MULTIPRODUCT ACTIVITY AND COMPETITION POLICY: 
THE TETRA PAK CASE*

Aurora García Gallego and Nikolaos Georgantzis**

WP-EC 95-04

* The research presented here was financed by the Instituto Valenciano de Investigaciones Económicas (IVIE), which Industrial Economics area is sponsored by IMPIVA. Part of the research was written when the authors were visiting the Faculdade de Economia of the Universidade Nova de Lisboa. We are grateful to the faculty members of that University, and especially to Luis Cabral and Pedro Pita Barros, for their hospitality and stimulating discussion. We are grateful to Francisco Caballero, Amparo Urbano, Gonzalo Olcina, Vicente Orts and Roberto Waldmann for encouraging comments. An earlier version was presented at the 1993 ASSET meeting in Barcelona, and the 1994 Economics Seminar at the University of Valencia, Spain. The paper benefitted substantially from comments and discussion by the participants in both events, as well as from comments by an anonymous referee. The authors are responsible for any remainig error.

**Universitat Jaume I, Castellón.
Editor: Instituto Valenciano de
Investigaciones Económicas, S.A.
ISBN: 84-482-1007-7
Depósito Legal: V-2105-1995
Impreso por Copistería Sanchis, S.L.,
Quart, 121-bajo, 46008-Valencia.
Impreso en España.
MULTIPRODUCT ACTIVITY AND COMPETITION POLICY:
THE TETRA PAK CASE

Aurora García Gallego and Nikolaos Georgantzis

ABSTRACT

The Bulow et al. [1985] framework is explored assuming demand and/or production relations. A multiproduct firm is assumed to have a single product rival in one of the two submarkets in which it participates. In the case of linear demand and cost functions, the direct and the strategic effects of the multiproduct firm’s actions with respect to the first product on the strategies of its rival in the supply of the second product cancel out entirely. In the presence of both demand and production relations and with more general specifications of demand and cost functions, we produce testable hypotheses concerning the market shares of each firm in each market. The price-setting version of the model produces similar results as far as market shares are concerned.

We suggest the Tetra Pak case as a real-world example.

KEY WORDS: strategic/production/demand complementarity,
strategic/production/demand substitutability.

RESUMEN

Se explora el método Bulow et al. presumiéndose relaciones de demanda y/o producción. Se presume que una empresa multiproducto tiene un rival en un único producto en uno de los dos submercados en los que participa. En el caso de funciones de coste y de demanda lineales, los efectos directos y estratégicos de las acciones de las empresas referidas al primer producto sobre las estrategias del rival en el suministro del segundo producto se anulan completamente. En presencia de relaciones de demanda y producción y de especificaciones más generales de funciones de demanda y coste, producimos hipótesis contrastables respecto a las participaciones de mercado de cada empresa en cada mercado. La versión para la fijación de precios del modelo produce resultados similares en lo que respecta a las participaciones de mercado.

Sugerimos el caso del Tetra Pak como un ejemplo real mundial.

PALABRAS CLAVE: Complementariedad estratégica/productiva/de demanda,
Sustituibilidad estratégica/productiva/de demanda
I Introduction

In this study we attempt a review of the characteristics of multiproduct activity, which have to be taken into account by policy-makers and case-handlers when assessing the behaviour of firms with a broad range of activities in terms of competition law.

Even in the USA, where anti-trust legislation has a much longer tradition than in Europe, the concern of competition authorities about the behaviour of multiproduct firms is still expressed in a somewhat naive anti-dumping law according to which (international) dumping is either defined as (international) price discrimination, or as marginal-revenue-below-marginal-cost pricing. It is a common place that to treat price discrimination as an illegal practice may have a negative effect on social welfare, whereas prices that yield a non-negative marginal profit are not sufficient to prove the legal, competitive behaviour of a multiproduct firm. The obvious conclusion would be that we need a point between price discrimination and below-cost pricing to define the limits between legal and illegal behaviour of multiproduct firms.

While earlier studies had dealt with the case of economies with many products, it is Bulow et al. [1985] that sets the bases for studying the strategic aspects of multiproduct oligopolies. A multi-market firm is assumed to have a single-market rival in one of the two markets which it supplies, being a monopolist in the other market. This set-up is the simplest that can be used to model a multiproduct firm and asymmetries in oligopolists' production patterns.

The paper argues that

---

1 We will use the term multiproduct as a synonymous of multi-service, with more than one activities, multi-plant, multi-location or multi-market. In that sense, multiproduct firms may be the ones that operate in more than one market (including geographical markets), have more than one plants and/or supply more than one good or service. However, the terms multiproduct and multi-market do not always mean the same thing, given that a multiproduct firm may participate in the market for a differentiated good, selling two different varieties.

2 Salop [1979], Dixit and Stiglitz [1977], etc.
“...changes in a firm’s opportunities in one market may affect its profits by influencing its competitors’ (or potential competitors’) strategies in a second oligopoly market.”

Strategic interaction is shown to be of great importance and that cross-market effects depend on joint economies or diseconomies and on whether rivals consider their products as strategic substitutes or complements.

While Bulow’s et al. [1985] article is a main reference of most theoretical studies on multiproduct oligopolies, it has meant little -if any- contribution to the design of competition law. This is so, despite the fact that the authors dedicate two sections of the article to competition policy issues. More specifically, they argue\(^3\) that governments may use subsidies to “dump” national products at low prices in foreign markets and that a small scale entry in a previously monopolised market is not always welfare improving, because it may force the multiproduct firm to reduce output in another market. Although these two hints for competition policy worth the attention of competition policy-makers, Bulow’s et al. [1985] results are probably too general and, therefore, anticipate significantly different outcomes for different points along the demand and the cost functions. The clearcut observation that everything will depend on whether oligopolists consider their products as strategic substitutes or complements can be interpreted to mean that “any outcome may be justified as the result of firms’ legal, competitive interaction”.

We do not make this last remark to argue that Bulow’s et al. [1985] work is of no interest for competition policy. We would rather like to point out that:

1. Important theoretical contributions are not always ready-to-use tools in the hands of the policy-maker and

2. Bulow’s et al. [1985] framework should be extended to produce easy-to-apply tests for firms’ competitive or anti-competitive behaviour, in order for concrete measures of competition policy to be suggested to the authorities.

A very similar set up is used by Kim et al. [1992], which is written as an ‘Addendum’ and ‘Corrigendum’ of Röller and Tombak [1990]. The two articles show that multi-market

\(^3\)Following the tradition of Eaton and Grossmann [1983] and Brander and Spencer [1984].
contact may be the equilibrium outcome in the case of two firms which consider investing in a technology that is flexible in the sense that it can be used to produce two goods whose demands are inter-related. In a more general set-up, Klemperer and Padilla [1993] study product choice and mixed industry configurations as a result of an exogenous asymmetry in firms’ production possibilities. Potential competition comes from incumbent firms. Like Bulow et al. [1985] argued that a little bit of competition is not always socially desirable, they argue that firms’ product lines may include more varieties than what the social optimum would be. This point could mean that there is an optimum of competition to be pursued by policy-makers and that we are wrong to believe that the more we promote competition the better.

Contrary to an extensive bibliography on the anti-competitive characteristics of multiproduct firms’ behaviour (Harrington [1987], Bernheim and Whinston [1989], Barros [1994], Parker and Röller [1994] etc.), our results are derived under the assumption that firms’ behaviour is pro-competitive. Therefore, our results aim at serving as a guide of what could be observed as the result of firms’ pro-competitive behaviour, rather than what could firms do in order to abuse a dominant position or reduce competition among them. We show that the role of a multiproduct firm is different under demand and under production relations as far as the cross equilibrium effects between two different markets are concerned. The direct effect of a firm’s actions with respect to one of its products on the strategies of its rival in the supply of another product is not trivial. We study the linear case focusing in both a quantity and price setting scenario, obtaining compatible results: a multiproduct firm’s advantage from selling complementary products is expressed in a higher market share of the multiproduct firm as compared to its single-product rival in a certain product’s market. Finally, we suggest de Tetra Pak case as a real-world example and give some testable hypothesis that could outline whether or not the Commission was right to accuse Tetra Pak of anticompetitive behaviour.
II Implementation of Competition Policy in Europe and the Tetra Pak Case

One of the recent cases subject to the EEC Competition Law concerns Tetra Pak, one of the world leaders in the field of cartons for liquid food and the equipment and technology for filling these cartons. Economic analysis has been applied to defend both the Tetra Pak and the EC Commission positions.

The decision of the EC Commission was based on information that seems to prove Tetra Pak’s anti-competitive behaviour and, in particular, that Tetra Pak has a dominant position in the non-aseptic sector due to its abusive practices with respect to its monopoly in the aseptic sector.

The Tetra Pak group makes cartons for packing both fresh or non-aseptic and aseptic liquids. It has also its own technology for the manufacturing of machines for both fresh and aseptic liquids.

Tetra has one main competitor in the production of cartons for non-a-septics liquids: Elopak. Elopak brought the case to the EC Commission. Elopak accused Tetra Pak for having infringed articles 85 and 86 of the EC Treaty.

Up to now, the Tetra Pak case has gone through two stages. In the first stage, “Tetra Pak I” [1988], the Commission found that Tetra Pak had infringed article 86 by preventing potential competitors from having access to a novel technique of filling cartons under aseptic conditions. In 1981 the British National Research and Development Council (NRDC) assigned an exclusive license for the use of this novel technique to Novus Corp, a member of the Liquipak group. In 1986 Tetra got hold of the exclusive license assigned

---

5This technique concerns the UHT treated or fully sterilized milk.
6The Liquipak group specializes in the development and manufacturing of filling equipment for liquid food products. In 1986 Tetra acquired Liquipak International Inc (St. Paul, Minnesota, USA), Liquipak International BV (Netherlands) and Pak Center Limited (UK). Tetra did not acquire Novus Corp which in 1983, however, had assigned its licence to the companies that Tetra subsequently acquired.
to Liquipak7.

In the second stage of the case, Elopak Italia (Milan) asked the Commission to investigate whether or not Tetra Pak Italia8 and its associated companies were infringing article 86 of the Treaty9. In its Decision "Tetra Pak II " [1991], the EC Commission charged Tetra Pak for having taken advantage of its dominant position in the aseptic sector in machines and cartons to commit abuses on the related sector of non-aseptics. Tetra Pak's alleged abuse includes the use of restrictive contracts, discriminatory and predatory pricing of its cartons and liquid packaging machinery, and even an exclusive contract preventing competitors from advertising in an Italian milk trade journal.

In July 1991, the European Commission fined the company a record Ecu 75 million for abusing its dominant market position in Western Europe claiming that Tetra Pak had pursued a deliberate policy aiming to eliminate actual or potential competitors. Tetra Pak has appealed to the European Court against the fine. The case is still under investigation.

II.1 Products and relevant market

Different definitions of the relevant market have been used concerning the Tetra Pak case. On one hand, in Tetra Pak I and II, the Commission distinguished four markets: the aseptic cartons, the supply of machines used for filling aseptic cartons, non-aseptic cartons, and the machines that fill them. On the other hand, Tetra claimed that the market is defined as the market of packaging, including carton, glass, plastic, etc.

We assume that the activity of Tetra Pak includes two main sectors, the aseptic10 and

7The British Technology Group (BTG), a public undertaking which had meanwhile taken over NRDC's activities, did not raise objections to that acquisition.

8With sede in Modena.

9In principle, Italy was the only country affected by the denounce from the beginning. However, the EC Commission considers the EEC as the geographic market and, therefore, the decision was extended to all Member States.

10Free from infection.
the non-aseptic\textsuperscript{11}. The non-aseptically packaged products can not be stored for a long
time so that they need a very rapid and regular distribution system and there is always
the risk of waste. The aseptic package gives to the product a 'shelf life' of several months.

Under this definition, there are varieties of an 'aseptic product' and varieties of a
'non-aseptic product'. Different varieties in the same sector are close substitutes. These
two sectors define the number of products as well as of relevant markets. As far as the
machines are concerned, they are part of the technology necessary for the production
of the different packaging modes and therefore\textsuperscript{12}, they should be treated as an input of
the packaging production process. Therefore, we distinguish two different markets in the
Tetra Pak case, the market for aseptics and the market for non-aseptics.

In addition, the two sectors are related. On one hand, there is a demand relation
between them since varieties of both sectors may be substitutes\textsuperscript{13}. On the other hand, we
would expect that a production relation exists between aseptics and non-aseptics. The
Commission does not explicitly refer to any effect of the co-existence of the two types of
products in the production line of the same firm. Nevertheless, it is plausible to assume
that a producer of both aseptic and non-aseptic products experiences some savings in
the fixed costs of production or that the production of one type of products affects unit
production costs of the other type (e.g. in the case of economies of scale in the use of one
input which is common to the production of both products\textsuperscript{14}).

\textsuperscript{11}Used for packaging of fresh products.

\textsuperscript{12}Since they are products that are in a stage prior to the process of production and normally not
destined to be sold.

\textsuperscript{13}For example, if I want to buy fresh milk but, at the moment of the purchase I consider the price too
high, or I find out that there is only UHT milk left, I am likely to consider buying UHT milk instead.

\textsuperscript{14}See Baumol et al. [1982].
II.2 Technological background

The technology which was the object of the licence\textsuperscript{15} assigned to Tetra in 1986 is based on the synergy of ultra violet light (UVL) and hydrogen peroxide\textsuperscript{16}. With the basic sterilization process as a first requisite, much development on both the filling machine and carton were still necessary. In fact, the only technically acceptable aseptic carton packaging machines commercially available in the EEC for long shelf life treated liquid food\textsuperscript{17} are the Tetra and PKL machines, both based on similar sterilization methods\textsuperscript{18}. This method of sterilization\textsuperscript{19} used by Tetra Pak and PKL is considered by Elopak as adequate for cartons supplied in continuous rolls (such as the Tetra brik) but it is less suited to gable-top cartons\textsuperscript{20} such as those in which Elopak has experience\textsuperscript{21}. Then, the 'non-entry' of Elopak in the production of aseptics may result from Elopak's inflexible

\textsuperscript{15}See EC Commission decision [1988], 'Tetra Pak I'.

\textsuperscript{16}UVL enhances greatly the sterilizing properties of the hydrogen peroxide.

\textsuperscript{17}Especially the UHT treated milk. In fact some packaging machines for UHT liquids being developed or currently available are not suitable for milk but may be adequate for fruit juices.

\textsuperscript{18}The other machines capable of packing aseptically UHT milk are either not available in the EEC and/or are no more than prototypes not effectively commercially exploited.

\textsuperscript{19}Which is not patented.

\textsuperscript{20}These cartons are lined with polyethylene and can be easily opened. They are used especially for the pasteurized or fresh milk. Machines developed to seal gable-top cartons cannot normally seal brik cartons. Even for a gable-top machine or a brik machine, the cartons must be adapted to fit the particular machine.

\textsuperscript{21}The key for entering the market for supplying cartons for long shelf life food products lies in the ability to supply aseptic packaging machines for these cartons. A machine producer must not only have an adequate sterilization technique for the cartons, but must also be able to incorporate this technique into a reliable filling machine capable not only of working continuously and reliably at high speeds but also of maintaining an aseptic-sterile environment in dairy conditions. These are the technical barriers to entry for production of machines.
technology\textsuperscript{22}.

Tetra has patented not only the technology concerning machines and carton packaging but also all the posterior modifications to those products\textsuperscript{23}. Tetra holds more than hundred patents concerning the cartons and also more than hundred patents for the machines.

Due to technological reasons, entry into the aseptic packaging sector is difficult for a non-aseptic packaging producer. However, it is relatively easy for a producer in the aseptic sector to enter the non-aseptic packaging for fresh liquid food.

In ‘Tetra Pak II’, the Commission clearly reiterates the idea that aseptic and non-aseptic cartons form distinct relevant markets. However, the decision goes on to argue that, though distinct, these two markets are ‘related’\textsuperscript{24}. It goes on to assert that, as a result, \textit{abuses can and have occurred on the non-aseptic market by virtue of dominance on the aseptic sector}, that this conclusion is legally and economically justified and that, \textit{as a result, Tetra Pak is dominant in the non-aseptic sector}.

\section*{III \ Production, Demand and Strategic Relations in Quantity and Price-Setting Multi-market Oligopolies}

The set-up suggested by Bulow \textit{et al.} [1985] seems to be the natural way to model the Tetra Pak case. They assume an exogenously given industry configuration in which a multiproduct firm monopolises one market while it has one competitor in another market. Despite the similarity between this set-up and the case in question, it is worth noting that Röller and Tombak [1992] show that in the presence of demand substitutability between the two products (or markets for the two products) such a situation would never be endogenously determined as the equilibrium of a game in which both firms are allowed

\textsuperscript{22}See Röller and Tombak [1990].

\textsuperscript{23}As, for example, the way of plaiting the carton.

\textsuperscript{24}The Commission does not specify the meaning of this ‘relation’.
to choose between multiproduct and single product activity. It is, therefore, necessary that one of the two firms is exogenously prevented from participation in both markets. However, we will not insist much on this point, given that in the abovementioned case it is true that the single product firm is exogenously impeded from entering into the aseptic sector. With the same demand and cost functions as Röller and Tombak [1992] we show that\textsuperscript{25}, under a broad range of conditions a barrier to entry into the second market for one of the two firms is sufficient for Bulow’s et al. [1985] set-up to emerge as an equilibrium configuration\textsuperscript{26}.

In the special case of demand relations and linear demand and cost functions, in equilibrium there is no effect of the multiproduct firm’s strategies in the monopolised market on its rival’s strategies in the duopolised market.

This last result follows Bulow’s et al. [1985] conjecture that demand relations would produce results similar to those obtained assuming strategic relations but (footnote No. 23) “...if demands are connected, firms must of course also account for any direct effect of their actions in one market on competitors’ behavior in another market”. Despite the aforementioned short reference to the direct effect, its magnitude seems to be underestimated.

**III.1 Some general characteristics of the Bulow et al. [1985] set-up and the role of multiproduct firms**

The model is developed as a static quantity-setting game with perfect and complete information. In the case in which there is a multiproduct firm, firm A is the sole producer of good 1 – which is produced in quantity $q_{A1}$ – and it competes with firm B in the production of good 2. Firm B produces only product 2. Therefore, product 2 is supplied

\textsuperscript{25}Appendix A.

\textsuperscript{26}Things become more complicated if one requires that the configuration in question is the free-entry long-run Cournot equilibrium of the industry. For a detailed discussion of the long-run implications of multiproduct activity see Georgantzis [1994].
by both firms $A$ and $B$ in quantities $q_{A2}$ and $q_{B2}$ respectively, and it is a perfectly homogeneous good. When there is no multiproduct firm, firm $A$ produces just product 1, firm $B$ produces product 2 and a third firm, firm $C$, competes with firm $B$ in the production of product 2 – which produces it in quantity $q_{C2}$.

When firm $A$ is a multiproduct firm, profit functions are expressed as:

$$\Pi_A = f_A(q_{A1}, q_{A2}, q_{B2})$$
$$\Pi_B = f_B(q_{A1}, q_{A2}, q_{B2})$$

(1)

Firms produce the quantity of product that maximise their profit. A Nash equilibrium will satisfy:

$$
\begin{pmatrix}
\frac{\partial^2 \Pi_A}{\partial q_{A1} \partial q_{A1}} & \frac{\partial^2 \Pi_A}{\partial q_{A1} \partial q_{A2}} & \frac{\partial^2 \Pi_A}{\partial q_{A1} \partial q_{B2}} \\
\frac{\partial^2 \Pi_A}{\partial q_{A2} \partial q_{A1}} & \frac{\partial^2 \Pi_A}{\partial q_{A2} \partial q_{A2}} & \frac{\partial^2 \Pi_A}{\partial q_{A2} \partial q_{B2}} \\
\frac{\partial^2 \Pi_A}{\partial q_{B2} \partial q_{A1}} & \frac{\partial^2 \Pi_A}{\partial q_{B2} \partial q_{A2}} & \frac{\partial^2 \Pi_A}{\partial q_{B2} \partial q_{B2}}
\end{pmatrix}
\cdot
\begin{pmatrix}
d q_{A1} \\
d q_{A2} \\
d q_{B2}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
$$

(2)

If there is no multiproduct firm, profit functions are expressed as:

$$\Pi_A = f_A(q_{A1}, q_{B2}, q_{C2})$$
$$\Pi_B = f_B(q_{A1}, q_{B2}, q_{C2})$$
$$\Pi_C = f_C(q_{A1}, q_{B2}, q_{C2})$$

(3)

for firms $A$, $B$ and $C$ respectively.

In this case, the Nash equilibrium satisfies:

$$
\begin{pmatrix}
\frac{\partial^2 \Pi_A}{\partial q_{A1} \partial q_{A1}} & \frac{\partial^2 \Pi_A}{\partial q_{A1} \partial q_{B2}} & \frac{\partial^2 \Pi_A}{\partial q_{A1} \partial q_{C2}} \\
\frac{\partial^2 \Pi_B}{\partial q_{B2} \partial q_{A1}} & \frac{\partial^2 \Pi_B}{\partial q_{B2} \partial q_{B2}} & \frac{\partial^2 \Pi_B}{\partial q_{B2} \partial q_{C2}} \\
\frac{\partial^2 \Pi_C}{\partial q_{C2} \partial q_{A1}} & \frac{\partial^2 \Pi_C}{\partial q_{C2} \partial q_{B2}} & \frac{\partial^2 \Pi_C}{\partial q_{C2} \partial q_{C2}}
\end{pmatrix}
\cdot
\begin{pmatrix}
d q_{A1} \\
d q_{B2} \\
d q_{C2}
\end{pmatrix}
= 
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix}
$$

(4)
It is clear that, in the case of production relations and no demand relation between the two products, the existence of a multiproduct firm is a necessary condition for an effect of the equilibrium in one of the two markets to be there.

Depending on whether there are demand relations and whether there is a multiproduct firm, we consider two different cases:

**Case 1**

Products are demand related and firm A is a multiproduct firm. In this case, profit functions for firms A and B are expressed as\(^{27}\):

\[
\Pi_A = P_1(q_{A1}, q_{A2}, q_{B2})q_{A1} + P_2(q_{A1}, q_{A2}, q_{B2})q_{A2} \\
\Pi_B = P_2(q_{A1}, q_{A2}, q_{B2})q_{B2}
\]  

(5)

Let the demand relation between products 1 and 2 be the only source of cross-market effects:

\[
\frac{\partial P_1}{\partial q_{A2}} = \frac{\partial P_1}{\partial q_{B2}} = - \frac{\partial P_2}{\partial q_{A1}} = -\lambda
\]  

(6)

which implies that:

\[
\frac{\partial \Pi_A}{\partial q_{A1} \partial q_{A2}} = -2\lambda
\]  

(7)

The Nash equilibrium conditions in this case result in:

\[
\begin{pmatrix}
  a_{11} & -2\lambda & -\lambda \\
  -2\lambda & a_{22} & a_{23} \\
  -\lambda & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
  dq_{A1} \\
  dq_{A2} \\
  dq_{B2}
\end{pmatrix} =
\begin{pmatrix}
  0 \\
  0 \\
  0
\end{pmatrix}
\]  

(8)

**Case 2**

Products are demand related and there is no multiproduct firm. Profit functions for firms A, B and C are of the form:

\[
\Pi_A = P_1(q_{A1}, q_{B2}, q_{C2})q_{A1}
\]

\(^{27}\)For simplicity, assume marginal costs equal zero.
\[ \Pi_B = P_2(q_{A1}, q_{B2}, q_{C2})q_{B2} \]
\[ \Pi_C = P_2(q_{A1}, q_{B2}, q_{C2})q_{C2} \]

(9)

The conditions for a Nash equilibrium are:

\[
\begin{pmatrix}
a_{11} & -\lambda & -\lambda \\
-\lambda & a_{22} & a_{23} \\
-\lambda & a_{32} & a_{33}
\end{pmatrix}
\begin{pmatrix}
dq_{A1} \\
dq_{B2} \\
dq_{C2}
\end{pmatrix}
= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}
\]

(10)

Comparison of cases 1 and 2 leads us to the following observation:

**With demand relations alone, there is some effect (direct complementarity or substitutability effect) of the equilibrium in one market on the equilibrium in the other market, even in the case that there are only single-product firms in the industry.**

The assumption of quantity competition is not necessary for the observation above to hold. However, we do not share Bulow's *et al.* [1985] opinion (see footnote 14, p. 501) on the way that price competition should be modelled. More specifically, they suggest that the multiproduct firm sets quantity in the first market and competes with the single-product firm in prices in the second product's market. Although in the standard monopoly case it makes no difference to assume price-setting instead of quantity-setting behaviour, in the presence of -say- demand substitutability between products 1 and 2, market 1 is not a standard monopoly any more, given that it faces competition from product 2. Due to this reason, it would not be indifferent for the results whether we assume price or quantity-setting behaviour in the first market. Then, assuming that the multiproduct firm sets quantity with respect to the first product and competes in prices in the market for the second product may not be the most obvious way to extend the analysis to price-setting oligopolies in the presence of demand cross-market relations.
III.2 Quantity-Setting Multiproduct Firms under Demand and/or Production Relations

As an specific application to the Tetra Pak case, we suggest a slightly more restricted version of the demand and cost functions, but allow for demand and production relations to co-exist. Firm A - Tetra Pak - is the only firm that has the proper know-how for the production of product 1 - aseptics - and, therefore, it is the only firm involved in the sector of product 1. On the other hand, the same firm produces product 2 - non-aseptics - which is also produced by firm 2 - Elopak -, the only competitor of firm A in the sector for product 2. From a technical point of view, the two modes of product 2 are considered as homogeneous and, from the demand point of view, they are perfect substitutes.

The inverse demand functions for products 1 and 2 are, respectively:

\[ p_1 = P_1(q_{A1},q_{A2},q_{B2}) \]  
\[ p_2 = P_2(q_{A1},q_{A2},q_{B2}) \]

in the regions in which prices and quantities are positive. These functions are continuous for all \( q_{A1} > 0, q_{A2} > 0, q_{B2} > 0 \) and such that demands are downward sloping:

\[ \frac{\partial P_1}{\partial q_{A1}} < 0, \frac{\partial P_2}{\partial q_{A2}} < 0, \frac{\partial P_2}{\partial q_{B2}} < 0 \]  
\[ (13) \]

and that \( q_{A2} \) and \( q_{B2} \) are quantities of a homogeneous good produced by different firms:

\[ \frac{\partial P_2}{\partial q_{A2}} = \frac{\partial P_2}{\partial q_{B2}} \land \frac{\partial P_1}{\partial q_{A2}} = \frac{\partial P_1}{\partial q_{B2}} \]  
\[ (14) \]

We also assume that direct demand effects are larger than cross-demand effects:

\[ \left| \frac{\partial P_1}{\partial q_{A1}} \right| > \left| \frac{\partial P_1}{\partial q_{i}} \right| \]
\begin{equation}
\left| \frac{\partial P_2}{\partial q_{A_1}} \right| < \left| \frac{\partial P_2}{\partial q_i} \right| \\
i = A2, B2
\end{equation}

In the production side, the cost structure of firms A and B is defined respectively by the following total cost functions:

\begin{align*}
C_A &= C_A(q_{A1}, q_{A2}) \\
C_B &= C_B(q_{B2})
\end{align*}

which are defined and continuous for all output levels \( q_{A1} \geq 0 \), \( q_{A2} \geq 0 \), and \( q_{B2} \geq 0 \). \( C_A \) and \( C_B \) have continuous first and second partial derivatives for all \( q_{A1}, q_{A2}, q_{B} \geq 0 \). Furthermore, the first partial derivatives of the cost functions, \( C_A \) and \( C_B \), are positive for all \( q_{A1}, q_{A2}, q_{B2} \geq 0 \). In addition, the cost function for firm A may be non-separable with respect to the two products, such that:

\begin{equation}
\frac{\partial^2 C_A}{\partial q_{A1} \partial q_{A2}} > 0
\end{equation}

if there is a relation of substitutability in the production of products 1 and 2, while:

\begin{equation}
\frac{\partial^2 C_A}{\partial q_{A1} \partial q_{A2}} < 0
\end{equation}

if the two products are complements in production.

Under this specification, profit functions are given by:

\begin{equation}
\Pi_A = P_1(q_{A1}, q_{A2}, q_{B2})q_{A1} + P_2(q_{A1}, q_{A2}, q_{B2})q_{A2} - C_A(q_{A1}, q_{A2})
\end{equation}

for firm A, and

\begin{equation}
\Pi_B = P_2(q_{A1}, q_{A2}, q_{B2})q_{B2} - C_B(q_{B2})
\end{equation}

for firm B.

The conditions of a Cournot-Nash equilibrium are:
\[ P_1^{A1} q_{A1} + P_1 + P_2^{A1} q_{A2} = C_A^{A1} \]  
\[ P_2^{A2} q_{A2} + P_2 + P_1^{A2} q_{A1} = C_A^{A2} \]  
\[ P_2^{B2} q_{B2} + P_2 = C_B^{B2} \]

where \( P_k^r \) represents the first derivative of the demand function for product \( k \) with respect to the variable \( q_r \) and \( C_k^r \) refers to the first derivation of the cost function for product \( k \) with respect to \( q_r \).

We assume that the interaction of the two products in consumer preferences and in the cost function of firm \( A \) is expressed in a term of the form \( \lambda q_{A1}(q_{A2} + q_{B2}) \) and \( \beta q_{A1}q_{A2} \) respectively.

Therefore, inverse demand functions can be written as follows\(^{28}\):

\[ p_1 = P_1(q_{A1}, q_{A2}, q_{B2}) = f_1(q_{A1}) + \lambda(q_{A2} + q_{B2}) \]  
\[ p_2 = P_2(q_{A1}, q_{A2}, q_{B2}) = f_2(q_{A2}, q_{B2}) + \lambda q_{A1} \]

where \( \lambda \) represents the relation of demand substitutability – if \( \lambda < 0 \) – or complementarity – if \( \lambda > 0 \) – between products 1 and 2.

In a similar way, the cost function of firm \( A \) can be expressed as:

\[ C_A = C_1(q_{A1}) + C_2(q_{A2}) + \beta q_{A1}q_{A2} \]

We assume that there are constant marginal costs which are equal for both firms. Therefore, differences in the costs of production of \( q_{A2} \) and \( q_{B2} \) derive from the fact that \( q_{A2} \) is produced jointly with \( q_{A1} \) or because of differences in the two firms' scales in the second market. The parameter \( \beta \) is a measure of production substitutability – if \( \beta > 0 \) – or complementarity – if \( \beta < 0 \) – between the two products.

\(^{28}\)Except for the standard properties of a demand function, we do not impose restrictions on the form of \( f_1 \) and \( f_2 \) functions.
Under the new specification the Nash equilibrium satisfies:

\[ P_{1}^{A1} q_{A1} + f_1 + \lambda (q_{A2} + q_{B2}) + \lambda q_{A2} = C_{A}^{A1} + \beta q_{A2} \]  
(28)

\[ P_{2}^{A2} q_{A2} + f_2 + 2\lambda q_{A1} = C_{A}^{A2} + \beta q_{A1} \]  
(29)

\[ P_{2}^{B2} q_{B2} + f_2 + \lambda q_{A1} = C_{B}^{B2} \]  
(30)

where \( C_{A}^{A1} \) and \( C_{A}^{A2} \) are the first derivatives of the functions \( C_1(q_{A1}) \) and \( C_2(q_{A2}) \) with respect to \( q_{A1} \) and \( q_{A2} \) respectively.

Using the fact that, due to homogeneity of the product in the second market, \( P_{2}^{B2} = P_{2}^{A2} \) and equations (29) and (30), we get:

**Proposition 1** Equilibrium quantities and prices of the quantity setting firms A and B in markets 1 and 2 satisfy

\[ (\lambda - \beta) q_{A1} + (C_{B}^{B2} - C_{A}^{A2}) = P_{2}^{A2} (q_{B2} - q_{A2}) \]  
(31)

One of the consequences of Proposition 1 is that demand and/or production relations between the two markets are directly related to the equilibrium market shares of the two firms in market 2. That is, the advantage (disadvantage) of the multiproduct firm, as expressed by the sum of production (and/or demand) complementarity (substitutability) between the two jointly produced goods increased (or decreased) by the cost advantage in the second products’ market, as expressed in the difference between the two firms’ marginal costs of producing 2, is expressed in the multiproduct firm’s larger (smaller in the case of disadvantage) market share in the second market as compared to that of its single-product rival in the same market. Since by expression (13) \( P_{2}^{A2} < 0 \), it is of interest to observe that if \( C_2 \) is a linear function:

\[ (\lambda - \beta) > 0 \implies q_{A2} > q_{B2} \]  
(32)

\[ (\lambda - \beta) < 0 \implies q_{B2} > q_{A2} \]  
(33)
Under this assumption, let us consider the case in which \( \beta = 0 \) so that there is no production relation between the products 1 and 2. In such a case, from Proposition 1 and considering that aseptics and non-aseptics are substitutes in demand (\( \lambda < 0 \)), firm \( A \) will have lower sales of product 2 than firm \( B \), for all positive quantities of product 1 – i.e. \( q_{B2} > q_{A2} \).

Similarly, if products 1 and 2 are substitutes in production as well as in demand (\( \beta > 0 \)), firm \( B \) will produce more of product 2 than firm \( A \) – i.e. \( q_{B2} > q_{A2} \).

If, on the contrary, products are substitutes in demand and complements in production, so that \( \lambda < 0 \) and \( \beta < 0 \), three results are possible:

(i) \( |\lambda| > |\beta| \). If the degree of demand substitutability between products 1 and 2 is larger than the degree of production complementarity between those products, firm \( B \) is expected to produce more than firm \( A \) of product 2 – i.e. \( q_{B2} > q_{A2} \).

(ii) \( |\lambda| = |\beta| \). If the degrees of demand and production relations between the two products are equal, both firms \( A \) and \( B \) will produce exactly the same quantity of product 2 – i.e. \( q_{B2} = q_{A2} \).

(iii) \( |\lambda| < |\beta| \). When the degree of production complementarity between the two goods is bigger than the existing demand relation of substitutability, then firm \( A \) will produce more than firm \( B \) of product 2 – i.e. \( q_{A2} > q_{B2} \).

In an exhaustive empirical analysis of the situation, this relation has to be taken into account. We propose first to estimate the demand system expressed in equations (25)-(26). Second, to estimate the cost function for firm \( A \), as it is expressed in (27). Third, to test demand and production substitutability or complementarity by looking at the estimated values of the parameters \( \lambda \) and \( \beta \). For example, it is very important to verify whether \( \hat{\lambda} < 0 \) or not, i.e. whether or not the two products are demand substitutes. Finally, by comparing the estimated values for \( \lambda \) and \( \beta \), to compare \( \text{sign}\{\lambda - \beta\} \) to \( \text{sign}\{q_{A2} - q_{B2}\} \).

Imagine, as an example, that it happens that \( \hat{\lambda} < 0 \) and \( \hat{\beta} < 0 \), so that demand substitutability and production complementarity are estimated between the two products, and that those values are such that \( |\hat{\lambda}| < |\hat{\beta}| \). Then, the observation of quantities of product 2 produced by firm \( A \) higher than quantities of product 2 produced by firm \( B \)
(\(q_{A2} > q_{B2}\)) may result from firm A's competitive behaviour. In that case, one cannot use the existing relation between the position of Tetra Pak in the market for non aseptics as an evidence of abuse of the firm's dominant position in the market for aseptics.

### III.3 A Linear Quantity-Setting Model with Demand Relations

We present here a variation of the model presented above, assuming linear demand and cost functions and demand relations alone. An interesting implication is that, in absolute terms, the equilibrium profits and the output of the single-product firm, as well as the price in the second market are not different from what they would be in the standard Cournot-duopoly case. In other words, the participation of the multiproduct firm in the first market affects only its own profits and strategies. The conclusions of the more general version of the model are compatible with this result, in the sense that differences in the two firms' market shares in the second market are due to the effect of multiproduct activity on the multiproduct firm's profits and strategies only.

Assume that firms A and B are the producers of two different commodities, 1 and 2. Firm A is the sole producer of good 1 and it competes with firm B in the production of good 2. Firm B produces only product 2.

Consider that demands for the two goods are interrelated through substitutability or complementarity, depending on the sign of the parameter \(\lambda\) in the system of the following inverse demand functions:\(^{29}\)

\[
p_1 = a - b_1Q_1 - \lambda Q_2 \tag{34}
\]

\[
p_2 = h - b_2Q_2 - \lambda Q_1 \tag{35}
\]

in the regions in which both prices \(p_1\) and \(p_2\) as well as quantities are positive, and where

---

\(^{29}\)In Röller and Tombak [1990] a symmetric system of demand functions similar to the ones used here is derived as the result of the maximisation of a quadratic utility function by the representative consumer. We follow their notation as much as possible. Necessary differences are due to the fact that we allow for asymmetries in supply and demand conditions for the two products.
\[ Q_1 = q_{A1} + q_{B1} \]  
\[ Q_2 = q_{A2} + q_{B2} \]

are, respectively, the total quantities of products 1 and 2.\(^{30}\)

In particular, \(q_{A1}\) and \(q_{A2}\) represent, respectively, the quantities of products 1 and 2 produced by firm \(A\). Since firm \(A\) is the only producer of commodity 1 we set \(q_{B1} = 0\). We assume that this is a result of some barrier to entry or that firm \(B\) does not possess the know-how for the production of product 1. The quantity of good 2 supplied by firm \(B\) is denoted by \(q_{B2}\). The parameters \(a, h, b_1\) and \(b_2\) are non-negative. The parameter \(\lambda\) is a measure of substitutability – if \(\lambda > 0\) – or complementarity – if \(\lambda < 0\) – between the two products. Furthermore, we assume that \(|\lambda| < \min\{b_1, b_2\}\), which in Röller and Tombak [1990] is interpreted as a higher own price effect as compared to the cross price effect.

Production technologies are described by the linear cost functions:

\[ C_A = c_1 q_{A1} + c_2 q_{A2} \]  
\[ C_B = c_2 q_{B2} \]

where \(C_A\) and \(C_B\) are, respectively, the total costs of production for firms \(A\) and \(B\) and \(c_1, c_2\) are the marginal costs of production for products 1 and 2.

Under this specification, profit functions are given by

\[ \Pi_A = (a - b_1 q_{A1} - \lambda (q_{A2} + q_{B2})) q_{A1} + \\
(h - b_2 (q_{A2} + q_{B2}) - \lambda q_{A1}) q_{A2} - \\
(c_1 q_{A1} + c_2 q_{A2}) \]  

\(^{30}\)This is a special case of the general model in which \(q_{A2}\) and \(q_{B2}\) do not represent quantities of an homogenous good. In this case, the demand system is expressed by the following three equations: \(p_{A1} = a_1 - b_1 q_{A1} - \lambda_1 q_{A2} - \lambda_2 q_{B2}\)  
\(p_{A2} = a_2 - b_2 q_{A2} - \lambda_1 q_{A1} - \lambda_3 q_{B2}\)  
\(p_{B2} = a_3 - b_3 q_{B2} - \lambda_2 q_{A1} - \lambda_3 q_{A2}\)
for firm $A$ and

$$\Pi_B = (h - b_2(q_{A2} + q_{B2}) - \lambda q_{A1})q_{B2} - c_2q_{B2} \quad (41)$$

for firm $B$.

Firms are risk neutral and set quantities to maximise profits. We assume competition in a quantity setting fashion with respect to product 2. From equations (40) and (41) we get a negative sign for the second derivatives $\partial^2 \Pi_A / (\partial q_{A1} \partial q_{A2})$ and $\partial^2 \Pi_B / (\partial q_{B2} \partial q_{A2})$. These two derivatives measure the effect of strategic substitutability (due to our assumption of quantity competition) on the profits of the firms. Following Bulow et al. [1985], this means that each firm regards its output of product 2 as a strategic substitute of its rivals’ output of the same product.

What remains is to study the cross-effect of firms’ strategies with respect to different products. From equation (40), we get $\partial^2 \Pi_A / (\partial q_{A1} \partial q_{A2}) = -2\lambda$. In Bulow et al. [1985] this sign depends on economies or diseconomies of scope. Here we find that in the presence of demand substitutability (complementarity) between products 1 and 2, the cross-effect of firm $A$’s quantity strategies $q_{A1}$, $q_{A2}$ on its profits is negative (positive). This confirms the conjecture of Bulow et al. [1985] (p. 493, footnote 5) that demand and production relations would be reflected in the sign of $\partial^2 \Pi_A / (\partial q_{A1} \partial q_{A2})$ in the same way.

Finally, from equation (41) we get $\partial^2 \Pi_B / (\partial q_{A1} \partial q_{B2}) = -\lambda$, which implies that the cross-effect of firm $A$’s strategies with respect to product 1 on firm $B$’s strategies with respect to product 2 on firm $B$’s profit is positive (negative) if products are demand complements (substitutes). Derivatives $\partial^2 \Pi_A / (\partial q_{A1} \partial q_{A2})$ and $\partial^2 \Pi_B / (\partial q_{A1} \partial q_{B2})$ measure the direct effect that the demand relation between the two products has on firms’ profits.

Each firm takes its rival’s output as given. Equilibrium is reached simultaneously with respect to the two products. The assumptions on $|\lambda|$, $b_1$ and $b_2$ guarantee that equilibrium is locally strictly stable.

Firms decide the output strategies that maximise their profit. Maximisation results in the following reaction functions:

$$2b_1q_{A1} + 2\lambda q_{A2} + \lambda q_{B2} = a - c_1 \quad (42)$$
\[ 2\lambda q_{A1} + 2b_2q_{A2} + b_2q_{B2} = h - c_2 \]  
\[ \lambda q_{A1} + b_2q_{A2} + 2b_2q_{B2} = h - c_2 \]  

Solution of the system of equations (42), (43) and (44) gives equilibrium quantities

\[ q_{A1} = \frac{b_2(a - c_1) - \lambda(h - c_2)}{2(b_1b_2 - \lambda^2)} \]  
\[ q_{A2} = \frac{(2b_1b_2 + \lambda^2)(h - c_2) - 3b_2\lambda(a - c_1)}{6b_2(b_1b_2 - \lambda^2)} \]

for firm A and

\[ q_{B2} = \frac{h - c_2}{3b_2} \]

for firm B. Equilibrium outputs are assumed to be positive in order for the industry configuration to be non trivial. We assume that \( a - c_1 > 0 \) and \( h - c_2 > 0 \), which guarantee that firm B’s output is positive. In Appendix A we discuss the conditions under which firm A finds it profitable to produce positive quantities of both products.

Equilibrium prices for the two goods are given by:

\[ p_1 = \frac{3b_2(a + c_1) - \lambda(h - c_2)}{6b_2} \]  

and

\[ p_2 = \frac{h + 2c_2}{3} \]

Equilibrium profits for firm A are given by:

\[ \Pi_A = \frac{9b_2^2(a - c_1)^2 - 18b_2\lambda(h - c_2)(a - c_1) + (5\lambda^2 + 4b_1b_2)(h - c_2)^2}{36b_2(b_1b_2 - \lambda^2)} \]  

\[ \Pi_B = \frac{(h - c_2)^2}{9b_2} \]
Consider the case where demands are independent, i.e. $\lambda = 0$. In this case the equilibrium strategies correspond to the monopoly and the standard duopoly cases respectively: $q_{A1} = (a - c_1)/2b_1$ and $q_{A2} = q_{B2} = (h - c_2)/3b_2$. We take the $\lambda = 0$ case as a reference. Before discussing the implications of our model with respect to competition it is worth observing that:

**Proposition 2** Firm A's production of products 1 and 2 will be higher in the presence of demand complementarity than in the case in which the two products are unrelated. When products are demand substitutes firm A will produce less of at least one of them as compared to what it would produce if products were unrelated.

**Proof:** See Appendix B.

**Proposition 3** In the linear case firm B's output of product 2 is the same in the presence of demand relations between products 1 and 2 as it would be if the two products were unrelated. Moreover, the price of product 2 is the same as if it were sold in a duopoly in the absence of demand relations with product 1.

**Proof:** Straightforward from equations (47) and (49) respectively.

In the Tetra Pak case, decreases in Elopak's market share and profits in the non-aseptic sector would contradict Tetra Pak's pro-competitive behaviour, only if real world data could be used to prove that the linear model presented here gives a close enough description of the debated situation.

**III.4 Price-Setting Multiproduct Oligopolies and Demand Relations**

In this section, we present the price-setting version of our model in the presence of demand relations. We assume that firm $A$ produces two products, 1 and 2, competing with firm $B$ which produces only the second product. Demand functions are given by:
\[
q_{A1} = f_1(P_{A1}) + \theta_1 P_{A2} + \theta_2 P_{B2} \tag{52}
\]
\[
q_{A2} = f_2(P_{A2}) + \theta_1 P_{A1} + \theta_3 P_{B2} \tag{53}
\]
\[
q_{B2} = f_3(P_{B2}) + \theta_3 P_{A2} + \theta_2 P_{A1} \tag{54}
\]

where \( q_{Ki} \), \( P_{Ki} \) are respectively firm \( K \)'s output and price of variety \( i \).

Varieties 1 and 2 are weak substitutes (complements) if the parameter \( \theta \) is positive (negative), while the two firms' outputs of the second varieties are close substitutes\(^{31}\). In all cases, a firm's own price for a certain variety has a larger own demand effect than any other variety's cross-demand effect. These rather standard assumptions are expressed in \( |\theta_n| < |f_j^K| \), with \( n, j \in \{1, 2, 3\} \), \( K \in \{A, B\} \), \( i \in \{1, 2\} \) and \( |\theta_3| > \max\{|\theta_1|, |\theta_2|\} \), where \( f_j^K \) denotes the derivative of a demand function with respect to firm \( K \)'s output in the market for product \( i \).

Assuming that firm \( A \)'s cost function is separable in varieties 1 and 2 (no production relation) and that prices are net of marginal costs (which are assumed to be constant), the two firms' profits are given by:

\[
\Pi_A = P_{A1} q_{A1} + P_{A2} q_{A2} \tag{55}
\]
\[
\Pi_B = P_{B2} q_{B2} \tag{56}
\]

Then, from equations (52)-(56), we get that the Bertrand-Nash equilibrium will satisfy the following first order conditions:

\[
f_1 + P_{A1} f_1^{P_{A1}} + 2\theta_1 P_{A2} + \theta_2 P_{B2} = 0 \tag{57}
\]
\[
f_2 + P_{A2} f_2^{P_{A2}} + 2\theta_1 P_{A1} + \theta_3 P_{B2} = 0 \tag{58}
\]
\[
f_3 + P_{B2} f_3^{P_{B2}} + 2\theta_3 P_{A2} + \theta_2 P_{A1} = 0 \tag{59}
\]

\(^{31}\)Unlike in the quantity-setting version of the model, we need to introduce less than perfect substitutability of firms' outputs in the second market, in order to avoid Bertrand's paradox.
Using equations (57) and (58) we get the following proposition:

**Proposition 4** The equilibrium prices of the two varieties sold by the multiproduct price-setting firm A satisfy:

\[
q_{A1} - q_{A2} = P_{A2}(f_2^{P_a2} - \theta_1) - P_{A1}(f_1^{P_{A1}} - \theta_1) 
\]  

(60)

**Proof:** Solving (52) and (53) for \( f_1^{P_{A1}}, f_2^{P_{A2}} \) and substituting in (57) and (58), we get (60).

From (58) and (59) we get:

**Proposition 5** The equilibrium outputs and prices of the two price-setting firms in the two markets satisfy:

\[
q_{A2} - q_{B2} = P_{B2}(f_2^{P_{B2}} - \theta_1) - P_{A2}(f_2^{P_{A2}} - \theta_1) 
\]  

(61)

**Proof:** Solving (53) and (54) for \( f_1^{P_{A1}}, f_2^{P_{A2}} \) and substituting in (58) and (59), we get (61).

From Proposition 4 we see that for negatively sloped demand functions, the multiproduct firm will tend to set higher price for the variety whose equilibrium output is larger, given that, under the assumptions we make concerning the parameters of the model, the two parentheses in (60) are negative. However, these parentheses, as well as the difference of the multiproduct firm's outputs in the two markets, tend to lower values, the more substitutable varieties 1 and 2 are. The intuition of this result is that the less firm A experiences competition in the \('monopolised\)' market, the more it benefits from its capacity to sell more of the most profitable of its products. In this context, price discrimination is the result of profit-maximising, pro-competitive behaviour of the multiproduct firm, while

\[\text{\footnotesize{In the presence of demand substitutability, we cannot really talk about a monopoly in market 1 because of competition from variety 2.}}\]
dumping may or may not be so, depending on the relative outputs of the multiproduct firm in the foreign and the domestic market and the demand functions' derivatives in equilibrium.

Proposition 5 shows that demand substitutability has a negative effect on the multiproduct firm's market share in the second market, whereas price differences in the second market will tend to reflect market share differences. This result is similar to Proposition 1, according to which demand substitutability (complementarity) means an advantage (disadvantage) to the multiproduct firm in the market in which it faces direct competition. Both Propositions 1 and 5 support the easily testable hypothesis that whether the multiproduct firm sets prices or quantities, its share in the second market -assuming not too different demand and cost conditions across markets and firms selling the same variety- will be higher (lower) the more complementary (substitutable) its two products are. Furthermore, the differences in the two firms' market shares will be affected in the standard way by differences in marginal costs (Prop. 1) and demand elasticities (Prop. 5).

IV Conclusions

Following a similar set-up to the one suggested by Bulow et al. [1985], we have studied the co-existence of demand, production and strategic relations in a multiproduct oligopoly. Studying demand and production relations separately, we have shown that the role of a multiproduct firm is different under demand and under production relations as far as the cross equilibrium effects between the two markets are concerned. When two products are demand substitutes or complements, there are two different effects of the equilibrium with respect to the first product on the equilibrium with respect to the second. The direct effect of a firm's actions with respect to one of its products on the strategies of its rival in the supply of another product is not trivial. In the case of linear demand and cost functions, this effect is of the same size and of opposite direction to the strategic effect of firm A's decisions with respect to the first product on its rival's strategies. The assumption of linear
cost and demand functions may play a significant role in the precision and the simplicity of this result. Similarly, a model in which production and demand relations exist together with strategic relations offers a more realistic scenario\textsuperscript{33}. There is a basic difference in modelling demand instead of production relations, that is, the former transmit the effects of the equilibrium in one market on the equilibrium in another, even in the absence of multiproduct firms. A price-setting version of the model with demand relations has shown that a multiproduct firm's advantage from selling complementary products is expressed in a higher market share of the multiproduct firm as compared to its single-product rival in a certain product's market. This last result is compatible with the result obtained from the quantity-setting version of the model, making the consequent policy implications robust with respect to the assumption of price or quantity-setting behaviour.

Beyond the restrictive assumptions of the linear model, we have produced testable hypothesis that could give some idea on whether the observed situation results from Tetra Pak's competitive behaviour. Unfortunately, the policy maker should engage in an excessively demanding -in terms of exhaustiveness- investigation before producing the evidence of a firm's anti-competitive behaviour.

A possible application of this study in the Tetra Pak case would require estimation of production and demand relation coefficients between the aseptic and the non-aseptic sector. In the absence of production relations and if the two firms involved act in the way that the linear version of the model implies, the Commission might be right to worry about the effect of Tetra Pak's market share in the aseptic sector on its rival's market share in the non aseptic sector. However, it would be necessary to prove first that these are the right assumptions to make in order to model the real world situation.

\textsuperscript{33}In the real world the co-existence of production and demand relations between the two products is the rule rather than the exception. The joint production of two goods that are close demand substitutes would probably lead to some economies of scope. The concept of strategic relations would prove a useful tool to express the overall effect of all other relations that exist between the two products. However, production or demand relations are more basic and result directly from quantifiable supply or demand conditions which are either directly observable or common knowledge to the participants in the industry.
Summarising our conclusions in a non-technical way, we would like to point out that the advantage of participating in more than one market (or producing more than one product) does not necessarily imply losses to those firms which do not have the possibility of multi-market activity. Not only demand but also production relations among the products of a multiproduct firm are responsible for the overall advantage (or disadvantage) from multiproduct activity.\footnote{In a quantity-setting oligopoly the sum of the coefficients of production and demand relations give us the coefficient of the overall effect of multiproduct activity.}

A Appendix: Conditions for Profitability of Multiproduct Activity

In Röller and Tombak [1990] and especially in Kim et al. [1992], firms choose between flexible and dedicated technologies. Flexible technologies can be used for the production of two products while dedicated technologies bind the firm with the production of only one product.

In the linear version of our model, firm $A$ is assumed to have access to multiproduct technologies while firm $B$ is bound to the production of product 2. In an implicit way this means that firm $A$ prefers multiproduct activity to production of only one of the two products. This requires that its total profits from participating in the two markets is higher than what it could earn as the sole producer of product 1 or as a duopolist in the supply of product 2. In the case of the duopoly, firms would compete in the absence of product 1. This requires that the parameters of our model satisfy inequalities $\Pi_A \geq D$ and $\Pi_A \geq Z$ where $Z$ denotes the profits of firm $A$ from producing only product 1 and $D$ is the profit of standard Cournot duopolist supplying product 2. $\Pi_A \geq D$ if:

$$\frac{(b_2t_1 - \lambda t_2)^2}{4(b_1b_2 - \lambda^2)} \geq 0$$  \hspace{1cm} (62)

which holds for all values of the parameters. Therefore, firm $A$ will never prefer to
become a duopolist in the supply of product 2 instead of producing both products. The only exception is when \( \lambda = \frac{b_2 t_1}{t_2} \) in which case firm A is indifferent.

Figure 1 is drawn to illustrate the case in which the expression above holds as an inequality for all values of \( \lambda \) and Figure 2 illustrates a case in which there is a value of \( \lambda \) that makes the expression to hold as an equality\(^{35}\).

The requirement that firm A's profits from multiproduct activity are higher than if it produced only product 1 implies that

\[
\Pi_A \geq \frac{b_1(\lambda t_2 - 2b_2 t_1)^2}{(4b_1 b_2 - \lambda^2)^2}
\]  

(63)

Analytical elaboration of the inequality above requires the solution of a fourth degree equation.

We solve graphically setting the same values of the parameters as in Figure 1 to draw Figure 3 and the same parameters as in Figure 2 to draw Figure 4.

An increase in the difference between the demand intercept and the marginal cost of one of the two commodities broadens the range of values of parameter \( \lambda \) for which firm A will prefer to specialise in the production of this particular commodity.

However, we have shown that there are always some values of \( \lambda \) – even in the case of demand substitutability – for which the set-up of our model will emerge. The divergence of this result from the result of Kim et al. [1992] is due to the fact that they assume that both firms have the possibility to become multiproduct.

B Appendix: Proof of Proposition 2

Consider the case of demand complementarity, that is \( \lambda < 0 \). Firm A's output of product 1 will be higher than in the absence of demand relations between products 1 and 2 if:

\(^{35}\)Figure 1 is constructed assuming that \( a = h = 5, b_1 = 0.8, b_2 = 1, c_1 = 2 \) and \( c_2 = 1 \). Figure 2 differs in that it is drawn assuming that \( h = 10 \).
\[
\frac{b_2(a - c_1) - \lambda(h - c_2)}{2(b_1b_2 - \lambda^2)} > \frac{a - c_1}{2b_1}
\] (64)

for product 1, and

\[
\frac{(2b_1b_2 + \lambda^2)(h - c_2) - 3b_2\lambda(a - c_1)}{6b_2(b_1b_2 - \lambda^2)} > \frac{h - c_2}{3b_2}
\] (65)

for product 2.

The two inequalities hold for \(\lambda^2(a - c_1) > b_1\lambda(h - c_2)\) and \(\lambda^2(h - c_2) > b_2\lambda(a - c_1)\) respectively, which hold for all negative values of \(\lambda\).

We consider now the case of demand substitutability, that is \(\lambda > 0\). Remember that

\[
\lambda < \min\{b_1, b_2\}
\] (66)

is a necessary condition for a strictly stable equilibrium. Firm A’s output of product 1 is higher than in the absence of demand relations between the two products if:

\[
\lambda > b_1 \frac{h - c_2}{a - c_1} = b'_1
\] (67)

Firm A’s output of product 2 is higher than in the absence of demand relations between the two products if:

\[
\lambda > b_2 \frac{a - c_1}{h - c_2} = b'_2
\] (68)

We want to show that there is no value of \(\lambda\) that makes the three last inequalities hold simultaneously.

Let \(a - c_1 = t_1\) and \(h - c_2 = t_2\). For each \(\{i, j\} = \{1, 2\}\), it is true that \(b'_i \leq b_i \implies b'_j \geq b_j\).

In that case, it must hold that \(\lambda > b'_j \geq b_j\), which violates \(\lambda < \min\{b_1, b_2\}\).
Figure 1: Multiproduct rather than duopolist: \( Pi_A \geq D. \) \((h = 5)\)

Figure 2: Multiproduct rather than duopolist: \( \Pi_A \geq D. \) \((h = 10)\)
Figure 3: Multiproduct rather than monopolist: $\Pi_A \geq \mathcal{Z}$. ($h = 5$)
Figure 4: Multiproduct rather than monopolist: $\Pi_A \geq Z$. ($h = 10$)
References


[34] SCHERER, F.M., [1980], Industrial Market Structure and Economic Performance, Rand McNally Chicago, IL.


DOCUMENTOS PUBLICADOS

WP-EC 90-01 "Los Determinantes de la Evolución de la Productividad en España"

WP-EC 90-02 "Mecanización y Sustitución de Factores Productivos en la Agricultura Valenciana"

WP-EC 90-03 "Productivity in the Service Sector"

WP-EC 90-04 "Aplicación de los Modelos de Elección Discreta al Análisis de la Adopción de Innovaciones Tecnológicas. El Caso del Sector Azulejero"

WP-EC 90-05 "Rentabilidad y Eficiencia del Mercado de Acciones Español"

WP-EC 90-06 "La Coordinación de Políticas Fiscales en el Marco de una Unión Económica y Monetaria"

WP-EC 91-01 "Medición de la Segregación Ocupacional en España: 1964-1988"

WP-EC 91-02 "Capital Adequacy in the New Europe"

WP-EC 91-03 "Determinantes de la Renta de los Hogares de la Comunidad Valenciana. Una Aproximación Empírica."

WP-EC 91-04 "Un Modelo para la Determinación de Centros Comerciales en España".

WP-EC 91-05 "Exchange Rate Dynamics. Cointegration and Error Correction Mechanism".

WP-EC 91-06 "Aplicación de una Versión Generalizada del Lema de Shephard con Datos de Panel al Sistema Bancario Español".

WP-EC 91-07 "Necesidades, Dotaciones y Deficit en las Comunidades Autónomas"

WP-EC 91-08 "Un Análisis del Racionamiento de Crédito de Equilibrio"

WP-EC 91-09 "Cooperación entre Gobiernos para la Recaudación de Impuestos Compartidos"

WP-EC 91-10 "El impacto del Cambio Tecnológico en el Sistema Bancario: El Cajero Automático"
WP-EC 91-11 "El Reparto del Fondo de Compensación Interterritorial entre las Comunidades Autónomas"

WP-EC 91-12 "Sobre la Distribución Justa de un Pastel y su Aplicación al Problema de la Financiación de las Comunidades Autónomas"

WP-EC 92-01 "Asignaciones Igualitarias y Eficientes en Presencia de Externalidades"

WP-EC 92-02 "Estructura del Consumo Alimentario y Desarrollo Económico"

WP-EC 92-03 "Preferencias de Gasto Reveladas por las CC.AA."

WP-EC 92-04 "Valoración de Títulos con Riesgo: Hacia un Enfoque Alternativo"

WP-EC 92-05 "Infraestructura y Crecimiento Económico: El Caso de las Comunidades Autónomas"

WP-EC 92-06 "Evolución y Estrategia: Teoría de Juegos con Agentes Limitados y un Contexto Cambiante"

WP-EC 92-07 "La Medición del Bienestar mediante Indicadores de ‘Renta Real’: Caracterización de un Índice de Bienestar Tipo Theil"

WP-EC 92-08 "Corresponsabilización Fiscal de Dos Niveles de Gobierno: Relaciones Principal-Agente"

WP-EC 92-09 "Labour Market and International Migration Flows: The Case of Spain"

WP-EC 92-10 "Un Análisis Microeconómico de la Demanda de Turismo en España"

WP-EC 92-11 "Solución de Pérdidas Proporcionales para el Problema de Negociación Bipersonal"

WP-EC 92-12 "La Volatilidad del Mercado de Acciones Español"

WP-EC 92-13 "Evidencias Empiricas del CAPM en el Mercado Español de Capitales"

WP-EC 92-14 "Economic Integration and Monetary Union in Europe or the Importance of Being Earnest: A Target-Zone Approach"

WP-EC 92-15 "Utilidad Expandida y Algunas Modalidades de Seguro"


"Capital Público y Eficiencia Productiva Regional (1964-1989)"

"Can the Previous Year Unemployment Rate Affect Productivity? A DPD Contrast"

"Comparing Cointegration Regression Estimators: Some Additional Monte Carlo Results"

"Factores Determinantes de la Innovación en las Empresas de la Comunidad Valenciana"

"Competencia Imperfecta y Discriminación de Precios en los Mercados de Exportación. El Caso del Sector de Pavimentos Cerámicos"

"Utilidad Expandida Estado Dependiente: Algunas Aplicaciones"

"El Efecto de las Nuevas Tecnologías de Transacción en la Demanda de Dinero en España"

"Desajustes en los Tipos de Cambio e 'Hysteresis' en los Flujos Comerciales: Las Exportaciones Españolas a EE.UU."

"Stock Prices and Macroeconomic Factors: Evidence from European Countries"

"Margen Precio-Coste Marginal y Economías de Escala en la Industria Española: 1964-1989"

"Temporal Links Between Price Indices of Stock Markets with Overlapping Business Hours"

"Competitive and Predatory Multi-Plant Location Decisions"

"Multiproduct Activity and Competition Policy: The Tetra Pack Case"

"Estudio Empírico de la Solvencia Empresarial en Comunidad Valenciana"

"El Método Generalizado de los Momentos"