to the EMS signifies a *commitment* to “behave well”; finally and more importantly, discipline implies the increasing *convergence* not only in the economic policies but also in the structural economic basis of the economies.

Convergence is the word which is at stake in the Maastricht Agreements and it is a prerequisite to carry out the EMU. Disciplined economic policies may not be enough to achieve convergence. In a context of economic union, the EMS has become a device to force convergence rather than an instrument to achieve stable exchange rates -as in its origins- in order to facilitate trade flows. Though discipline and convergence is what underlies the final success of the target zone (and of EMU) the institutional setup includes other elements which facilitate the endurance of the zone. These elements can be grouped in two categories, credibility and what I will call buffers.

**Buffers and credibility**

The “buffers” that appear in the scheme are the reserves stock and capital controls. I call them buffers because they are particular instruments in the hands of the authorities to absorb exchange rate tensions, hence maintaining the exchange rates within the bands.

The role of *reserves* in the support of the exchange rate is well known. The interventions are carried out by selling or buying reserves as previously explained and they affect the stock of reserves (in fact reserves are a *buffer stock*). The use of reserves is limited by its quantity. The relation between the position of reserves and the exchange rate sustainability were first studied by Krugman (1979) who then extended the analysis to the target zones with Rothenberg (1990). The main conclusion is that it is just an occassional instrument\(^9\) which, as we will see, tends to disappear in the long run and/or in a context of free movement of capital.

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\(^9\) See Alberola (1992), pgs.18-22 for a simplified exposition of the role of reserves and further references.
The importance of capital controls will be explored in the next section. At this moment it is enough to point out the intuition. The introduction of capital controls isolates the financial markets of the various countries and constitute a recurrent way of disrupting the link between capital flows and differentials of rentability. Once the controls have been placed, the differentials can increase up to a certain extent or, in other words, the national authorities are able to conduct a less constrained monetary policy. Imposing these capital controls is no longer possible for the main countries in the Community\textsuperscript{10} and the degrees of freedom of the national economic policies is reduced.

Whereas the buffers are instruments which are disappearing (capital controls) or which cannot be extensively used (reserves), there is a feature in the target zones that may contribute to the support of the exchange rate: the credibility of the system. This conveys two very closed concepts which will be properly defined in section 4: credibility and reputation.

Here it is enough to consider the reputation of the system as a stock variable and the credibility as a flow arising from the gained reputation. A target-zone which suffers too frequent realignments is not a reputed system. A lasting target-zone increases in general terms its reputation with time because the maintenance of the existing parities, mainly in objectively difficult circumstances, increases the confidence of the agents regarding the existing bands. Then, when the system faces new tensions, past performances play an important role in the actual expectations of the exchange rates: if the effective bands are highly credible the agents will consider that the probability of surpassing the band is very low and this helps to carry out more flexible policies to the authorities. This gives raise to a self-feeding mechanism (credibility $\rightarrow$ preservation of the zone $\rightarrow$ +reputation $\rightarrow$ +credibility). This completes the scheme laid out in Figure 3. A rich set of relationships is displayed there. The one we are interested in is also displayed: there may exist a trade-off between removal of capital controls and credibility, since the absence of controls renders more demanding and strenuous the support of the zone and that may erode the reputation of the system and reduce its credibility.

\textsuperscript{10} The two Directives on financial integration set July 1990 as the general deadline for the removal of all the barriers to free movement of capital. This process finished in January and June 1990 respectively for France and Italy, the two big countries which typically used to impose them. For Greece, Ireland, Spain and Portugal the deadline was extended. Spain was allowed to keep such barriers until December 1992, but they are effectively removed from February 1992. In December 1992 the safeguard clauses for these last countries will be revised.
S-shaped curve and the scheme

The assertion just made is to be investigated within the target-zone approach. Though both issues have yet to be integrated in the framework and it will be done in the next two sections, it may be useful to outline the results graphically on the standard S-curve.

Figure 4 displays an S-curve and the arrows indicate the effects of the different factors. Let us place ourselves in a point close to the upper bound. The traditional defence of the band consists of selling reserves (R) reducing the stock, as explained in the basic model\textsuperscript{11}. The parameter related to the capital mobility issue is \( \beta \). We will see that increasing the barriers to capital mobility (reducing \( \beta \)) allows the fundamental (and interest rates differential) to diverge more than with perfect mobility, stretching out the S-curve. Finally, increases in credibility are symbolised by a larger \( \pi \); it will be shown that the S-curve rotates away from the free float as the system becomes more credible.

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\textbf{FIGURE 4}

S-CURVE AND DEFENCE ELEMENTS

\textsuperscript{11} Since it is an unsterilized intervention there is a direct link between the sell of reserves and what we have called discipline, shown by the white arrow in figure 3: selling reserves implies a one-to-one reduction in the quantity of money and this binds the monetary policy, disciplining it.

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3.- CAPITAL CONTROLS.

The role that capital controls have played in the success of the EMS in the last years has been mainly stressed by the literature when the prospective of their definitive suppression was getting near.

Though there are occasional opinions dismissing the effectiveness of capital controls, the general conclusion is that capital controls have been important in maintaining exchange rate stability and in providing the authorities with a safety belt while -for instance-implementing inflation-convergence policies\textsuperscript{12}. In our context it means that it has contributed to build up the reputation of the ERM and has facilitated the convergence process (even to the point of delaying it -critics would conclude).

The importance of capital controls in the EMS countries was first stressed by Rogoff (1985). Their existence was evident from the differential between domestic (on-shore) and Euromarket (off-shore) interest rates; I will refer to that differential as $g$. Then, a more formal test was carried out by studying the existence of a shift in the volatility of interest rates between pre-EMS and post-EMS regimes\textsuperscript{13}. The various tests on volatility (parametric in Rogoff (1985), no-parametric in Artis & Taylor (1988)) support the effectiveness of the capital controls imposed by Italy and France.

There is an alternative way to analyse the importance of the restrictions to the capital flows which better suits our purposes. Let us denote by $id$ the domestic or on-shore interest rate and by $ie$ the Euromarket or off-shore interest rate. Giavazzi & Giovannini (1989), ch.7 exploit the on-shore off-shore differential ($id$-$ie$=$g$), to make a descriptive but very clarifying analysis of the working of capital controls in the EMS. The differential $g$ can be seen as non-exploited arbitrage opportunities. Given the current development of the financial markets and assuming efficiency in them, the existence of these “lost chances” may be evidence of

\textsuperscript{12} The Spanish case is paradigmatic: the fight against inflation has been based on a tight monetary policy which has created too wide an interest differential to be sustainable without capital controls within the EMS. An overvalued currency (in real terms) helps to reduce inflation and belonging to the EMS adds credibility to that effort; imposing capital controls may have welfare-improving effects (in the sense pointed out by Dornbusch (1986)) by allowing the exchange rate to be managed in the lower part of the band without any risks of surpassing it.

\textsuperscript{13} The argument is the following: EMS performance has greatly reduced exchange rate variability; but the variability may have been shifted to the other variable i.e. interest rates as long as it is accepted that there exists a "lump of uncertainty"-see Artis & Taylor (1988) pg. 198-9 for a discussion; then, the interest rate volatility should be expected to be higher in the post-EMS regime. This will be so in the off-shore market; on the other hand, if in the on-shore market this larger volatility hypothesis is rejected (or, equivalently, if there is a shift in the volatility of $g$) the existence of effective capital controls will be empirically supported.
effective capital controls along with other factors. Frenkel & MacArthur (1988) study the importance of these factors (transaction costs, political risks) in a covered parity (CP) approach but underline the preeminence of the capital controls element in the European countries.

Capital controls are not binding in the Euromarkets, hence we can rewrite the UCP in equation (1) with the Euro-rates (ie) and substituting a risk premium net of capital controls (nr) for the original risk premium (r)\(^{14}\):

\[
ie_i - ie^* = E_i \frac{ds}{dt} + nr_i
\]  

(10)

As Frenkel & McArthur show the CP and the UCP with the on-shore rates (id) leads to the rejection of the hypothesis in some EC countries: Spain, France, Italy, Portugal... despite the fact that some of them, significatively France, have a developed financial system; the most plausible explanation for this result is given by the existence of capital controls.

Consequently, the gap between on-shore and off-shore interest rates can be used to examine the importance of capital controls in the EMS\(^{15}\): If \(g\) were greater than zero (less than zero)-id-ie > 0, there would be an incentive to borrow in the Euromarket and lend at home and that would be an indication of controls on capital inflows (capital outflows). The larger that gap, the stricter are the capital controls.

The empirical evidence found by Giavazzi & Giovannini for Italy and France and Viñals (1990,a) for Spain is self-explaining: there is a manifest relationship between pressures in the EMS and the imposition of capital controls. Now, within this framework we proceed to suggest a route for the insertion of capital controls in our basic model.

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\(^{14}\) If the CP holds for the Euromarket, as the empirical evidence shows, nr is equal to the exchange risk premium. For a discussion on the fulfilling of the parity in the Euromarkets see Dooley & Isard (1980), pg.373-75; For empirical evidence, see Ailiber (1973) or Herring & Marston (1976).

More generally, an overall risk premium which conveyed, in addition to the premium derived from the appreciation risk, all those elements pointed by Frenkel & McArthur could be added to the scheme: exchange rate risk and political risk premia in the terminology of Ailiber (1973). The former could subsequently be divided into a risk premium due to capital controls and to uncertainty (on future controls on failure, in the service,etc.,i.e the actual political risk). So the final expression after the removal of the capital controls effect could also be stated with a modified overall risk premium composed of: exchange rate risk and "uncertainty" political risk i.e nr.

\(^{15}\) In fact, Giavazzi & Giovannini use the differential between differences in bid and ask rates to account more accurately for the arbitrage opportunities.
Degree of capital mobility and target zones

We assume for simplicity that the foreign country has no capital controls so \( g_{t} = 0 \);

\[ id_{t} - ie_{t} = g_{t} \]
\[ id^{\ast}_{t} - ie^{\ast}_{t} = g^{\ast} = 0 \]  \hspace{1cm} (11)

then, we can make the substitution in (10):

\[ id_{t} - g_{t} - id^{\ast}_{t} = E_{t} \frac{ds_{t}}{dt} + nr_{t} \]  \hspace{1cm} (12)

Returning to our basic model, the \textit{domestic interest rate} (\( id \)) is the determinant rate in the money market, that is, the LM curve in (1). Taking this into account and comparing the UCP in (1) with (12) and (10) we can see that \( g \) can alternatively be expressed as the difference between the “gross” and “net” risk premia \( (g=r-rn) \). We could now substitute (12) for (1) to include capital controls in the model; there is however a more appropriate way of doing it. Let us define the coefficient \( \beta \) and propose the transformed UCP equation:

\[ \beta(id_{t} - id^{\ast}_{t}) = E_{t} \frac{ds_{t}}{dt} + nr_{t} \]  \hspace{1cm} (13)

Then we just have to infer its equivalence to (12) given by

\[ \beta(id_{t} - id^{\ast}_{t}) = E_{t} \frac{ds_{t}}{dt} + nr_{t} = (id_{t} - id^{\ast}_{t}) - g_{t} \]

\[ (\beta - 1)(id_{t} - id^{\ast}_{t}) = -g_{t} \]  \hspace{1cm} (14)

Let us contemplate the case in which the restrictions are on capital inflows as has been the situation in Spain in recent years. Apart from the commented positive gap, it usually
also implies that the (given) onshore differential is also positive. Hence, when no capital controls exist the gap is zero and \((\beta-1)=0\), that is \(\beta=1\), being the interest rate differential equal to the expected deprecation plus the net risk premium; as the controls become stricter, the gap widens and \((\beta-1)\) increases in absolute value; in a complete isolated market the domestic differential would have no relation at all with the devaluation expectation \((\beta=0)\): \(\beta\) is resulting bounded between zero (no capital mobility) and one (perfect capital mobility).

Consequently, capital controls can be incorporated into the model by substituting

\[
id_i = i_i; \quad id^*_{-i} = i^*_i \\
\beta(i_i - i^*_i) = E\left(\frac{ds_t}{dt}\right) + nr_t \\
0 \leq \beta \leq 1
\]  

(15)

for the UCP in (1). The recomputation of the results having taking this modification into account (and setting again the (net) risk premium equal to zero) is carried out in the appendix.

Let us here simply recall the new expression for the basic equation and the free rate solution:

\[
s_t = f_t + \frac{\alpha}{\beta} E\left(\frac{ds_t}{dt}\right) \\
(16)
\]

\[
s_t = f_t + \frac{\alpha}{\beta} \mu \\
(17)
\]

\[\text{From (14) it can be shown that is would be so when the sum of } g \text{ were greater than the expected revaluation. In fact, Giavazzi & Giovannini stress the fact that it is immediately before expected devaluation when the gap -negative in this case- is bound to widen dramatically in short-term maturity assets. This "punctual" devaluation risk can also be incorporated to the model following the approach of Svensson (1989). A devaluation risk shifts the curve to the left, reducing the maximum interest differential and eventually changing its sign; TRAZOS allows for the consideration of this devaluation risk and it is possible to show with it how introducing capital controls as shown in this section offsets the effects of the devaluation risk.}\]
On the one hand, when the fundamental process is driftless the free float solution does not change, since the exchange rate expectation is zero (implying a zero differential for any degree of capital mobility). On the other hand, observe that the role of expectations increases as $\beta$ goes to zero, that is, as there are more capital controls. That means that the discrepancy between the exchange rate and the fundamental widens and the scope for the interest differentials to diverge becomes larger: the existence of strict controls allows the authorities to manage more easily the zone, so the agents' expectations that the band will not be surpassed are reinforced. In graphical terms the upper section of the S-curve stretches to the right as Figure 5 shows for $\beta=1$ and $\beta=0.5$, to encounter a further $f_1$. We can then conclude that applying capital controls eases the management of the exchange rate, allowing for a larger variation of the fundamentals and interest rate differentials.

FIGURE 5
IMPOSITION OF CAPITAL CONTROLS IN THE ZONE
Figure 6 displays the bounds of the interest rates (left) and fundamentals corresponding to the different degrees of capital mobility conveyed in the parameter $\beta$. An isolated market would allow for a completely autonomous policy: the values of $\delta$ and $f$ will tend to infinity (the plot displays $\beta=0.02$ as lower limit). As controls relax ($\beta \to 1$), the restraints on the policy become larger.

**FIGURE 6**

**DEGREE OF CAPITAL MOBILITY AND BOUNDS FOR INTEREST DIFFERENTIALS AND FUNDAMENTALS**

_Capital controls, reserves and integration_

The interaction between the buffers in the last section has been explored by Wyplosz (1986) in the context of speculative attacks. Disregarding this extreme possibility, it is anyway clear from the current setup that the difficulties of keeping the exchange within the band or the drains of reserves which it implies can lead to the imposition of capital controls as a plain method of overcoming the tensions. In terms of Figure 5 it would imply a downwards jump from the perfect capital mobility curve to the imperfect mobility curve, without any change in the fundamentals and without any cost in reserves. In addition to that, in the absence of capital controls sterilized intervention turns to be ineffective (see Rogoff (1984)): now, the exchange rate responds immediately to changes in the fundamentals and a variation in reserves ($R$) cannot be compensated with a variation in the credit ($D$), to keep
f unchanged, since it would drive the currency out of the band. This compels the authorities to a more disciplined behaviour.

Regarding the European integration process, by now the possibility of imposing explicit capital controls has practically disappeared ($\beta = 1$) and that could provoke strong speculative pressures on a currency expected to be devalued; that tension cannot be offset by tightening capital flows anymore, introducing a potential element of instability to the system (See Giavazzi & Giovannini and footnote 15). Due to this, the debate about financial integration has stressed, on the one hand the dangers of getting rid of this instrument and, on the other hand, the necessity of a higher coordination and commitment for convergence (our definition of discipline) to increase the credibility and counterweight the perils of the new situation\(^{17}\).

4.- CREDIBILITY AND REPUTATION.

The EMS can be seen as a game among private agents and policymakers (national and foreign) in which the latter are obliged by the ERM to cooperate in order to maintain a fixed exchange rate. Credibility and reputation are essential issues for the system to succeed and they find their natural consideration in the field of policy games. Both concepts are defined following Weber (1990,a):

-Reputation is the extent to which beliefs concerning a policy conform to the way in which the policy has actually been conducted.

-Credibility is defined as the extent to which beliefs concerning a policy conform to official announcements about that policy.

\(^{17}\) The basic reference are the experts' articles in European Economy (1988); for the Spanish case see Viñals (1990,b) and Viñals et al. (1990).
Reputation depends then on the past performance of the EMS (in this case) and its interaction with credibility can be compared to the building up of a stock (reputation) which delivers increased credibility, which contributes to keeping the exchange within the band, which expands the stock of reputation and so on, as it appears in Figure 3. In an EMS context, the "announcement" corresponds to the current bands to be defended.

The purpose of this section is to show how increases in credibility help to manage the target zone in the same way that credibility in counterinflation policies facilitates the effective deflation. The work of Weber is essential in this context.

On the one hand, he extends the policy coordination existing models on credibility (basically Cukierman & Meltzer’s models) which deal with monetary announcements, to allow for exchange rate (and interest rate) announcements (Weber (1990,b)). Previously, he had formulated two measures of credibility (average and marginal credibility) whose application in a learning process from monetary announcements had shown that the deflation policies had gained credibility in the EMS (Weber 1988); however, he did not find a similar pattern in the analysis of credibility of exchange rate commitments and -he concluded- it was anyway far from being perfect (Weber 1990,a). More recently, he has employed a similar analysis to explicit target zones models, but to this last contribution we will come back later.

The problem with the policy game framework is that the models are too stylized to represent the complex processes which the integration mechanisms convey\textsuperscript{18}. Furthermore, we are interested in a target model which combines the credibility issue with the integration of capital markets and the convergence process.

\textit{Credibility in target-zone models}

As stated in the introduction of section 2, devaluation risks and credibility have been introduced in the target zone framework in response to the empirical failure of the first generation of models. Simple tests of credibility have been constructed by Svensson (1990) and Frenkel & Phillips (1991). Their empirical results highlight the importance of the (lack of) credibility\textsuperscript{19}.

\textsuperscript{18} See Driffill (1988),pg 213.

\textsuperscript{19} The tests are too simple, I would say; for instance Svensson’s test measures the risk of devaluation and finds that it exists even when the currencies are strong. For an updating of the tests, see Torres (1992).
Svensson (1989) and Bertola & Svensson (1990) introduce explicitly credibility in the model by considering the devaluation risk\textsuperscript{20} inside the band, though the process which drives the devaluation risk is exogenously given. Following the speculative attack literature, the variable that has been used to endogenize that risk has been reserves: Delgado & Dumas (1990) study the relation between the reserves position and sustainability; Flood & Garber (1991) and Krugman & Rothenberg (1990) link target zones and speculative attacks models. More interestingly, Bertola & Caballero (1991) explore the probability of realignments as dependent on the past interventions (performance) in the zone, hence taking into account reputation.

In a recent paper, Weber (1991) has reexamined the failed empirical evidence concerning EMS previously studied in Flood, Rose & Mathieson (1990): once credibility (embodied in a time-varying devaluation risk) is taken into account, the data match the theory, giving a strong push to the course of research of this second generation of models.

All of these articles highlight three important points displayed in the scheme in Figure 3: the role of credibility in the sustainability of the zone, the limitations of our buffer stock (reserves) in the long run and the close relationship between both elements.

\textit{Allowing for credibility in the model}

I will make use here of an alternative way of accounting for credibility, though. It is given by Pesenti (1991) and it is analytically very simple to implement and provides insights on the 'tangency solution problem', which other approaches fail to do. The model presented in I.1 assumes complete credibility in the band. The bounds of variation in the fundamentals are solved for the tangency point of the curve with the exchange rate bands, by applying the smooth pasting conditions. Strictly, if the band is completely credible, then it does not matter that the fundamentals are surpassed. This is what I call the 'tangency solution problem'. This discomforting feature can be overcome by observing that in the EMS or in any fixed regime in general, a band can never be absolutely credible: there is always some uncertainty left.

Actually, the bounds given by the value matching conditions along with the free float line determine, depending on the credibility degree, the range of variation of fundamentals as the additional curve in Figure 7 shows. This degree of credibility will be given by a parameter $\pi$ fluctuating between zero (null credibility) and one (complete credibility). The

\textsuperscript{20} A more detailed description of the models mentioned here can be found in Alberola (1992) s.4.
tangency of the S-curve to the band is achieved only with complete credibility and it is what is assumed in the basic model. This limiting case is too extreme, as mentioned above; thus, the S-curve will always outdo the exchange rate bound when the maximum fundamental bound is surpassed\textsuperscript{21}. Let us develope this argument analytically.

\textbf{FIGURE 7}

\textbf{DEGREE OF CREDIBILITY IN THE BANDS}

Consider Figure 7 again. When the exchange rate reaches its upper limit at point C the authorities can decide either to \textbf{defend} the band and push the exchange rate into the band \((s'(f') \rightarrow s(f'))\), in which case the exchange rate jumps downwards; or to \textbf{abandon} the target zone, pushing the exchange rate up to its free float solution \((s'(f) \rightarrow f')\)\textsuperscript{22}.

\textsuperscript{21} Krugman & Rothenberg (1990) show that when the danger of a speculative attack exists there is no tangency either. Observe that what underlies that approach is a question of no credibility in the band which triggers the attack.

\textsuperscript{22} A third possibility is that a realignment were carried out and it would be equivalent to a devaluation. This could be expressed by a jump from \(s'\) to \(s^d\), where \(s^d = f + d\), being \(d\) the size of the devaluation centered on the free float level.
\( \pi \) can now be defined as the probability that the band is defended when it reaches \( C \). The no arbitrage condition implies that the expected jump in the exchange rate is zero:

\[
\pi [s(f^e) - s^c(f^e)] + (1 - \pi)[f^e - s^c(f^e)] = 0 \tag{18}
\]

or

\[
\pi s(f^e) + (1 - \pi)f^e = s^c(f^e) \tag{19}
\]

\( s(f^e) \) is given by the complete credibility solution in (8). Let \( B \) be the fraction term in that equation. We ignore the solution for \( s^c(f^e) \) since we do not know the expression for the curve to which \( C \) belongs -\( s^c(f) \). We just know that the relation is not linear and lies between the free-float and the complete credibility lines, so it can be expressed as follows:

\[
s^c(f) = f + B^* \tag{20}
\]

where \( B^* \) is to be determined.

Substituting now both values for the exchange rate into (19) we get

\[
\pi (f^e + B) + (1 - \pi)f^e = f^e + B^* \tag{21}
\]

then:

\[
B^* = \pi B \tag{22}
\]

which renders the solution for the \( s^c(f) \) curve extremely simple; after substituting the actual value of \( B \) we have

\[
s^c = f - \pi \frac{\sinh(\lambda f)}{\lambda \cosh(\lambda f^e)} \tag{23}
\]
The derivative of the curve at point C is not zero now but

\[ s_f |_{s=s_0} = 1 - \pi \frac{\cosh(\lambda f^c)}{\cosh(\lambda f_0)} \]

(24)

\[ 0 \leq s_f \leq 1 \Leftrightarrow 1 \geq \pi \geq 0 \]

due to \( \pi \) and to the fact that in this case \( f^c \) is different from \( f_+ \). Actually the incomplete credibility line represents a coefficient \( \pi = 0.8 \) and now the relation becomes more linear. Figure 8 illustrates the effects of credibility on the bounds of a given band: under no credibility and the assumption of a driftless process for the fundamentals, the interest rate differential is zero like in the free float case. As credibility increases (\( \pi \to 1 \)), the weight of the nonlinear element in (23) and the fundamentals and the interest differentials are allowed to drift away more towards the limit imposed by the complete credibility bounds.

**FIGURE 8**

**DEGREE OF CREDIBILITY AND BOUNDS FOR INTEREST Differentials AND FUNDAMENTS**

The self-feeding process pointed out in section 2.2. can also be observed: If, as mentioned in Bertola & Caballero (1990) the coefficient \( \pi \) depends on the past performance of the zone, i.e the reputation, the success in having defended the zone makes easier -ceteris paribus- its support in the future, since \( \pi \) would increase, approaching the actual curve to the complete credibility limiting case.
5.- THE TRADE-OFF BETWEEN CREDIBILITY AND CAPITAL CONTROLS.

Once the model is completed, we can examine the relationship between capital controls and credibility. We can do this with the help of an example and then consider more generally the claimed trade-off.

Consider a member of the EMS pursuing anti-inflationary policies by means of a tight monetary policy. That implies a positive interest differential (in our example, \( i-i' > 2 \)). A process of financial integration is under way, though the country still keeps some controls over capital flows (\( \beta = 0.7 \)); furthermore, our country belongs to the ERM and has been able to defend its central parity while the EMS has suffered no realignments. Thus the target zone enjoys a good reputation, which gives a high level of credibility to the system (\( \pi = 0.8 \)).

The currency is strong due to the interest rate differential and this is beneficial for the objectives of the authorities, since it allows the country to "import deflation". Let us take a look at Figure 9. The current position is B on the less concave curve; the interest rate differential is 2.1%, below the maximum of 2.28% (see table).

<table>
<thead>
<tr>
<th>( \beta )</th>
<th>( \pi )</th>
<th>0.8</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>2.28</td>
<td>4.48</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1.93</td>
<td>2.27</td>
<td></td>
</tr>
</tbody>
</table>

The integration of the financial markets implies the removal of all capital controls (\( \beta: 0.7 \rightarrow 1 \)). That would signify, without any change in the fundamentals, an exceeding of the band (jump to T) since the sustainable differential falls now to 1.93%. Then, in order to defend the band, the government should buy reserves and, consequently relax its monetary policy (remember that without capital controls the intervention is unsterilized); that would endanger the prices objective and the badly needed convergence in the indicators.

There is nonetheless a way out of this vicious circle: gaining credibility; if, before removing the last controls, the continued good behaviour of the system had allowed a slightly higher degree of credibility (e.g. \( \pi: 0.8 \rightarrow 0.85 \)), the new curve -the most concave curve which crosses B- would allow the same level of discipline in the monetary policy as before with no controls, since it is possible to keep the interest rate differential above 2%.
FIGURE 9
EXAMPLE OF THE TRADE-OFF BETWEEN CREDIBILITY AND CONTROLS

Departing from the concrete example, two more effects regarding this trade-off could be mentioned:

A: On the negative side, the removal of capital controls itself adds uncertainty to the system since the agents know that one buffer is lost and consequently that the future defence of the band will be more difficult, reducing the credibility\textsuperscript{23}.

B: On the positive side, the irreversible free mobility of capitals removes any fear of future capital controls. As suggested by Frankel & MacArthur (1988) pg.1096, this is a risk element and as such should be incorporated in the risk

\textsuperscript{23} This suggests that the credibility degree not only depends on the gained reputation. In this case the reputation is not eroded and anyway credibility is lost. In the conclusion I will give some more insights.
premium (see footnote 14). Its suppression would reduce 'ceteris paribus' the value of the fundamentals in (3).

Trade-off and the integration process

Figure 10 is intended to introduce the next discussion; the upper bounds for the interest rate differential are considered. Take the graph on the left. Each line represents a value of $\pi$; the higher the line, the larger the degree of credibility. Then the plot displays the interest differential for the different degrees ($0.02 \leq \beta \leq 1$) of capital mobility. The graph on the right presents an equivalent picture for different degrees of credibility ($0 \leq \pi \leq 1$). The interpretation is straightforward at this point.

FIGURE 10

CONTROLS AND CREDIBILITY TRADE-OFF AND BOUNDS FOR THE INTEREST RATE DIFFERENTIALS

Consider now Figure 11. It displays the same relations in a three dimensional graph. The space underneath the surface is the range of variation of the interest rate differential at any combination $\beta, \pi$. Let us suppose that the country of the example is placed at a point on the perpendicular, dashed line from $x$ to zero, being $x$ the maximum possible interest rate differential $(i-i')_{x}$, for the given combination $\beta_{x}, \pi_{x}$. The financial integration process is
supposed to be irreversible. That means that the space under the shaded area to the right of x is no longer attainable; furthermore, the process is under way, so that area is spreading to the left (the arrows point west). At the limit the feasible area will collapse into the curve corresponding to $\beta = 1$.

**FIGURE 11**

INTEGRATION PROCESS AND POSSIBLE TRAJECTORIES

Given the integration process the actual range of variation of the interest rate will be ultimately given by the degree of credibility of the area. Then from x the paths of the interest differential bounds can be summarized by the arrows departing from point x$^{24}$. Trajectory y represents the *virtuous path*, trajectory z represents the *perverse path*. I am assuming that immediately after releasing some capital controls the system loses credibility (in terms of the digression above, effect A is supposed to be stronger than effect B). Subsequently, if the zone is well defended the system recovers and can even acquire more credibility, as effect B strengthens. Thus, in this case the path would be given by trajectory y. It can be shown by simulation in TRAZOS that it could be possible to attain the same range of variation in (i-i*) at x than in a point on the $\beta = 1$ curve with enough credibility.

---

$^{24}$ The actual process of integration in the EC does not imply total irreversibility. There are still applicable safeguard clauses (to be discussed again by December 1992) and countries like Spain which have liberalised the movements of capital can still re impose them. In the graph it would mean that the arrow would point southeast: the cause of reimposing the controls would coincide with an excessive flow of capital and a subsequent devaluation risk which would erode the credibility. In the long run however the arrow would turn west probably with an additional loss of credibility.
Now suppose that after the removal of the controls the currency is under a strong pressure which gives rise to fears either of a realignment or of the final feasibility of the convergence process; in this case the effect A drives the path to direction southwest, as in z, rendering more and more difficult the defence of the band and eventually making the situation collapse into the form of a realignment or, worse a withdrawal of that country from the band.

In the light of this argumentation, which is the more convenient strategy to dismantle the capital controls without losing grip on the target zone?, The authorities will try to choose a y-type path; the question is whether the dynamic trade-off is manageable. Capital controls should be stripped as the authorities feel that they are able to accommodate the shock (discipline) and that this confidence can be transmitted to the markets (credibility), which will itself be possible only as long as the system has reputation.

“Shock accommodation” implies degrees of freedom in the hands of the authorities and that may denote several things: on the one hand, in the case of a weak currency it means a healthy position of reserves; on the other hand it is advisable that prior to the removal of the controls, pressure on the currency is not too strong: that means staying as far as possible from x on the perpendicular line.

Let us take the recent Spanish case. On February 1st, 1992 the remaining explicit capital controls were finally removed. At that time the peseta was around the middle of the lower part of the band (halfway between x and zero in the figure). Following that, interest rates were decreased (it had to be so according to the scheme if we move west) and doubts about the opportunity of the measure arose. Nevertheless the peseta strengthened in the ERM and reserves flowed in. All in all the band has been defended with no problems. So far, so good. The problem is that the relaxation of the monetary policy has had to be made with an inflation rate which was still high relative to the EMS countries and that puts at stake the very same credibility of the convergence process and consequently the long-run credibility of the system.

25 Fortunately enough (and probably not by chance) the measure has coincided with inflationary pressures in Germany which have tightened the monetary policy there. As a result the interest differential has been "exogenously" reduced.
6.- CONCLUSION: THE IMPORTANCE OF BEING EARNEST.

The previous example concerning Spain has tried to highlight the difficulties involved in the process of economic integration. There is still an element which has not been introduced: the Monetary Union (EMU), which can be considered the final stage of the overall integration process. Recent literature has underlined from different angles the various concerns on EMU. Hughes-Hallet & Viner (1991) point out the dangers of losing all the tools capable of adjusting an external shock in an EMU from a dynamic game context; Frattiani, Von Hagen & Waller (1992) review the various proposals to implement EMU and show the common underlying worry of being excessively fast and the doubts that the convergence process is not mature enough to get to EMU; in the several opinions collected in European Economy (1988) or in Viñals (1991a) the removal of controls is related to the future performance of EMS. As a consequence of all these evaluations and other political conditionings, Maastricht Agreements are subject to the fulfilling of a set of conditions summed up in the accomplishment of convergence indicators to enter the final phase of monetary union.\textsuperscript{26}

In terms of Figure 11 EMU implies that the surface will rotate towards the origin, eventually collapsing to zero.\textsuperscript{27} Since throughout this process the countries will be placed on the $\beta=1$ line (perfect capital mobility), we can see that preserving the credibility will be an essential feature in the years ahead. Furthermore, since the 'buffers' are lost the only instrument in the hands of the authorities is discipline or, to use a more fashionable term, convergence. As long as the authorities achieve convergence the gained reputation in the EMS will be kept; this will still allow for certain degree of autonomy in the conduct of the policies. Otherwise the country will enter a vicious circle: failure in convergence $\rightarrow$ loss of credibility $\rightarrow$ stronger exigency of discipline.... On the road to EMU, a firm commitment to discipline is the only way to exploit the advantages of the reputation by now acquired, making the adjustment process the least costly possible. All in all, this is- as Mr. Wilde would put it- the importance of being Earnest.

\textsuperscript{26} See The Economist (1991) for an explanation on the indicators. Greece has no chance but there are other countries with difficulties: Italy has a unsurmountable deficit and no political power to skip it; Spain's inflation has a hard-core component very difficult to erode despite the fast progresses achieved recently; quite ironically, last events in Germany may make it unfit for EMU....

\textsuperscript{27} It is supposed that before the single currency, the bands will narrow to 1,5%; Spain and Portugal still have to enter the narrow band.
This paper has intended to show how to endogenize some of the issues concerning the process of European integration: credibility of the EMS and the role of the capital controls. In my view, target-zone models offer an ideal framework to study the convergence process. Before doing that it may be convenient to examine the empirical evidence. Regarding Spain, the more than two years of membership can be considered at first as a success, since no realignment has taken place. However the underlying costs are not difficult to identify: piling up of a huge reserves stock which may endanger the managing of the monetary policy, loss of competitiveness of our industries, etc.; moreover, some uncertainties are present at the moment: how to reconcile a strong peseta with a still wide inflation differential -in terms of PPP- and which consequences will have this in the long run; the feasibility of entering a narrower band and eventually a monetary union and so on. In order to account for all these questions in a target-zone framework it is necessary to extend the model in several directions. If it can be done, a wide and promising field of study will open up to venture on.
APPENDIX.


The tools of stochastic calculus borrowed from the theory of options are used here. Basic tough references are Karatzas & Schreive (1988) and Harrison (1985). More centered in the resolution of our setup are Dixit (1989) and above all Froot & Obstfeld (1991) and Smith (1991).

Let us rewrite (2) for convenience

\[ s_t = f_t + \alpha E_t \frac{ds_t}{dt} \]  

(2')

So the current exchange rate depends on the current level of fundamentals and on the future exchange rate (rational) expectations, which will depend on the future levels of fundamentals. There is an unique exchange rate path which satisfies (2')- the saddlepath solution; It can be written in integral form as a function of the present and future fundamentals’ levels, that is, of the current fundamentals and the present discounted value of future fundamentals-being \( \frac{1}{\alpha} \) the discount factor:

\[ s_t = \frac{1}{\alpha} E_t \int_{t}^{\infty} e^{\alpha(t-r)} f_r dr \]  

(6')

That expression implicitly rules out bubbles so the transversality condition holds:

\[ \lim_{r \to \infty} e^{-\frac{\alpha}{c} r} s_r = 0 \]  

(1.7) \hspace{1cm} (A.1)

Free float

In order to solve the dynamic model we have to make explicit the behaviour of the fundamentals. We have to specify a process for each of its components: money (m) and velocity (v). At the moment we will assume that m, the monetary growth, is not affected by the variations in velocity under free float, so it can be considered as given (e.g \( m = 0 \), for instance, as in Svensson (1989)). On the other hand, the velocity and hence the fundamentals are assumed to follow.
a Brownian motion process\textsuperscript{28} (vid. Dixit(1989)); Consequently:

\begin{align}
\text{df} &= \text{dm} + \text{dv} \\
\text{m} &= 0 \Rightarrow \text{dm} = 0; \\
\text{dv} &= \mu dt + \sigma dz \\
\text{df} &= \text{dv} = \mu dt + \sigma dz
\end{align}

where \(dz\) is a standard Wiener process, such that \(E(dz) = 0\), \(E(dz^2) = dt\), so \(\sigma\) is the variance in the growth of \(v\) and \(f\) in this case by unit of time. \(\mu\) is the drift term. In fact, the Brownian process can be seen as a random walk with drift in continuous time. It also follows from this specification that:

\begin{align}
(f_t | f_0 = f_0) &\sim N(f_0 + \mu t, \sigma^2 t) \\
\text{So} \hspace{2cm} E(f_t | f_0) &= f_0 + \mu(\tau - t) \\
\forall \tau \geq t
\end{align}

Integrating then the saddlepath solution (6'), this simply is

\[ s_t = f_t + \alpha \mu \tag{A.4} \]

that is, the exchange rate equals the level of fundamentals plus its (weighted) drift, yielding the straight "free float" line in Figure 1\textsuperscript{29}. Comparing (2') and this expression it can be seen that the expectation equals the drift of fundamentals.

\textit{Target zone}

The simplest approach to introduce the target zone framework is assuming marginal interventions\textsuperscript{30}. These interventions can be considered infinitesimal and assumed to be non-sterilized (in terms of (5), there is a variation in the reserves not compensated by an increase in deposits). They are represented by regulators of the form \(dL\) (lower), \(dU\) (upper). \(dL\) represents, then, increases in the quantity of money (\(dL = dR = dm > 0\)) and it is activated when \(f\) reaches its lower bound \(f_l\). \(dU\) on

\textsuperscript{28} Alternative processes can be contemplated and its solution to be found. An interesting case is when fundamentals follows a Ornstein-Uhlenbeck process, implying mean-reverting fundamentals (vid. Froot & Obstfeld (1991), pg.249.

\textsuperscript{29} The solution is like (7) with drift.

\textsuperscript{30} For a resolution with intramarginal interventions see Pesenti (1991), pg.19-24.
the other hand, stands for decreases in m \((dU=dm=dR<0)\) and it is activated by \(f=f_+\). Consequently, the fundamental follows now a **regulated Brownian Motion**\(^{32} \) and (1.8) is modified to give:

\[
\begin{align*}
    dm &= dL - dU; \\
    df &= \mu dt + \sigma dz + dU - dL
\end{align*}
\]  

\((A.5)\)

It can be seen that there is a **stochastic process shift** in the fundamentals when the fundamentals make the exchange rate hitting the band. The essential point in the resolution is to find solutions which render continuous path solution when a process switch is present. The procedure consist of two steps: firstly, functions of the form \(s=s(f)\) which satisfy (2') when fundamentals follow (A.2); then, the member of the family which satisfies the boundary conditions of the second process is found and it constitutes the saddlepath solution of the problem.

An Ito process can be set for the exchange rate (for details in a simplified way see Bertola(1991)) and subsequently apply Ito's lemma on \(s\), to express:

\[
\begin{align*}
    E_i \frac{ds}{dt} &= E_i \frac{ds(f)}{dt} = \alpha \mu s(f) + \frac{\sigma}{2} \sigma^2 s(f); \\
    So from (2') &
\end{align*}
\]  

\((A.6)\)

\[s = f + \alpha \mu s(f) + \frac{\sigma}{2} \sigma^2 s(f)\]

which is a second order differential equation\(^{32} \):

---

\(^{31}\) Actually, allowing for imperfect capital mobility implies that \(f=f(\beta)\), and altering the capital mobility parameter affects the fundamentals and its bounds, as will be evident later, so the lower bound under imperfect capital mobility can be expressed as \(f(\beta) = \phi(\beta, \lambda)f;\ \phi_\beta < 0, \phi_\sigma > 0, \phi_Y > 0\).

\(^{32}\) The theory of regulated Brownian motions can be found in Harrison(1985).

\(^{33}\) If we compare (A.6) and (A.4), we can observe that to achieve the previous free float result the derivative \(s_t\) should be one and the second derivative \(s_{tt}\) zero; namely, fundamentals and exchange rates are linearly (one to one) related. When there is a compromise of intervening to prevent the currency from surpassing the bands, that lineal relation breaks down, because the expectations discount that fact. Intuitively that is what is going on; analytically we have yet to give a step beyond to achieve the result.
\[ s = f + \alpha \mu + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \]
\[ \frac{\alpha \sigma^2}{2} \lambda^2 + \alpha \mu \lambda - 1 = 0 \]  
\[ \lambda_1 = -\frac{\mu}{\sigma^2} + \frac{B}{\alpha \sigma^2}; \quad \lambda_2 = -\frac{\mu}{\sigma^2} - \frac{B}{\alpha \sigma^2} \]  
\[ B = \sqrt{(\alpha \mu)^2 + 2 \alpha \sigma^2} \]

where \( \lambda_1, \lambda_2 \) are the roots of the characteristic equation and \( A_1, A_2 \) are constants of integration, and the last two terms are non linear. These terms will be binding when there is a expected regime switch.

Now the constants \( A_1, A_2 \) have to be solved for the boundary conditions. Let us take the upper band condition, derived from the application of Ito’s lemma to the regulated brownian motion.

\[ E[ds(f_\cdot)] = \alpha \mu s(f_\cdot) dt + \frac{\sigma^2}{2} s(f_\cdot) dt + s(f_\cdot) dU \]  
(A.8)

Comparing (A.8) and the first expression in (A.6), it can be seen that guaranteeing the continuity of the expectation and, consequently of the exchange rate-fundamental relationship implies that \( s(f_\cdot) = 0 \) (= \( s(f) \)). These are the value matching conditions\(^{34}\). Deriving the differential equation in (A.7)

\[ 0 = 1 + A_1 \lambda_1 e^{\lambda_1 t} + A_2 \lambda_2 e^{\lambda_2 t} \]
(A.9)

\[ 0 = 1 + A_1 \lambda_1 e^{\lambda_1 t} + A_2 \lambda_2 e^{\lambda_2 t} \]

from which \( A_1, A_2 \) can be solved. Then the saddlepath value of the target zone is defined by the following expression:

\[ s = f + \alpha \mu + \lambda_1 (e^{\lambda_1 t} - e^{\lambda_2 t}) + \lambda_2 (e^{\lambda_2 t} - e^{\lambda_1 t}) \]
\[ \frac{\lambda_1 \lambda_2 (e^{\lambda_1 t} - e^{\lambda_2 t})}{\lambda_1 \lambda_2 (e^{\lambda_2 t} - e^{\lambda_1 t})} \]  
(A.10)

This can be highly simplified if we take a process with drift zero (\( \mu = 0 \)). On the one hand from (A.7) \( \lambda_1 = \lambda_2 = \lambda \) and \( A_1 = -A_2 = A = [\lambda(e^{\lambda t} - e^{\lambda t})] \). On the other hand, since the bands are supposed to be symmetric \( f = -f_\cdot \); finally, the exponential equations are substituted by hyperbolic

\(^{34}\) Also called 'smooth pasting' in the literature. Here I have followed Froot & Obstfeld’s (1989) explanation, but there are other kind of derivations like those formal of Dixit (1989), the heuristical of Krugman (1989) or even, relating it to the reserve position and attacks (Flood & Garber(1989), Krugman & Rotemberg (1990)).
trigonometric functions\textsuperscript{35} and (A.10) yields now

\[ s = f - \frac{\sinh(\lambda f)}{\lambda \cosh(\lambda f)} \]  
(A.10')

where it is immediate to check that the derivative at the boundaries is zero.

Finally, let us express the interest rate differential in terms of the fundamentals. Letting \((i-i^*)=\sigma\) stand for the interest rate differential and substituting the UCP condition in (2) we obtain:

\[
\delta(f) = \frac{(s(f) - f)}{\sigma} \quad (1.17)
\]

\[
\delta f = \frac{(s - 1)}{\sigma} \quad (1.18)
\]

Then it is straightforward to derive a relationship between exchange rate and interest rate differentials as Figure 2 shows.

\textit{Capital controls}

The parameter \(\beta\) is added to allow for constrains to the capital mobility changes the model in the following way. Firstly, the UCP is modified by \(\beta\):

\[ UCP \rightarrow \beta(l, \frac{dE_t}{dt}) = \frac{ds}{dt} \quad (A.11) \]

\[ 0 \leq \beta \leq 1 \]

and that implies a change in the basic equation \((2')\) and in the saddlepath solution \((6')\):

\textsuperscript{35} We make use of the expressions \(2\sinh(x) = [e^x - e^{-x}]\) and \(2\cosh(x) = [e^x + e^{-x}]\).
\[ s_i = f_i + \frac{\alpha}{\beta} E_i \frac{ds_i}{dt} \quad \text{(A.12)} \]

\[ s_i = \frac{\beta}{\alpha} E_i \int_0^\infty e^{\alpha t - \eta} f_i d\tau \quad \text{(A.13)} \]

The solution derived for the free float case becomes now:

\[ s_i = f_i + \frac{\alpha}{\beta} \mu \quad \text{(A.14)} \]

so in the driftless case the solution does not change. The solutions in a target zone change in two ways: firstly, from (A.12) it can be observed that the wedge which the expectation term reflects widens as \( \beta \) tends to zero shifting the upper bound to the right in Fig 5. Secondly the differential equation and consequently its roots and solutions are modified as follows:

\[ s_i = f_i + \frac{\alpha}{\beta} \mu + A_1 e^{\lambda_1 t} + A_2 e^{\lambda_2 t} \]

\[ \frac{\alpha \sigma^2}{2\beta} \lambda^2 + \frac{\alpha \mu}{\beta} \lambda - 1 = 0 \]

\[ \lambda_1 = -\frac{\mu}{\sigma^2} + \frac{\beta}{\alpha \sigma^2}; \lambda_2 = -\frac{\mu}{\sigma^2} - \frac{\beta}{\alpha \sigma^2} \]

\[ B = \sqrt{\frac{(\alpha \mu)^2}{\beta^2} + 2 \frac{\alpha \sigma^2}{\beta}} \]

\[ s = f_i + \frac{\alpha}{\beta} \mu + \frac{\lambda_2 (e^{\lambda_2 t} - e^{\lambda_1 t}) + \lambda_1 (e^{\lambda_1 t} - e^{\lambda_2 t})}{\lambda_1 \lambda_2 (e^{\lambda_1 t} - e^{\lambda_2 t})} \quad \text{(A.15)} \]

The non-linear expression (A.10') for the driftless change does not change in its formulation, but, of course the values for \( \lambda \) and \( f + \) do; that is also the case of the interest rate-fundamentals relation in (A.11).
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