ARE BUSINESS CYCLES ASYMMETRIC?
SOME EUROPEAN EVIDENCE

Amado Pérez

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ABSTRACT

Economic thought has often regarded business cycles as asymmetric. This paper examines the existence of asymmetries over the business cycle in three European countries: France, Germany and the United Kingdom. To analyze this issue, industrial production in these countries from 1957 to 1994 is examined, and quarterly contractions and expansions in this variable are compared. The results obtained with both parametric and nonparametric methods allow the existence of asymmetries in these countries to be questioned.

KEY WORDS: Business cycle; symmetry
JEL classification: E32

RESUMEN

El pensamiento económico ha considerado frecuentemente que los ciclos económicos son de naturaleza asimétrica. Este trabajo examina la existencia de asimetrías en los ciclos económicos de tres países europeos: Francia, Alemania y el Reino Unido. Para analizar este tema, se estudia la producción industrial en estos países desde 1957 a 1994 y se comparan las contracciones y expansiones en esta variable. Los resultados obtenidos, tanto con métodos paramétricos como con métodos no paramétricos, cuestionan la existencia de asimetrías en estos países.

PALABRAS CLAVE: Ciclos económicos; simetría
Clasificación JEL: E32
1. INTRODUCTION

The presumption that important economic variables present asymmetric behavior over the business cycles has a long tradition in economic thought. According to this view, contractions are steeper and shorter than expansions. Numerous economists have maintained this conviction, including such distinguished figures as Keynes (1936) and Hicks (1950). Nevertheless, until recent times, this hypothesis had not been studied in depth. Given the importance of this issue in business cycle modeling, several researchers have examined different economic variables throughout the business cycle.

The conclusions reached are far from being unanimous. Neftçi (1984), Rothman (1991), Sichel (1993), McQueen and Thorley (1993) or Ramsey and Rothman (1996) find asymmetric behavior in a number of economic series, but DeLong and Summers (1986), Sichel (1989) or Westlund and Öhlén (1991) do not confirm the existence of asymmetry over the business cycle. Besides the different countries, series and periods examined in all these contributions, the disparity of conclusions may also be due to the variety of methodologies used. Neftçi (1984) compared the transition probabilities in the framework of finite state Markov processes. A similar approach was followed by Falk (1986), Sichel (1989) and McQueen and Thorley (1993). DeLong and Summers (1986), Sichel (1993) and Hassler et al. (1994) base their results on the sample skewness, the third central moment divided by the cube of standard deviation. In addition to these two procedures, other researchers have addressed this topic from the perspective of nonlinear time series (Tiao and Tsay, 1991) or with semiparametric methods (Hussey, 1992 and Brunner, 1992).

But none of these methods is free from problems and difficulties, and the limitations are not always explicit. The method proposed by Neftçi (1984) implies the dichotomization of the series under analysis and the consequent loss of information (see Westlund and Öhlén, 1991). Moreover, DeLong and Summers (1986) and Sichel (1989) have pointed out the low power of this procedure. On the other hand, the use of sample skewness embodies some potential problems. The distribution of this statistic is very sensitive to the underlying distribution, a common feature of high-order statistics, and moderate departures from normality may significantly affect its distribution. Nevertheless, the main problem may be that most techniques, if not all, are intended to cope with specific kinds of asymmetry; they can hardly detect other asymmetries, although, evidently, there are innumerable possibilities. With regard to this point, Sichel (1993) has distinguished two types of asymmetry: steepness and deepness, and proposes
two tests that use the sample skewness to detect them. Steepness occurs when contractions are steeper than expansions. Depthness occurs when troughs are deeper than peaks are tall. In a similar way, Ramsey and Rothman (1996) distinguish between longitudinal and transversal asymmetry. By longitudinal asymmetry they mean asymmetries in the direction of the movement of the business cycle. By transversal asymmetry they mean asymmetries orthogonal to the direction of the movement of the business cycle. Steepness would be a longitudinal asymmetry while depthness would be an example of transversal asymmetry.

In an attempt to generalize and formalize these classifications, we will distinguish between unconditional and conditional asymmetries. A (stationary) series presents unconditional symmetry about μ if its unconditional density function is symmetric about μ; that is, if for any a:

\[ f(μ + a) = f(μ - a) \]  

where \( f \) is the unconditional density function. If the series is symmetric about μ, then μ is the mean of the distribution and coincides with the median. Otherwise, if (1) is not true for some \( a \), then the series presents unconditional asymmetry.

On the other hand, a (stationary) series presents conditional symmetry about μ if for any \( n > 0 \) and for any \( a_i, i = 0, 1, 2, ..., n \),

\[ f(X_i = μ + a_i | X_{i-1} = μ + a_{i-1}, X_{i-2} = μ + a_{i-2}, ..., X_{i-n} = μ + a_{i-n}) = f(X_i = μ - a_i | X_{i-1} = μ - a_{i-1}, X_{i-2} = μ - a_{i-2}, ..., X_{i-n} = μ - a_{i-n}) \]

where \( f \) now denotes the conditional density function. Otherwise, if (2) is not true for some \( a_i \), then the series presents conditional asymmetry. Intuitively, a series presents conditional asymmetry when the change of the preceding \( n \) realizations for their mirror images implies the change of the distribution of current realization for its mirror image. The concept of unconditional asymmetry formalizes the concept of transversal asymmetry, proposed by Ramsey and Rothman (1996), and includes the type of asymmetry of steepness, proposed by Sichel (1993). Analogously, the concept of conditional asymmetry is a formalization of the concept of longitudinal asymmetry, proposed by Ramsey and Rothman (1996), and includes the type of asymmetry of steepness proposed by Sichel (1993).

An economic variable may present one of these two types of asymmetry, both, or neither. In a special case, both types coincide; when \( n = 0 \), (2) becomes (1). If a series is independent and identically distributed, then its conditional distribution, given its past, is equal to its unconditional distribution. In this particular case, as will be seen later, it would suffice to analyze the unconditional distribution to conclude its symmetry or asymmetry.

A final important point in the analysis of symmetry is the extraction of the cyclical component. The problem of the decomposition of a non-seasonal economic variable into trend and cycle is deeply related to the problem of non-stationarity. The solution to these problems involves the detrending of the non-stationary series or the use of different decomposition methods, and these tasks have been carried out in different ways although not always with enough care. Presumably, the diversity of the aforementioned results is also due to the diversity of methods used; it would be preferable for the conclusions obtained not to rely on an arbitrary (or, simply, wrong) decomposition method.

The objective of this paper is to investigate the symmetry of European business cycles. To carry out this study, section 2 presents the data used, industrial production in France, Germany and the United Kingdom. These series exhibit peculiarities that are taken into account when facing the decomposition problem. In section 3, parametric and non-parametric methods are used to study the issue of symmetry, and the results are discussed. Finally, section 4 summarizes the main results and conclusions.

2. DATA AND DETRENDING

When studying the issue of asymmetries over the business cycle, the question arises of which variable to consider. Industrial production is a clear pro-cyclical variable and, while it may be the most frequently analyzed variable, there is no agreement on its symmetric or asymmetric behavior. Furthermore, it is plausible that asymmetries may be better observed in industrial

\[ * \text{In the U.S., industrial production is a component of the Bureau of Economic Analysis coincident index, and shows the highest coherence and the highest correlation with the index of coincident indicators (ICI) compiled by Stock and Watson (1990). They examined the comovements of 163 monthly series with the ICI in the period 1959-1989. The correlation between (filtered) industrial production and the (filtered) ICI is 0.59, and the coherence between the rates of growth in industrial production and the rates of growth in the ICI moves from 0.89 to 0.99 over different frequency bands.} \]
production than in variables like GNP, a very broad variable with possible counter-cyclical components. For these reasons, quarterly data on industrial production from France, Germany and the United Kingdom have been used. For the purpose of comparison, some results from Japan and the U.S. are also reported. All these quarterly observations were collected from International Financial Statistics, International Monetary Fund. They cover the period 1957.1-1994.4, and are seasonally adjusted. In these series, only one anomalous value was observed in French industrial production in the second quarter of 1968, and can be attributed to the social problems which occurred in May of that year.

In order to isolate the cyclical component as well as to induce stationarity in these variables, they must be detrended. The selection of a detrending procedure is a complex task because there are several possible ways of carrying out this process, and each of them is pertinent in certain circumstances and has its own implications. In the last years, abundant literature has investigated this point, but without reaching a complete consensus. At the same time, there is also considerable empirical evidence on the consequences of the detrending method on the (time-series) properties of the detrended series. Specifically, the detrending method may induce spurious periodicities, as Nelson and Kang (1981) showed. To assure that this is not the case, the three most habitual methods (the Hodrick and Prescott filter, the Beveridge and Nelson decomposition and first differencing) will be examined carefully.

A first detrending method is the procedure proposed by Hodrick and Prescott (1980), HP. In this method the original series, seasonally adjusted, is decomposed into two components: trend and cycle, the latter component being stationary. This method has been widely used in the last years, but recently several authors have warned of the consequences of applying this filter in certain circumstances. King and Rebelo (1993) note that the dynamics of an HP-filtered series may be entirely different from the dynamics of the differenced series. Moreover, Cogley and Nason (1995) show that applying the HP filter to difference-stationary time series is equivalent to differencing the data and smoothing the differenced data with an asymmetric moving average filter that amplifies the fluctuations in business cycle frequencies and damps the others. In particular, if the original series follows a random walk, Jaeger (1994) and Cogley and Nason (1995) show that the HP-filtered cyclical component will present autocorrelations at lag 1 as high as 0.72, which will decrease to a minimum of -0.28 at lag 11.

Another detrending technique is the method proposed by Beveridge and Nelson (1981), BN. In this method, the original non-stationary series, seasonally adjusted, is decomposed into a permanent (random walk with drift) and a transitory or cyclical component (stationary series

with zero mean). This decomposition has been simplified computationally by Cuddington and Winters (1987) and Miller (1988). It must be stressed here that when the original series to be decomposed follows a random walk, the BN decomposition yields only the permanent component, as can be expected. The cyclical component is absent, which means that all movements are permanent.

Before choosing the proper detrending method, the series were closely examined. (Augmented) Dickey-Fuller and Phillips-Perron unit root tests, whose results are available on request, confirmed that the logarithms of these series are non-stationary. Therefore, first differences were taken to induce stationarity, Table 1 shows some basic statistics of the growth rates of industrial production. In order to obtain the dynamic characteristics of these series, their autocorrelation functions were estimated and the results are shown in Table 2. Surprisingly, the relative changes in industrial production in the European countries display very weak or non-existent temporal dependence. This is clearly the case for France and the U.K., where the changes in industrial production behave like white noise. Germany is the only country where the changes in industrial production display a certain temporal dependence, but this is rather weak and is basically due to the second and third order correlations. It is interesting to notice that, with reference to these points, the European countries behave in a similar fashion, but are in sharp contrast with Japan and the U.S., where the growth rates of industrial production are strongly autocorrelated.

<table>
<thead>
<tr>
<th>Table 1: Basic Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>France</strong></td>
</tr>
<tr>
<td>Observations</td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Standard Deviation</td>
</tr>
</tbody>
</table>

Basic statistics on the quarterly growth rates in industrial production from the second quarter of 1957 to the fourth quarter of 1994. The anomalous observations corresponding to the second and third quarter of 1968 in France (-15.0% and +18.4%, respectively) have been excluded.

When computing the sample autocorrelations for France, the anomalous observations corresponding to the second and third quarters of 1968 (-15.0% and +18.4%, respectively) have been made equal to the mean of the other changes (+0.71%). If the true values had been considered, the sample autocorrelation corresponding to the first lag would have been negative and significant; an even harder issue to explain.
Table 2: Autocorrelation in Changes in Industrial Production

<table>
<thead>
<tr>
<th>Lag</th>
<th>France</th>
<th>Germany</th>
<th>U.K.</th>
<th>Japan</th>
<th>U.S.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.077</td>
<td>0.031</td>
<td>0.045</td>
<td>0.661</td>
<td>0.446</td>
</tr>
<tr>
<td>2</td>
<td>0.134</td>
<td>0.207</td>
<td>0.042</td>
<td>0.497</td>
<td>0.065</td>
</tr>
<tr>
<td>3</td>
<td>0.107</td>
<td>0.172</td>
<td>0.068</td>
<td>0.318</td>
<td>0.031</td>
</tr>
<tr>
<td>4</td>
<td>-0.108</td>
<td>0.062</td>
<td>-0.141</td>
<td>0.120</td>
<td>-0.095</td>
</tr>
<tr>
<td>5</td>
<td>-0.039</td>
<td>-0.074</td>
<td>-0.025</td>
<td>0.029</td>
<td>-0.124</td>
</tr>
<tr>
<td>6</td>
<td>-0.074</td>
<td>0.110</td>
<td>0.035</td>
<td>0.018</td>
<td>-0.060</td>
</tr>
<tr>
<td>7</td>
<td>0.066</td>
<td>-0.077</td>
<td>-0.178</td>
<td>-0.018</td>
<td>-0.160</td>
</tr>
<tr>
<td>8</td>
<td>-0.050</td>
<td>-0.082</td>
<td>0.050</td>
<td>0.009</td>
<td>-0.258</td>
</tr>
</tbody>
</table>

Sample autocorrelations and Ljung-Box statistics corresponding to the quarterly growth rates in industrial production from the second quarter of 1957 to the fourth quarter of 1994. The anomalous observations corresponding to the second and third quarter of 1968 in France (+15.0% and +18.4%, respectively) have been replaced by the means of the other changes (+0.716%). The (Bartlett) standard error of the sample autocorrelations is equal to 0.081. The values in parentheses are P-values for the corresponding Ljung-Box statistics.

To confirm this fact from a different point of view the Schwarz identification criterion was used. ARMA(p,q) models, with 0≤p≤4 and 0≤q≤4, were estimated for the relative changes in industrial production in each country, and the Schwarz criterion was then computed for each model. The selected models were ARMA(0,0), i.e. white noise, for France and the United Kingdom, and an ARMA(3,0), with the first two autoregressive parameters equal to zero, for Germany. These results, in conjunction with the sample autocorrelations displayed in Table 2, lead to the conclusion that the logarithm of industrial production follows a random walk in France and the U.K. In Germany, the dynamics is rather weak, and is well represented by an ARIMA(3,1,0).

In spite of all the results mentioned above, when these series are HP-filtered, the cyclical component is very strong, with high autocorrelations at low lags that decrease to a minimum at

about three years (12 lags). This happened when the smoothing parameter was equal to 1600 (a usual choice with quarterly data) or when it was set at other plausible values. However, nothing at all in the original series or in their first differences allows the existence of this component to be suspected. Doubtless, we face the problem of spurious cycles generated by this filter, exactly in the way described by Cogley and Nason (1995) and Jaeger (1994). Therefore, we reject this decomposition, and move on to consider other methods.

As previously stated, when the BN filter is applied to random walks, the only component obtained is the permanent one, which coincides with the original series. The cyclical component cannot be extracted, as there is no evidence of its existence. This is clearly the case for France and the United Kingdom. Both series are identified as random walks, and, therefore, there is no cyclical component to deal with. For Germany, the situation is not so clear. The Schwarz criterion indicates an ARIMA(3,1,0) for the logarithm of German industrial production, with the first two autoregressive parameters equal to zero. To perform the BN decomposition of this series, the computational procedure proposed by Miller (1988) was implemented. It is interesting to remember that when the ARIMA model involves only an AR, but no MA polynomial, this procedure yields results identical to those of the original BN computational method. The results of this decomposition are also surprising. Although the model estimated for German industrial production is not too different from a random walk, the dynamics of the cyclical component obtained with the BN filter is very strong, with high autocorrelations at low lags. This feature motivates the suspicion that the BN filter also induces spurious dynamics in the cyclical component.

Given these facts, we may cast doubt on the appropriateness of the HP and BN decompositions. The purpose of the consideration of these filters was twofold: the detrending of the series examined and the extraction of the cyclical component. The first goal can be achieved by taking first differences. With respect to the second, these first differences will be regarded as the cyclical component, with the peculiarity that this component is not autocorrelated for France and the United Kingdom (i.e. they are white noise), and it is slightly autocorrelated for Germany.

The problem of symmetry in the business cycle will be tackled by examining these first differences in the logarithm of industrial production, that is, by examining the growth rates of

3 The model selected is X_t=0.007+0.173X_{t-1}+\eta_t, where X_t denotes the rate of growth in German industrial production in quarter t, and \eta_t is a white noise process. The t-statistics corresponding to the autoregressive parameter is 2.330 and the coefficient of determination is R^2=0.036. In spite of this low coefficient of determination and the fact that the t-statistic is significant at the 5% level but not at the 1% level, the sample autocorrelations of the cyclical component are as high as 0.77 and 0.57 for the two first lags.
industrial production. Each particular quarterly rate may be below or above the mean of the rates in the whole period. The changes that are below the mean will be called 'contractions' and the changes above the mean will be called 'expansions'. The issue to address is the comparison of both types of movements. If they are symmetric, they will be so about the mean; that is, contractions and expansions will be reciprocal mirror images, with the mean being the axis of symmetry. For this reason, deviations from the mean have been taken, thus obtaining the set of contractions,

\[ C : \{ X_t - \bar{X} \leq 0 \}, \]  

and the set of expansions,

\[ E : \{ X_t - \bar{X} > 0 \}, \]

where \( X_t \) is the growth rate in industrial production in quarter \( t \), and \( \bar{X} \) is the mean over the whole period. It must be observed that both types of movements are taken in absolute values. Thus, if in a certain quarter the growth rate in industrial production is higher than the mean rate, this quarter will present an expansion, and its measure will be given by the difference between these quantities. Conversely, if in another quarter the growth rate of industrial production is below the mean rate, this quarter will present a contraction, and its measure will be given by the difference, in absolute value, between these quantities.

More generally, only movements of a certain strength could be considered when defining the sets of contractions and expansions in the following way:

\[ C(k) : \{ X_t - \bar{X} - ks \leq 0 \}, \]

\[ E(k) : \{ X_t - \bar{X} > ks \}, \]

where \( k > 0 \), and \( s \) is the sample standard deviation of the growth rates. When \( k = 0 \), (5) and (6) become (3) and (4), respectively. As \( k \) increases, only more extreme movements are taken into account. In what follows, two values will be considered: \( k = 0 \) and \( k = 1 \). Higher values of \( k \) would imply an excessive reduction in the number of contractions and expansions to be considered. All these movements are shown in Figure 1 for the three European countries in question. The marks below zero indicate contractions, \( C(0) \), and those over zero indicate expansions, \( E(0) \). As two horizontal lines are drawn for each country at \( +s \) and \(-s\), strong contractions, \( C(1) \), are those values under the horizontal lines, while strong expansions, \( E(1) \), are those above the lines.

3. EVIDENCE ON SYMMETRY USING DISTRIBUTION-FREE TESTS

Given that the growth rates in industrial production in these three European countries are independent or almost so, the research on the issue of symmetry must focus on unconditional asymmetries. This type of asymmetries has usually been tested with the use of the coefficient of skewness. Nevertheless, this approach presents two serious problems. On the one hand, the distribution of the test statistic, the third central moment divided by the cube of standard deviation, is very sensitive to the underlying distribution, a common feature of statistics that involve high order moments. Moderate departures from normality or the existence of outliers may alter the sizes of the tests carried out drastically. On the other hand, if this test rejects the null of symmetry, no clear indication is provided of the characteristics of the asymmetry.

To avoid these problems, the distribution of contractions will be compared with the distribution of expansions. This comparison will be carried out in two ways: firstly, with conventional \( t \)-tests for the equality of means and \( F \)-tests for the equality of variances; secondly, with nonparametric or distribution-free methods. The distribution-free methods may be especially suitable in these cases because the distribution of the test statistic does not depend on the specific distribution function of the population; these methods only require minimal assumptions about the underlying distribution. If, in addition, these methods provide an indication of the type of the hypothetical asymmetry, they could be superior to the conventional method that rely on the coefficient of skewness.

Therefore, in order to check the symmetry of contractions and expansions, these movements will be considered symmetric when: 1) the probability of occurrence of a contraction

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4 Two types of business cycles have been distinguished: the classical cycle, measured by the NBER in the U.S., and the growth cycle, measured by the Japanese and several European governments. Although both types are closely related, here, as we consider fluctuations around the trend we are closer to growth cycles. (See, for example, Moore and Zarnowitz, 1991 and Cooley and Prescott, 1955).

5 Only the sign of the skewness coefficient suggests the existence of extreme values with this sign.
is equal to the probability of occurrence of an expansion, and 2) the statistical distribution of contractions in absolute value is equal to the statistical distribution of expansions.

If the probability of occurrence of a contraction is equal to the probability of occurrence of an expansion, then both the number of contractions and expansions will follow a binomial distribution with parameters equal to the number of observations and 1/2. Using this distribution, Table 3 shows that the equal probability of these movements cannot be rejected for any country with fairly high $P$-values. This occurs for $k=0$ as well as for $k=1$; contractions and expansions happen with the same probability, in the same way that strong contractions and strong expansions are equally likely.

Table 3: Number of Contractions and Expansions in Industrial Production

<table>
<thead>
<tr>
<th></th>
<th>France ($k=0$)</th>
<th>France ($k=1$)</th>
<th>Germany ($k=0$)</th>
<th>Germany ($k=1$)</th>
<th>U.K. ($k=0$)</th>
<th>U.K. ($k=1$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contractions</td>
<td>77</td>
<td>22</td>
<td>82</td>
<td>22</td>
<td>76</td>
<td>16</td>
</tr>
<tr>
<td>Expansions</td>
<td>72</td>
<td>21</td>
<td>69</td>
<td>19</td>
<td>75</td>
<td>16</td>
</tr>
<tr>
<td>$P$-value</td>
<td>0.682</td>
<td>0.879</td>
<td>0.290</td>
<td>0.639</td>
<td>0.935</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Number of contractions and expansions in industrial production from the second quarter of 1957 to the fourth quarter of 1994. The $P$-values refer to the hypothesis that both changes are of equal probability.

To test for the equality of distributions, the mean and the variance of contractions will be compared with the mean and the variance of expansions through the usual $t$- and $F$-tests. Table 4 shows the results of these tests. The equality of means cannot be rejected for any country, neither for $k=0$ nor for $k=1$. The equality of variances cannot be rejected for France or the United Kingdom. However, the results of the $F$-tests for Germany suggest a different dispersion between contractions and expansions, both for $k=0$ and $k=1$, with a higher variability in expansions than in contractions. This is the only sign of asymmetry that can be observed with these tests.

Although $t$-tests for the equality of means are rather robust to distributional assumptions, $F$-tests are rather sensitive to these assumptions (see, for example, Stuart and Ord, 1987). Therefore, it would be desirable to corroborate all these results, especially the different dispersion observed for Germany, with distribution-free methods. Three distribution-free methods will be used: the Kolmogorov-Smirnov two sample test, the Wilcoxon rank-sum test and the Siegel-Tukey test. These are two-sample tests which will allow the comparison of the distributions of contractions and expansions. In all of them the null hypothesis establishes the equality of the populations underlying the two samples. But, while the Kolmogorov-Smirnov test is sensitive to any difference in the distribution of the two samples, the Wilcoxon rank-sum test is especially appropriate for detecting differences in location, and the Siegel-Tukey test is especially appropriate for detecting differences in dispersion (see Gibbons and Chakraborti, 1992).

In the Kolmogorov-Smirnov two-sample test, the test statistic, $KS$, is obtained by computing the maximum absolute difference between the empirical distributions of both sets of movements,

$$KS = \max_{0 < x < 1} |F_C(x) - F_E(x)|,$$

where $F_C$ and $F_E$ are the empirical distribution functions of contractions and expansions, respectively. The critical values of the asymptotic distribution of $KS$ under the hypothesis of equal distributions are tabulated in Gibbons and Chakraborti (1992).
In the Wilcoxon rank-sum test, the absolute values of contractions and expansions are combined. The test statistic, $W$, is given by the sum of the ranks of the absolute values of the contractions in the ordered combined sample,

$$W = \sum_{i=1}^{r} r(\bar{X} - X_i), \quad [8]$$

where

$$I_i = \begin{cases} 1 & \text{if } X_i < \bar{X} \\ 0 & \text{if } X_i \geq \bar{X} \end{cases} \quad [9]$$

$T$ is the number of observations and $r(\cdot)$ is the rank operator. Under the null hypothesis of equal distributions, the asymptotic distribution of $W$ is given by

$$W \to N \left( \frac{T_1(T+1)}{2} \cdot \frac{T_2(T+1)}{12} \right), \quad [10]$$

where $T_1$ is the number of contractions (first sample), $T_2$ is the number of expansions (second sample) and $T_1 + T_2 = T$.

In the Siegel-Tukey test the absolute values of contractions and expansions are also combined and ordered. The test statistic is

$$ST = \sum_{i=1}^{r} I_i w_i, \quad [11]$$

where $I_i$ is defined as in (9); that is, $I_i$ has value 1 if the place $i$ in the ordered combined sample is occupied by an observation coming from the first sample (contractions) and 0 otherwise. For $T$ even, the weights are

$$2t \quad \text{if } t \leq \frac{T}{2}, \quad t \text{ even}$$

$$2t-1 \quad \text{if } t > \frac{T}{2}, \quad t \text{ odd}$$

$$w_i = \begin{cases} 2(T-t)+2 & \text{if } t \leq \frac{T}{2}, \quad t \text{ even} \\ 2(T-t)+1 & \text{if } t > \frac{T}{2}, \quad t \text{ odd}. \end{cases}$$

If $T$ is odd, then the middle observation is dropped, and these weights are applied to the resulting number of observations. Thus, the lower weights are assigned to the extremes and the higher weights to the middle of the ordered combined sample. Under the null hypothesis of equal distributions, the asymptotic distribution of $ST$ is the same as that of $W$,

$$ST \to N \left( \frac{T_1(T+1)}{2} \cdot \frac{T_2(T+1)}{12} \right). \quad [13]$$

Table 5 shows the results of these tests for the three European countries and for $k=0$ and $k=1$. The null hypothesis of equal distributions can never be rejected. No test detects differences between contractions and expansions; beyond their signs, these movements are similar. More specifically, it does not seem that they differ in intensity or in variability. These results are in accordance with the conventional tests shown in Table 4, with the only exception of the different dispersion for Germany previously mentioned. The non-parametric tests do not detect this fact. In particular, the Siegel-Tukey tests, especially sensitive to differences in dispersion, do not allow the rejection of the null of equal distributions. A detailed examination of contractions and expansions in German industrial production revealed that the results of the $F$-tests for Germany were very influenced by three strong expansions occurring in the late fifties (see Figure 1). These three strong expansions determined not only the results of the $F$-tests for $k=1$, with 41 observations, but also the results for $k=0$, with 151 observations. In fact, when the sample period covers from 1960:1 to 1994:4 instead of 1957:2 to 1994:4, the $F$-tests can no longer reject the null hypothesis of equal variances, and, what's more, the sample variance of contractions becomes higher than the sample variance of expansions. This feature leads to the conclusion that, for Germany, the results of the $F$-tests were due to a few anomalous values, but were not characteristic of the whole sample at all. Conversely, the distribution-free tests are not unduly influenced by extreme values and yield different results.
Figure 1

Expansions and contractions in industrial production

Table 5: Distribution-free Tests

<table>
<thead>
<tr>
<th></th>
<th>France ((k=0))</th>
<th>France ((k=1))</th>
<th>Germany ((k=0))</th>
<th>Germany ((k=1))</th>
<th>U.K. ((k=0))</th>
<th>U.K. ((k=1))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(K_S)</td>
<td>0.176</td>
<td>0.251</td>
<td>0.152</td>
<td>0.172</td>
<td>0.100</td>
<td>0.187</td>
</tr>
<tr>
<td>(P)-value</td>
<td>0.198</td>
<td>0.507</td>
<td>0.351</td>
<td>0.923</td>
<td>0.840</td>
<td>0.941</td>
</tr>
<tr>
<td>(W)</td>
<td>5.678</td>
<td>4.60</td>
<td>5.964</td>
<td>4.45</td>
<td>5.761</td>
<td>2.60</td>
</tr>
<tr>
<td>(Z)-value</td>
<td>-0.368</td>
<td>-0.583</td>
<td>-1.003</td>
<td>-0.444</td>
<td>-0.056</td>
<td>-0.151</td>
</tr>
<tr>
<td>(P)-value</td>
<td>0.713</td>
<td>0.560</td>
<td>0.317</td>
<td>0.657</td>
<td>0.955</td>
<td>0.880</td>
</tr>
<tr>
<td>(ST)</td>
<td>5.743</td>
<td>4.97</td>
<td>6.053</td>
<td>4.38</td>
<td>5.752</td>
<td>2.20</td>
</tr>
<tr>
<td>(Z)-value</td>
<td>-0.122</td>
<td>0.316</td>
<td>-0.669</td>
<td>-0.627</td>
<td>-0.089</td>
<td>-1.658</td>
</tr>
<tr>
<td>(P)-value</td>
<td>0.903</td>
<td>0.752</td>
<td>0.504</td>
<td>0.530</td>
<td>0.929</td>
<td>0.097</td>
</tr>
</tbody>
</table>

\(K_S\) is the Kolmogorov-Smirnov two sample test statistic. \(W\) is the Wilcoxon rank-sum test statistic. \(ST\) is the Siegel-Tukey tests statistic. The \(Z\)-values are the standardized statistics. In all cases, the first sample is formed by quarterly contractions in industrial production, and the second sample is formed by quarterly expansions. The sample period is from 1957:2 to 1994:4.

An additional interesting fact lies in the \(Z\)-values of the tests carried out. Although the means are not significantly different, all the \(t\)-statistics are negative; and, although the variances are not significantly different, all the \(F\)-statistics are below one. Similar results are obtained with the distribution-free tests. Although no significant differences are found, all the \(W\)- and \(ST\)-statistics, except the \(ST\)-statistic corresponding to France and \(k=1\), are below their expected values, as reflected by their negative \(Z\)-values. These results are practically common to the three countries, and indicate a slightly (not significant at the usual statistical levels) higher intensity and variability of expansions over contractions.

As the growth rates of industrial production in Germany are slightly autocorrelated, one could question the results obtained for this country, arguing that the tests carried out presuppose random samples, and that this weak autocorrelation could affect the results. In order to analyze this possibility, 10,000 replications with a sample size equal to 150 were generated of the following time series: a white noise, an \(AR(3)\) process equal to the model estimated for Germany and several \(AR(1)\) processes with different parameters. Two samples were then built with an equal number of observations, \(T_1=T_2=75\). The first sample was formed by the negative deviations from the mean that were taken in absolute values, and the second sample by the positive deviations. Wilcoxon rank-sum and Siegel-Tukey tests were run with these series to compare.
both samples. Table 6 shows the 0.950, 0.975 and 0.995 quantiles of the sample distribution of the test statistics in these simulations. The quantiles of the AR(3) process are practically equal to the values of the white noise. This is not the case for the AR(1) processes, whose quantiles depart from those of the white noise as the autoregressive parameter increases. Similar results were obtained when $T_1$ and $T_2$ were set equal to the different numbers of contractions and expansions reflected in Table 3. The conclusion is clear: the dependence of the changes in German industrial production is too weak to affect the results obtained with the distribution-free tests.

Finally, it must be repeated that the approach followed in this article has been designed specifically to study the issue of symmetry in industrial production in France, Germany and the United Kingdom, and such a tailor-made approach presents clear advantages over other procedures. Evidently, it may require certain modifications, and may even be inappropriate in other circumstances.

<table>
<thead>
<tr>
<th>Table 6: Sample Distribution of $W$ and $ST$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>$X = \alpha$</td>
</tr>
<tr>
<td>$X = 0.173Y_{t-1} + \alpha$</td>
</tr>
<tr>
<td>$X = 0.5X_{t-1} + \alpha$</td>
</tr>
<tr>
<td>$X = 0.7X_{t-1} + \alpha$</td>
</tr>
<tr>
<td>$X = 0.9X_{t-1} + \alpha$</td>
</tr>
</tbody>
</table>

The table shows the quantiles of the sample distribution of Wilcoxon rank-sum (W) and Siegel-Tukey (ST) test statistics under several processes: a white noise, an AR(3) and three AR(1) with different parameters. 10,000 replications with a sample size equal to 150 were generated, and two samples were then built with an equal number of observations, $T_1 = T_2 = 75$. In all cases $\alpha$ is i.i.d. $N(0, 1)$.

4. CONCLUSIONS

While traditional economic thought has regarded business cycles as clearly asymmetric, recent empirical research on this point does not yield unanimous conclusions. The disparity of results is due, at least partly, to the different methodologies used. Each of these different methods is suitable to detect specific types of asymmetry in certain circumstances, but may be unable to detect other types in distinct circumstances. Therefore, when studying this topic, it is important to distinguish between two broad types of asymmetries: asymmetries in the unconditional density function and asymmetries in the conditional (given its past) density function.

In order to analyze the question of symmetry over the business cycle, quarterly industrial production has been examined in France, Germany and the United Kingdom from 1957 to 1994. The logarithms of these series behave as random walks. In other words, the growth rates are white noise. This is clearly the case for France and the United Kingdom, and almost so for Germany. In these circumstances, both types of asymmetries coincide, and the analysis of the unconditional density function covers all possible asymmetries. Contractions and expansions in industrial production have been compared, and the comparisons are made for two different degrees of magnitude in these movements. These comparisons attempt to establish whether both kinds of movements are equally likely and qualitatively similar. With regard to the first point, it is found that contractions and expansions happen with the same probability in these European countries. To test for the equality of the distributions of contractions and expansions, conventional $t$- and $F$-tests, as well as distribution-free tests, have been used. None of these tests found any evidence of asymmetry. The only exception is that the $F$-tests indicate a different variance in German contractions and expansions, both for $k=0$ and $k=1$. However, this result is not ratified by distribution-free tests, as it is due to a few anomalous values at the very beginning of the sample.

The main conclusion of this article is that contractions and expansions do not seem to be asymmetric; each of these two kinds of movements is indistinguishable from the mirror image of the other. Nevertheless, it is interesting to note that expansions almost always present more intensity and variability than contractions. However, in spite of being a common feature of the three countries, the magnitude of these differences is not important enough to be statistically significant.
REFERENCES


Keynes, J.M. (1936): The general theory of employment, interest and money, (Macmillan, London)


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