OIL PRICE SHOCK: A NONLINEAR APPROACH

Rebeca Jiménez-Rodríguez

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Correspondence: University of Alicante. Departamento de Fundamentos del Análisis Económico. 03080 Alicante (Spain). Tel.: +34 590 36 14 Ext.: 2629 / Fax: +34 96 590 38 98 / E-mail: rebecaj@merlin.fae.ua.es.

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ABSTRACT

Nowadays, the importance of crude oil goes beyond simple economic aspects and affects social life in general. As such, it is imperative that we should know what the relationship between GDP growth and oil price changes is like. This paper presents evidence of a nonlinear relationship between the two things. We also argue that this non-linearity is not merely due to the use of data from the mid-1980s onwards, as most authors, so far, seem to believe. In fact, we find the existence of non-linearity with the use of data earlier than 1984, and even before 1977. Furthermore, we question that the nonlinear transformations of oil prices proposed in the Literature are able to reflect such non-linearity. We therefore use a non-linear function that relates GDP growth to oil prices, and estimate this function by means of kernel methods, avoiding any assumptions about its form. This kernel estimation improves on the linear estimation, and also improves on both Hamilton’s (2001b) estimation and those of the nonlinear transformations.

Keywords: Macroeconomic fluctuations; Oil price shock.

RESUMEN

Hoy día la importancia del petróleo sobrepasa los aspectos meramente económicos, afectando de manera generalizada a nuestra vida social. Por ello, es muy importante saber cuál es la relación existente entre el crecimiento del PIB y los cambios en el precio del petróleo. Este artículo presenta evidencia de la existencia de una relación no-lineal entre ambos. Esta no-linealidad se debe no solamente al uso de datos desde mitad de los años ochenta, como la mayoría de los autores parecen creer. De hecho, se puede encontrar la existencia de no-linealidad con el uso de datos anteriores a 1984, e incluso anteriores a 1977. Este artículo adicionalmente cuestiona que las transformaciones no-lineales propuestas en la literatura sean capaces de capturar dicha no-linealidad. Por todo ello, se utiliza una función no-lineal que relaciona el crecimiento del PIB con el precio del petróleo, estimándola a través de métodos “kernel”, evitando así cualquier supuesto sobre su forma. Esta estimación “kernel” mejora la estimación lineal, así como aquellas derivadas de las transformaciones no-lineales y aquella propuesta por Hamilton (2001b).

Palabras clave: Fluctuaciones macroeconómicas, shock del precio del petróleo.
1 Introduction

From the middle of twentieth century onwards, crude oil has become one of the main indicators of economic activity worldwide, due to its outstanding importance in the supply of the world’s energy demands.

Nowadays, the importance of crude oil as the main source of energy has waned somewhat, due to the appearance of alternative forms of energy (such as wind, water, and solar power). Notwithstanding this, the importance of oil exceeds economic aspects and affects social life in general. One of the issues that the public has been particularly concerned about is oil-price fluctuations, so that these fluctuations have become one of the current affairs published on the front pages by the vast majority of the world’s newspapers, mainly from the Yom Kippur War (October 5, 1973) on. Thus, the prevailing view among economists is that there is a strong relationship between the growth rate of a country and oil-price changes\(^1\). Precisely what form this relationship takes, and how it might be modified, and other such questions are issues of outstanding value.

As such, the relationship between the macroeconomic variables and the oil-price shocks has been extensively analyzed in the Literature, but especially so over the last twenty-five years. Hamilton (1983), Burbidge and Harrison (1984), Gisser and Goodwin (1986), Mork (1989), Hamilton (1996), Bernanke, Getler, and Watson (1997), Hamilton (2001b), Hamilton and Herrera (2000), and several others have concluded that there is a negative correlation between increases in oil prices and the subsequent economic downturns in the United States. The relation seems weaker, however, when data from 1985 is included\(^2\). Notwithstanding this, the role of the breakpoint 1985-86 has only been taken into account by very few researches, most of whom argue that the instability observed in this relationship may be due to a mis-specification of the functional form used. The linear specification\(^3\) might well mis-represent the relationship between GDP growth and oil prices.


\(^2\)Note that there was a decline in oil price of more than 50% in 1986:I.

\(^3\)We should highlight that all of these authors except Hamilton (2001b) consider the GDP-Oil price relationship in a linear multivariate context. In particular, they consider VAR specifications.
The mis-representation of the linear specification has led to different attempts to redefine the measure of the oil-price changes. These measures are based on non-linear transformations of the oil prices. They try to re-establish the correlation between GDP growth and these new measures. In fact, they are, actually, attempts to restore the Granger-causality between oil prices and GDP, which disappears when data from 1985 onwards (periods of oil-price declines) are included. On the one hand, Mork (1989) finds the existence of asymmetry between the responses to oil-price increases and decreases by the GDP. He proposes an asymmetric specification in which only the increases were taken into account. Thus, his results confirm that the above-mentioned negative correlation remains when data from 1985 onwards is included. Lee, Ni and Ratti (1995), on the other hand, report that the response to an oil-price shock by the GDP depends on the environment of oil-price stability. An oil shock in a price stability environment is more likely to have greater effects on GDP than those occur in a price volatile environment. These authors propose a measure that takes this volatility into account. They find asymmetry in the effects of positive and negative oil-price shocks, but they also manage to re-establish the above negative correlation. In the same way, Hamilton (1996) points out that it seems more appropriate to compare the current price of oil with what it was during the previous year, rather than during the previous quarter. He therefore proposes to define a new measure, NOPI, what restores the negative correlation between GDP and oil-price increases.

In such a context, Hooker (1996) perceives the existence of a breakpoint in 1973:III, observing the existence of Granger-causality in the first subsample (1948:I-73:III), although not in the second one (1973:IV-94:II) nor in the full sample (1948:I-1994:II). Thus, he concludes that the oil price-GDP relationship changes when data from the 1980s is considered, since a simple oil-price increase/decrease asymmetry is not enough to represent

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4 Specified, they capture these features through a GARCH-based on oil price transformation that scales estimated oil price shocks by their conditional variance.

5 The percentage increase in oil price if the quarter's price exceeds the previous year's maximum, and zero otherwise.

6 He argues that 1973 marks the beginning of the productivity slowdown, the period of the floating exchange rate, and several years of unusually low real interest rates. Furthermore, there have been different institutional regimes that have been determining oil prices since 1973.
it accurately. Likewise, Hooker (1999) argues that Lee-Ni-Ratti’s (1995) and Hamilton’s (1996) transformations do not, in fact, Granger-cause GDP in post-1980 data\textsuperscript{7}, but that their apparent success is due to an improved..t in the 1950s data. Finally, Hamilton (2001b) reports evidence of non-linear representation and states that the functional form that relates GDP to oil prices looks very much like what has been suggested in earlier parametric studies. He specifically analyzes the non-linear transformations of oil prices proposed in the Literature, and he points out that, on the basis of the non-linearity test (Hamilton, 2001a), the Lee-Ni-Ratti’s formulation does the best job of summarizing the non-linearity.

This paper presents evidence of non-linearity, and also argues that despite the fact that the above non-linear specifications do not take oil-price decreases into account they continue to give problems with the out-of-sample forecast. Thus, we propose a different non-linear approach.

The paper challenges the non-linear transformations previously mentioned, since they are ad hoc and only consider the oil-price increases. There seems, therefore, to be some form of data-mining. It is clear that the oil-price declines do not have the same role on the economy as the increases do, but their effects on the economy can not be ignored. It does not seem logical to treat a decline in oil price as if nothing has happened at all. Do not such decreases affect anything in the least? If so, then why are decreases desirable? These and other such questions do not make sense in a framework where the declines do not have consequences. We believe, however, they do, and, as such, should be taken into account.

The aim of our study, therefore, is to analyze the relationship between oil prices and GDP growth, taking both positive and negative changes into account. To do so, we use a non-linear function that relates GDP growth to oil prices, and which we estimate by kernel methods, avoiding any assumption about its form. In-sample, this kernel estimation improves the linear estimation, and also improves both the Hamilton’s (2001b) estimation and those of the above non-linear transformations, considering data from 1960s onwards\textsuperscript{8}.

\textsuperscript{7}He now considers the existence of a breakpoint around 1980.

\textsuperscript{8}Hooker (1999) highlights the fact that the Lee-Ni-Ratti (1995) and Hamilton’s (1996) specifications derive much of their apparent success from data in the 1950s. It is worth noting that the 1950s period is
Furthermore, the one-period ahead out-of-sample kernel forecasting improves those of the above-mentioned non-linear transformations, in the sense that it has a smaller Mean-Square Forecast Error.

We develop the paper on three different parts. We first take the traditional linear approach as a starting point, summarizing the economic activity through a seven-variable system, in particular a VAR specification. In this model, we consider the effects of a positive oil shock through the orthogonalized impulse-response functions, which are obtained by simulation. We also observe both the in-sample and out-of-sample accuracy of this approach. Secondly, we challenge that the non-linear transformations can summarize the non-linearity, and check the linear specification and such transformations with the non-linearity test proposed by Hamilton (2001a). Finally, we propose a semiparametric specification to represent the above relationship. We estimate the model by means of kernel methods, avoiding any assumption about the form of the function that relates GDP to oil prices.

The paper is organized as follows. Section 2 describes the linear approach. Section 3 presents non-linear transformations. Section 4 reports evidence of non-linearity. Section 5 considers the non-linear approach, considering a semiparametric specification. Concluding remarks are offered in Section 6.

2 First Approach: Linear Model

2.1 Previous Considerations

We begin by modelling the economy of the United States, considering the financial, output and price variables, that summarize the economic activity. Our aim is to analyze the relationship between output variables and oil-price changes. One of the main problems one of relative stability in the oil price, with the only smooth movements in Suez Crisis (1956).

At the beginning, we considered a bivariate model with GDP growth and oil price changes as variables. When we observe Figure 1, we notice that this specification forecasts important decreases in GDP in the mid-1970s and notable increases in GDP in the mid-1980s, which do not appear in the GDP. We perform the omitted variables test, and we observe that it is necessary to include more variables to improve the model.
Figure 1
One-period ahead out-of-sample LINEAR forecasting (Bivariate Model)

Note: This figure plots the one-period ahead out-of-sample forecasting for GDP growth in a bivariate model with GDP growth and oil price changes as variables. The forecast runs from 1957:II to 2001:III.

Figure 2
In-sample Linear Forecasting (Multivariate Model: Seven-variable system)

Note: This figure plots the estimation of GDP growth in a seven-variable system. The sample period runs from 1960:I to 2000:III (Notice that the available sample for unemployment rate starts in 1960:I).
of this sort of modelling, however, is the choice of the specific variables that should be included in the model. We have chosen the ones we consider to be most relevant for our goal.

We consider the “chain-weighted real GDP”, $g_t$, and the unemployment rate, $u_t$, as output variables; the long-run interest rate, $lr_t$, and the Federal fund rate, $fed_t$, as financial variables; wage, $w_t$, consumer price index, $p_t$, and measure of oil-price change, $o_t$, as price variables$^{10}$.

It is our belief that an oil-price shock has both direct and indirect effects on macroeconomic variables. The indirect effects come from the responses of the monetary policy to this shock, so that we have included two monetary variables. Our belief in the indirect effects of an oil-price shock is based on the movements observed in the monetary variables after the shocks, especially after increases, as well as on the fact that there are several papers that support this belief. Bohi (1989), among others, argues that the economic downturns observed after oil-price shocks are caused by both the price-shocks themselves and the monetary responses to them. Along these lines of thought we ..nd Bernanke, Getler and Watson (1997), who state that the effects of an oil-shock in isolation (i.e. without responses from monetary policies) is considerably smaller than when monetary variables are considered. Hamilton and Herrera (2000), on the other hand, challenge the Bernanke-Getler-Watson conclusion on two basic grounds: (a) the feasibility of the policy proposed, and (b) the short lag length used in their specification ($p = 7$, considering monthly data). They conclude that the contractionary consequences of the monetary responses to oil-price increases are not as great as Bernanke et al. suggest, although they could not disregard the Bernanke-Getler-Watson conclusions on the effects caused by the Monetary Policy undertaken after an oil shock.

From what has been said, it can be seen that the use of monetary variables makes sense. But we do not extend more about it, leaving it as an open question, since it can be an outstanding issue for future researches.

$^{10}$We have chosen these variables, considering the six-variable dynamic system developed by Sims (1980), as a reduced-form of macroeconomic reality.
2.2 Linear Macroeconomics Model

We denote 

\[ y_t = (y_{1t}; y_{2t}; y_{3t}; y_{4t}; y_{5t}; y_{6t}; y_{7t})^0 = (g_t; u_t; o_t; p_t; f t; l r_t; w_t)^0, \]

which is a \((7 \times 1)\) vector.

One way of summarizing the economic activity is to represent it through a seven-variable system. Specifically, we model it as a \(p\)th-order vector autoregression, \(VAR(p)\),

\[ B_0 y_t = K + B_1 y_{t-1} + B_2 y_{t-2} + \cdots + B_p y_{t-p} + u_t. \] (2.1)

The matrix \(B_0\) is taken to be lower triangular with 1's along diagonal. With this assumption, we are guaranteeing a \(VAR(p)\) just identified. Therefore, we can rewrite it as follows:

\[ y_t = c + \Theta_1 y_{t-1} + \Theta_2 y_{t-2} + \cdots + \Theta_p y_{t-p} + \tau_t, \] (2.2)

where \(c = (c_1; \cdots; c_7)^0\) is the \((7 \times 1)\) intercept vector of the \(VAR\), \(\Theta_i\) is the \(i^{th}\) \((7 \times 7)\) matrix of autoregressive coefficients for \(i = 1; 2; \cdots; p\), and \(\tau_t = (\tau_t; \cdots; \tau_t)^0\) is the \((7 \times 1)\) generalization of a white noise.

Assuming that \(\tau_t\) is a Gaussian White Noise Process, the \(VAR\) can be estimated by Maximum Likelihood, since, even though the true innovations are non-Gaussian, the estimates obtained are consistent. But it is well known that it is enough to estimate the system by OLS, equation by equation, to get such estimators. The estimate sample used (including the lagged initial values) runs from 1960:I to 2000:III\(^{11}\), for a total of \(T = 163\) usable quarterly observations. To find the suitable lag length, we implement Akaike Information Criterion, Schwartz Criterion and the Sims' modification (1980) of the Likelihood Ratio Test. We choose, therefore, a lag length of four on the basis of these criteria. Hereafter we consider a fourth-order \(VAR\).

Now, we briefly comment on the results of the different tests performed in the \(VAR\) context. Firstly, note that the oil prices do not appear as significant variables in the GDP equation in the multivariate \(VAR\), although when we consider the bivariate \(VAR\) the fourth lag of oil price is statistically significant in the GDP equation. Secondly, all of the equations, except the one for oil prices, are jointly significant in explaining

\(^{11}\)We have used this sample size because the available sample for the unemployment rate starts in 1960:I.
the dependent variable (See Table 1\textsuperscript{a}). There is a clear intuitive explanation for this: oil prices are fixed\textsuperscript{12} in the world-wide crude oil market, considering both demand and supply aspects. As such, although the US might be an important part of that demand, they are no longer able to fix oil prices as they wish. Thirdly, the Wald test, whose null hypothesis is “all lags of oil-price change in GDP equation are zero”, shows us (See Table 2)\textsuperscript{13} that all lags of oil-price change are not statistically significant as a whole in the multivariate VAR context for any sample considered.

Most of the studies on the matter have overlooked the testing of the normality they assume, as such we have performed the Jarque-Bera test equation by equation to check the normality of residuals. The results are presented in Table 1\textsuperscript{b}. The residuals depart significantly from normality in all of the cases but CPI case. Thus, while the estimation gives us consistent estimators, it does not provide efficient estimators\textsuperscript{14}.

The estimation of GDP growth and its one-period ahead out-of-sample forecasting can be seen in Figures 2 and 3. As we can see, the problem of the 1980s was not a very important one, but the linear out-of-sample forecast is not very accurate. As such, we can tend to believe that there is a structural change in the GDP equation of the multivariate VAR. But can we verify this? The answer depends on whether we consider the existence of a structural change in the oil price coefficients or whether we consider a structural change in all of the regression coefficients. We look for the existence of a breakpoint in the period that runs from 1970:III to 1992:IV. Figure 5 presents the p-values for a test of the null hypothesis that all oil price coefficients are stable in the Chow’s sense against the alternative hypothesis that these coefficients change on indicated date in the horizontal axis. We note that there is no evidence of a structural change for

\textsuperscript{12}Right up to 1973, oil prices were controlled mainly by the Texas Railroad Commission and other institutions. From this date on, however, and right through to the 1980s, the OPEC countries began to dominate the worldwide petroleum market, and, from then on, the forces of the free market have been establishing the price of crude oil.

\textsuperscript{13}Following the indications of other authors, we first assume 1973:IV and 1985:IV to be breakpoints, although we do question the validity of these dates later on.

\textsuperscript{14}It must be remembered that if the relationship among the variables is non-linear, the Jarque-Bera results cannot be right (See Section 4).
### Table 1

Joint Significance and Jarque-Bera Test

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>F-Statistic&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Jarque-Bera Statistic&lt;sup&gt;b&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>3.52601&lt;sup&gt;*&lt;/sup&gt;</td>
<td>18.5481&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Unemployment Rate</td>
<td>218.896&lt;sup&gt;***&lt;/sup&gt;</td>
<td>40.7168&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Oil price</td>
<td>1.24928&lt;sup&gt;b&lt;/sup&gt;</td>
<td>885.578&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.202)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>CPI</td>
<td>28.7279&lt;sup&gt;***&lt;/sup&gt;</td>
<td>2.1752</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.337)</td>
<td></td>
</tr>
<tr>
<td>Fed fund rate</td>
<td>79.0425&lt;sup&gt;***&lt;/sup&gt;</td>
<td>370.501&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Long rate</td>
<td>158.073&lt;sup&gt;***&lt;/sup&gt;</td>
<td>21.5170&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
<tr>
<td>Wage</td>
<td>14.7536&lt;sup&gt;***&lt;/sup&gt;</td>
<td>105.022&lt;sup&gt;***&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.000)</td>
<td>(0.000)</td>
<td></td>
</tr>
</tbody>
</table>

Note.- These tests are performed equation by equation in the VAR (4) framework (1960:I-2000:III). P-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.  
<sup>a</sup>The F-statistic of joint significance whose null hypothesis is “all of the coefficients, except the constant term, are zero”.  
<sup>b</sup>The Jarque-Bera test whose null hypothesis is “the existence of normality of the residuals”.

### Table 2

Wald test (F-statistic)

<table>
<thead>
<tr>
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<th></th>
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<tbody>
<tr>
<td>VAR</td>
<td>0.297123&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.959682&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.070917&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.880302&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.011177&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.879389)</td>
<td>(0.448236)</td>
<td>(0.990653)</td>
<td>(0.480322)</td>
<td>(0.417297)</td>
<td></td>
</tr>
</tbody>
</table>

Note.- The lag length used is 4. The null hypothesis is \“all lags of oil price in GDP equation are zero\”. P-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.

### Table 3a

Bivariate Granger-Causality Test (F-statistic)  
(Linear Case )

<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>1.659&lt;sup&gt;b&lt;/sup&gt;</td>
<td>3.297&lt;sup&gt;***&lt;/sup&gt;</td>
<td>2.260&lt;sup&gt;***&lt;/sup&gt;</td>
<td>1.075&lt;sup&gt;b&lt;/sup&gt;</td>
<td>0.705&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.867&lt;sup&gt;b&lt;/sup&gt;</td>
</tr>
<tr>
<td>(0.162)</td>
<td>(0.012)</td>
<td>(0.065)</td>
<td>(0.370)</td>
<td>(0.589)</td>
<td>(0.119)</td>
<td></td>
</tr>
</tbody>
</table>

Note.-<sup>b</sup> denotes ‘does not Granger cause’. P-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.
Note: This figure plots the one-period ahead out-of-sample forecasting for GDP growth in a bivariate model with GDP growth and oil price changes as variables. The forecast runs from 1957:II to 2001:III.

Note: This figure plots the estimation of GDP growth in a seven-variable system. The sample period runs from 1960:I to 2000:III (Notice that the available sample for unemployment rate starts in 1960:I).
Figure 3
One-period ahead out-of-sample Linear Forecasting
(Multivariate model: Seven variable-system)

Note: This figure plots the one-period ahead out-of-sample forecasting for GDP growth in a seven-variable system. The forecast runs from 1975:II to 2000:III.

Figure 4
Orthogonalized impulse-response function
(Multivariate model: Seven variable-system)

Note: This figure plots the orthogonalized impulse-response function, over 24 quarters, to one standard deviation oil-price innovation by GDP in the multivariate model.
any date $t_0$, with $t_0^2[1970:III \text{ to } 1992:IV]$. When we consider the possibility of a structural change in all regression coefficients (See Figure 6), however, the results show the existence of a breakpoint on the following dates: 1978:II, 1979:III, 1979:IV, 1980:I, and any date included in the interval $[1980:III \text{ to } 1983:II]$. We conclude, therefore, that there is stability of price coefficients in GDP equation. This is confirmed when we consider Andrews’ test (1993), Andrews and Ploberger’s (1994) tests (both the average and exponential specifications) (See Table 6, first line). The instability of the linear GDP equation comes from other variables, although if we consider Andrews’ test and Andrews and Ploberger’s tests (See Table 5, three first lines), they indicate that there is stability in all of the coefficients.

Finally, as we are interested in observing whether an increase in the price of oil Granger-causes the recession, and also whether a decrease Granger-causes the economic boom, we will observe the results of a bivariate Granger causality test. We perform this test for each variable of the VAR with respect to oil prices for the full sample (See Table 3a), and for GDP growth with respect to oil prices for different sub-samples (See Figures 9 and 10). In the full sample, the oil price only Granger-causes the unemployment rate at a 5% critical level and CPI at a 1% critical level. Moreover, if we consider the first subsample that runs from 1947:II to the date indicated $t_1$ in the horizontal axis (Figure 9), we obtain that oil-price changes Granger-cause GDP growth when $t_1$ is any date between 1974:III and 1986:III. On the other hand, if we consider the second subsample that runs from the date indicated $t_1$ in the horizontal axis to 2001:III (Figure 10), we obtain that oil-price changes do not Granger-cause GDP growth on any date at all. It is clear, therefore, that oil-price changes do not Granger-cause GDP growth neither in the full sample nor in the second subsample. Although, causality appears when we consider

15 If we consider the bivariate VAR, we obtain that neither the oil price coefficients nor all of the regression coefficients have changed at any date (See Figure 7 and 8).

16 When we consider the sample used for the multivariate VAR, the results are as follows: on the one hand, if we consider the first subsample, which runs from 1960:I to the date indicated $t_1$ in the horizontal axis, we obtain that oil price changes Granger-cause GDP growth when $t_1$ is any date between 1970:I and 1982:II (with exceptions) or any date between 1983:II and 1986:III. On the other hand, if we consider the second subsample, which runs from the date indicated $t_1$ in the horizontal axis to 2000:III, we obtain that oil price changes do not Granger-cause GDP growth on any date at all.
Figure 5
Chow test for stability of coefficients on oil prices
(Multivariate model) (Ho: Stability)

Note: This figure represents the p-values for a test of the null hypothesis that all oil price coefficients in
the GDP equation (in the multivariate model) are stable in the Chow's sense against the alternative
hypothesis that these coefficients change on indicated date in the horizontal axis.

Figure 6
Chow test for stability of all regression coefficients
(Multivariate model) (Ho: Stability)

Note: This figure represents the p-values for a test of the null hypothesis that all regression coefficients
in the GDP equation (in the multivariate model) are stable in the Chow's sense against the alternative
hypothesis that these coefficients change on indicated date in the horizontal axis.
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<td>158.073^**</td>
<td>21.5170^**</td>
</tr>
<tr>
<td>Wage</td>
<td>14.7536^**</td>
<td>105.022^**</td>
</tr>
</tbody>
</table>

Note.- These tests are performed equation by equation in the VAR(4) framework (1960:I-2000:III). p-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.

^aThe F-statistic of joint significance whose null hypothesis is "all of the coeft cients, except the constant term, are zero".

^bThe Jarque-Bera test whose null hypothesis is "the existence of normality of the residuals".

Table 2  
Wald test (F-statistic)

<table>
<thead>
<tr>
<th></th>
<th>Full sample</th>
<th>1st subsample</th>
<th>2nd subsample</th>
<th>1st subsample</th>
<th>2nd subsample</th>
</tr>
</thead>
<tbody>
<tr>
<td>VAR</td>
<td>0.297123</td>
<td>0.956629</td>
<td>0.070917</td>
<td>0.880302</td>
<td>1.011177</td>
</tr>
<tr>
<td>(0.879389)</td>
<td>(0.448236)</td>
<td>(0.990653)</td>
<td>(0.480232)</td>
<td>(0.417297)</td>
<td></td>
</tr>
</tbody>
</table>

Note.- The lag length used is 4. The null hypothesis is \"all lags of oil price in GDP equation are zero\". p-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.

Table 3a  
Bivariate Granger-Causality Test (F-statistic) (Linear Case)

<table>
<thead>
<tr>
<th></th>
<th>Oil9 GDP</th>
<th>Oil9 UR</th>
<th>Oil9 CPI</th>
<th>Oil9 Fed</th>
<th>Oil9 LR</th>
<th>Oil9 W</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full Sample</td>
<td>1.659</td>
<td>3.297^**</td>
<td>2.260^**</td>
<td>1.075</td>
<td>0.705</td>
<td>1.867</td>
</tr>
<tr>
<td>(1960-I-2000:III)</td>
<td>(0.162)</td>
<td>(0.012)</td>
<td>(0.065)</td>
<td>(0.370)</td>
<td>(0.589)</td>
<td>(0.119)</td>
</tr>
</tbody>
</table>

Note.- denotes 'does not Granger cause'. p-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.
Figure 7
Chow test for stability of coefficients on oil prices
(Bivariate model) (Ho: Stability)

Note: This figure represents the p-values for a test of the null hypothesis that all oil price coefficients in the GDP equation (in the bivariate model) are stable in the Chow's sense against the alternative hypothesis that these coefficients change on indicated date in the horizontal axis.

Figure 8
Chow test for stability of all regression coefficients
(Bivariate model) (Ho: Stability)

Note: This figure represents the p-values for a test of the null hypothesis that all regression coefficients in the GDP equation (in the bivariate model) are stable in the Chow's sense against the alternative hypothesis that these coefficients change on indicated date in the horizontal axis.
Figure 9

Bivariate Granger-causality test: First Sub-sample
(Ho: Oil price does not Granger-cause GDP)

Note: This figure presents the p-values for the Granger-causality test of the null hypothesis that oil price change does not Granger-cause GDP growth for the first subsample, which runs from 1947:II to the date indicated in the horizontal axis that starts in 1959:IV.

Figure 10

Bivariate Granger-causality test: Second Sub-sample
(Ho: Oil price does not Granger-cause GDP)

Note: This figure presents the p-values for the Granger-causality test of the null hypothesis that oil price change does not Granger-cause GDP growth for the second subsample, which runs from the date indicated in the horizontal axis that ends in 1989:I to 2001:III.
subsamples that end before 1986:III.

### 2.3 The effects of an oil price shock

In order to appreciate the effects of an oil-price shock in the VAR context, we use impulse-response functions. We have represented the orthogonalized impulse-response functions\(^{17}\).

Figure 4 shows the responses, over 24 quarters, to one standard deviation oil-price shock. We only comment on the response to oil-price innovations by GDP growth. An oil-price innovation has a negative influence GDP growth, and its major negative influence occurs during the fourth quarter following it. This is entirely consistent with the result obtained by most of studies carried out on the topic.

We have observed that the linear model creates some problems, basically, in out-of-sample forecasting. And the linear specification indicates that oil-price changes do not Granger-cause GDP growth in the full sample. In trying to solve such problems, different non-linear transformations of oil prices have appeared. In the following section, we briefly point out the main non-linear transformations proposed in the Literature.

### 3 Non-linear transformations


\(^{17}\)Since we do not consider the contemporaneous influence of oil price innovation over GDP growth, the ordering of variables used is \((g; o; p; f; e; d; l; r; w; u)\). Although, if our aim were also to observe the contemporaneous effects of an oil price innovation, it would be seemed appropriate to place it at the top of the ordering of variables \((o; p; f; e; d; l; r; w; u; g)\). In any case, we have verified that the impulse-responses do not substantially change. Only the contemporaneous effect changes, begging zero in the first case and negative in the second one.
(1996a, 1996b, 1999) also contributes non-linear evidence, and although he criticizes both Mork's (1989) and Hamilton's (1996) specifications, he has not been able to find the "right" transformation for oil prices. The common conclusion is, therefore, that positive oil-price changes affect GDP growth, whereas declines do not, and also that oil-price increases after a long period of stability in the price had more dramatic consequences than those that were merely corrections to greater oil-price declines during the previous quarter.

We now look at the non-linear transformations proposed in the Literature.

Mork (1989) shows the existence of asymmetry between the GDP's responses to oil-price increases and decreases. He concludes that oil-price decreases are not statistically significant. We refer to Mork's specification as one in which only increases are considered.

\[ o_t^m = \begin{cases} \alpha_t & \text{if } o_t > 0 \\ 0 & \text{otherwise} \end{cases} \]

Lee, Ni and Ratti (1995), and Hamilton (1996), observe that oil-price increases after a long period of price stability have more dramatic consequences than those that are merely corrections to greater oil-price decreases during the previous quarter. Thus, the first authors consider a GARCH representation of oil-prices to reflect the above fact. We refer to Lee, Ni and Ratti specification as SOPI (scaled oil price increase).

\[
\begin{align*}
    z_t & = \beta_0 + \beta_1 z_{t-1} + \beta_2 z_{t-2} + \beta_3 z_{t-3} + \beta_4 z_{t-4} + \epsilon_t \\
    e_{t1} & \sim \text{N}(0; h_t) \\
    h_t & = \omega_0 + \omega_1 \epsilon_{t1}^2 + \omega_2 h_{t-1} \\
    \text{SOPI}_t & = \max(0; \epsilon_t = \sqrt{\hat{h}_t})
\end{align*}
\]

where \( z_t \) is the real oil-price changes.

Hamilton (1996) proposes the non-linear transformation, known as net oil price increase (NOPI). We refer to Hamilton's specification as NOPI (the amount by which oil prices in quarter \( t \) exceed the maximum value over the previous 4 quarters; and 0 otherwise).

\[
\text{NOPI}_t = \max f(0; \alpha_t; \max f(\alpha_{t1}; \alpha_{t2}; \alpha_{t3}; \alpha_{t4}))
\]
There is a variation of the above measure that considers the previous 12 quarters. We refer to this specification as NOPI\(3\) (the amount by which oil prices in quarter \(t\) exceed the maximum value over the previous 12 quarters; and 0 otherwise).

\[
\text{NOPI}_3(t) = \max \{0; \alpha_t \} \cdot \max \{\alpha_{t-1}; \alpha_{t-2}; \ldots; \alpha_{t-12}\}
\]

Note that all of these non-linear transformations are the one that have been proposed in the literature for restoring Granger-causality and avoiding the forecasting of a non-existent GDP increase when oil prices decrease. But these specifications are rather ad hoc, and ignore the effects of oil-price decreases.

We observe that the Granger-causality is re-established in the full sample (1947:II-2001:III) when these specifications are employed (See Table 3b). We also note, however, that when we split the sample, the above result does not hold. To be more specific, if the subsample runs from 1947:II to any date beyond 1974:II, there is Granger-causality. On the other hand, if the subsample is from any date beyond 1974:I to 2001:III, the Granger-causality disappears, suggesting that the success of the Granger-causality is due merely to the first dates considered.

None of these transformations, however, succeed in solving the problem of the linear specification in out-of-sample forecasting\(^{18}\) (See Figures 11 and 12).

Table 4 reports the results of Diebold and Mariano test, whose null hypothesis is that there is equal forecast accuracy. We set up this statistic such that a positive value means that the linear specification fits better than the other specifications considered. We then find that in-sample and out-of-sample the non-linear specifications considered have a smaller MSE/MSFE, but we cannot reject the null hypothesis of DM test.

Furthermore, to verify that the problem is not one of a structural change, we performed different tests for stability of coefficients on oil prices and for stability of all coefficients in the multivariate model with the non-linear specifications. The results of the Chow test indicate that there is stability in any specification considered. The results of the Andrews' test and those of Andrews and Ploberger are reported in Tables 5 and 6. We obtain that all of the specifications are essentially stable with these tests.

\(^{18}\)In Figure 12 we make a direct comparison between the SOPI specification, which is the one with the lowest MSE, with the linear specification.
Table 3b
Bivariate Granger-Causality Test (F-statistic)
(Longest available sample: 1947:II-2001:III)

<table>
<thead>
<tr>
<th>Oil price Measure</th>
<th>Oil nominal</th>
<th>Mork</th>
<th>NOPI</th>
<th>SOPI</th>
<th>NOPI3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0:127)</td>
<td>(0:011)</td>
<td>(0:008)</td>
<td>(0:001)</td>
<td>(0:004)</td>
</tr>
</tbody>
</table>

Note.-9 denotes 'does not Granger cause'. p-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.

Table 4
Diebold and Mariano Test
(MULTIVARIATE MODEL)
(In-sample and Out-of-sample)

<table>
<thead>
<tr>
<th>Diebold-Mariano Test</th>
<th>DM-S_1 test</th>
</tr>
</thead>
<tbody>
<tr>
<td>H_0 : equal forecast accuracy</td>
<td>(relative to Linear model)</td>
</tr>
<tr>
<td>Statistics</td>
<td>(p, value)</td>
</tr>
</tbody>
</table>

| Linear | IN | (MSE: 0.4618) | i |
|        | OUT | (MSFE: 0.9698) | i |
| Mork   | IN | (MSE: 0.4615) | 0:0467 (0:962) |
|        | OUT | (MSFE: 0.9651) | 0:2050 (0:834) |
| SOPI   | IN | (MSE: 0.4599) | 0:2105 (0:832) |
|        | OUT | (MSFE: 0.8970) | 0:9182 (0:358) |
| NOPI   | IN | (MSE: 0.4594) | 0:4029 (0:686) |
|        | OUT | (MSFE: 0.9550) | 0:6929 (0:488) |
| NOPI3  | IN | (MSE: 0.4542) | 0:8444 (0:398) |
|        | OUT | (MSFE: 0.9431) | 1:1784 (0:238) |

Note.- Mean-Square Error and Mean-Square Forecast Error are defined as follows: \( M S F E = E [(\tilde{y}_{T+1} - Y_{T+1})^2 | I_T] \) and \( M S E = E [(y_T - \tilde{y}_T)^2 | I_T] \), respectively, where \( \tilde{y}_T \) is the in-sample estimation, \( \tilde{y}_{T+1} \) is the one-period ahead out-of-sample forecasting, and \( I_T \) is the available information in \( T \). In-sample refers to the period that runs from 1961:I to 2000:III. Out-of-sample refers to the period that runs from 1975:II to 2000:III. The DM statistic tests the null hypothesis that there is any statistically significant divergence between the in-column model and the linear model. p-values based on two-sided tests appear in parenthesis (One/two/three asterisks mean a p-value less than 10%/5%/1%).
Figure 11
One-period ahead out-of-sample forecasting
(Non-linear transformations)

Note: This figure plots the one-period ahead out-of-sample forecasting for GDP growth for different non-linear transformations in a seven-variable system. The forecast runs from 1975:II to 2000:III.

Figure 12
Comparison between Linear forecasting and SOPI forecasting

Note: This figure plots the one-period ahead out-of-sample forecasting for GDP growth in the linear and SOPI cases.
Table 5
Test for stability of ALL coefficients
(MULTIVARIATE MODEL)

<table>
<thead>
<tr>
<th>Oil price measure</th>
<th>Test statistic</th>
<th>Asymptotic p-value</th>
<th>Homoskedastic bootstrap p-value</th>
<th>Heteroskedastic bootstrap p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal oil price change</td>
<td>Sup F</td>
<td>0.088*</td>
<td>0.178</td>
<td>0.383</td>
</tr>
<tr>
<td>Mork spec.</td>
<td>Avg F</td>
<td>0.252</td>
<td>0.281</td>
<td>0.059*</td>
</tr>
<tr>
<td>NOPI spec.</td>
<td>Avg F</td>
<td>0.285</td>
<td>0.330</td>
<td>0.064*</td>
</tr>
<tr>
<td>SOPI spec.</td>
<td>Avg F</td>
<td>0.362</td>
<td>0.375</td>
<td>0.007**</td>
</tr>
<tr>
<td>NOPI3 spec.</td>
<td>Avg F</td>
<td>0.323</td>
<td>0.451</td>
<td>0.479</td>
</tr>
</tbody>
</table>

Note.- Like Hamilton (2001b) we have performed these tests using $\lambda = 0.15$ and 28 restrictions. Asymptotic and bootstrap p-values were calculated as in Hasen (1997) and Hansen (2000), respectively. One/two/three asterisks mean a p-value less than 10%/5%/1%.

Table 6
Test for stability of coefficients on oil prices
(MULTIVARIATE MODEL)

<table>
<thead>
<tr>
<th>Oil price measure</th>
<th>Sup F (date)</th>
<th>Avg F</th>
<th>Exp F</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal oil price change</td>
<td>8.646 (1992:I)</td>
<td>4.271</td>
<td>2.703</td>
</tr>
<tr>
<td>Mork spec.</td>
<td>5.966 (1990:III)</td>
<td>3.115</td>
<td>1.935</td>
</tr>
<tr>
<td>SOPI spec.</td>
<td>8.156 (1992:I)</td>
<td>6.022</td>
<td>3.186</td>
</tr>
<tr>
<td>NOPI spec.</td>
<td>5.327 (1979:III)</td>
<td>2.881</td>
<td>1.701</td>
</tr>
<tr>
<td>NOPI3 spec.</td>
<td>5.137 (1974:I)</td>
<td>2.717</td>
<td>1.517</td>
</tr>
</tbody>
</table>

Asymptotic 5% critical values | 16.45 | 7.67 | 5.23 |

Note.- Like Hamilton (2001b) we have performed these tests using $\lambda = 0.15$ and four restrictions. Critical values were taken from Andrews (1993) and Andrews and Ploberger (1994). One/two/three asterisks mean a p-value less than 10%/5%/1%.
It seems only natural, therefore, that doubts should arise with regard to the ability of these non-linear transformations to accurately reflect non-linearity.

4 Non-linearity test

As we have seen in the previous section, the Literature “offers” evidence of a non-linear relationship between GDP growth and oil-price changes, but the most contribution to this evidence is the result of the non-linearity test\(^{19}\) proposed by Hamilton (2001a)\(^{20}\).

Hamilton (2001b) has already performed this test for the full sample (1947:II-2001:III) and for the different specifications mentioned above. We, in contrast to Hamilton, have performed this test for different sub-sample, in an effort to identify where the non-linearity appears\(^{21}\). We too have established different window sizes to achieve to identify where the non-linearity appears.

4.1 Test description

We have followed Hamilton’s indications in testing the null hypothesis that the true relationship between GDP growth and oil-price changes is linear, considering a non-linear regression model of the following form:

\[
y_t = ¹(x_t) + ²z_t + ß_t
\]

where \(y_t\) is the real GDP growth; \(x_t\) is a \(k\)-dimensional vector which contains lags in oil price growth, with \(k = 4\), \(x_t = (o_{t-1}; o_{t-2}; o_{t-3}; o_{t-4})\), for which linearity is not assumed; \(^1(\cdot)\) is a function, whose form is unknown\(^{22}\); \(z_t\) is a \(p\)-dimensional vector with lags in GDP.

\(^{19}\)There are several tests for neglected non-linearity, among which we find: the Regression Error Specification Test, also called the Ramsey’s Reset test (Ramsey, 1969), Tsay’s test (Tsay, 1986), the V23 test (Terasvirta-Lin-Granger, 1993), the neural network test (White, 1989, and Lee-White-Granger, 1993), and others.

\(^{20}\)Dahl (1999) finds that this test has good size and power properties.

\(^{21}\)It is noteworthy that practically all of the authors referred to above attribute the non-linearity to declines in oil prices during the mid-1980s.

\(^{22}\)The Hamilton (2001a) approach considers the function \(^1(\cdot)\) itself as being the outcome of a random field. He uses the generalization of the finite-differenced Brownian motion.
growth, with \( p = 4, z_t = (y_{t,1}; y_{t,2}; y_{t,3}; y_{t,4})^0 \), for which linearity is assumed\(^{23}\); and \( \varepsilon_t \) is an error term. To implement the test, \( g_i \) is defined as

\[
g_i = 2[k(T - 1)X_t (x_i t - x_i)]^1 \varepsilon; \quad (3.2)
\]
governing the variability of the non-linear component with respect to the \( i \)th explanatory variable. Using these values for \( g_i \), calculate \( h_{st} = (1 - 2)[P_i = 1 g_i^2 (x_i t - x_i s)^2]^{1 - 2} \) and construct the \((T \times T)\) matrix \( H \) whose row \( t \), column \( s \) element is given by

\[
H_{hts} = 1 \cdot (2 - 4)! (2 - 3) h_{st} (1 - h_{st}^2)^{3 - 2} h_{st} (1 - h_{st}^2) + \sin^1 (h_{st}) \quad (3.3)
\]
when \( 0 \leq h_{st} \leq 1 \) and by zero when \( h_{st} > 1 \).

We perform an OLS linear regression\(^{24}\) of \( y_t \) on \( x_t, z_t \) and a constant, \( y_t = X - \cdot \varepsilon \):

\[
y_t = 0.747219_i + 0.0048260_{t,1} + 0.0064580_{t,2} \quad (3.4)
\]

\[
i = 0.0064820_{t,3} + 0.0119680_{t,4} + 0.275762y_{t,1} + 0.121160y_{t,2} + 0.077809y_{t,3} + 0.125788y_{t,4} \quad (0.006622)
\]

\[
\varepsilon = (0.006574) (0.069062) (0.068425)
\]

Calculate the OLS residuals, \( \varepsilon \), regression squared standard error, \( \frac{\varepsilon^2}{\varepsilon^2} = (T - k - p - 1)^{1/2} \), and \((T \times T)\) projection matrix \( M = I_T \cdot X (X' X)^{-1} X \).

We then calculate the Lagrange multiplier statistic for neglected non-linearity:

\[
\lambda^2 = \frac{[\frac{\varepsilon^2}{\varepsilon^2} + \frac{\varepsilon^2}{\varepsilon^2} tr(M H M)]^2}{\frac{\varepsilon^2}{\varepsilon^2} (2trf[M H M] \cdot (T - k - p - 1)^{1/2} tr(M H M)^2) \varepsilon^2 (g)}; \quad (3.5)
\]

Hamilton (2001a) shows that this statistic has an asymptotic \( \lambda^2(1) \) distribution under the null hypothesis of linearity.

### 4.2 Empirical results

We carried out this test twice, first with our own set of data and then with Hamilton's, so that we could make a direct comparison.
Table 7 shows the results of the non-linearity test performed with both sets of data (See Appendix A). We observe that the null hypothesis that the relationship between oil prices and GDP growth is linear is rejected with either set of data. We also observe the acceptance of the null hypothesis that the non-linear specification is correct with both sets of data\textsuperscript{25} for any non-linear transformation considered in the previous section.

We now wish to see what happens if we consider different sub-samples that run from any date beyond 1947:II up to 2001:III (See Figure 13).\textsuperscript{26} We accept the existence of linearity at a 5% critical value if the subsample starts after 1948:III. Furthermore, we accept that all of the non-linear specifications considered are correct at any initial date.

Note that we have considered different sub-samples of different sizes, so that we should perform this test with different sub-samples, although they might all be of the same size. We first establish a window with a fixed number of observations. We consider different window sizes ($T = 55; 60; 70; 80; 90; 100; 110; 120; 130; 140; 150; \text{ and } 160$). We then displace this window over time and perform the non-linearity test.

a) For the linear case (See Figure 15), the results are as follows:
- For window sizes of less than 110 and for observations that do not contain data beyond 1973:II\textsuperscript{27}, we obtain the acceptance of the linear relationship between GDP growth and oil-price changes.
- For window sizes of less than 110 and for observations that do not contain data beyond 1976, we achieve the rejection of the above linear relationship. To attribute the non-linearity of the GDP-Oil price relationship to the mid-1980s data as the previous authors have done, would therefore, on the basis of this test, seem inappropriate. Furthermore, when we consider window sizes of 120; 130 or 140 and observations that do not contain data beyond 1984, we obtain the existence of non-linearity again.

\textsuperscript{25} Note that this is the Hamilton’s interpretation (2001b) when he applies the non-linear test to non-linear transformations.

\textsuperscript{26} When we consider Hamilton’s data set (Figure 14), we should consider sub-samples that run beyond 1973:III to be able to accept the linearity, even though we accept linearity for any date between 1963:II and 1967:IV, and in 1971:II. Moreover, we accept that SOP1 specification is correct for any subsample, and that Mork, NOPI3 and NOPI (with 2 exceptions: 1969:I and 1970:I) specifications are correct for any initial date beyond 1948:II.

\textsuperscript{27} Note that oil-price changes are not important up to 1973:II.
### Table 7
**Non-linearity Test**

<table>
<thead>
<tr>
<th>Spec.</th>
<th>Our data set</th>
<th>Hamilton’s data set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full Sample</td>
<td>Full Sample</td>
</tr>
<tr>
<td>Linear</td>
<td>7.47$^{***}$ (0.00623)</td>
<td>39.99$^{***}$ (2.5AE$^{100}$)</td>
</tr>
<tr>
<td>Mork</td>
<td>0.05 (0.62044)</td>
<td>2.99$^{a}$ (0.08434)</td>
</tr>
<tr>
<td>SOPI</td>
<td>0.0084 (0.92681)</td>
<td>0.51 (0.47652)</td>
</tr>
<tr>
<td>NOP1</td>
<td>0.21 (0.64823)</td>
<td>3.84$^{a}$ (0.05008)</td>
</tr>
<tr>
<td>NOP13</td>
<td>0.81 (0.36798)</td>
<td>1.58 (0.20826)</td>
</tr>
</tbody>
</table>

**Note.** This Table reports the statistic value and the p-value of the non-linearity test performed in the full sample (1947:II-2001:III). p-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.

### Table 8
**Mean-Square Errors**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Linear</td>
<td>0.9288</td>
<td>0.7129</td>
<td>1.109119</td>
</tr>
<tr>
<td>Mork</td>
<td>0.9032</td>
<td>0.6844</td>
<td>0.993695</td>
</tr>
<tr>
<td>SOPI</td>
<td>0.8860</td>
<td>0.6842</td>
<td>1.051724</td>
</tr>
<tr>
<td>NOP1</td>
<td>0.9031</td>
<td>0.6846</td>
<td>0.974486</td>
</tr>
<tr>
<td>NOP13</td>
<td>0.8943</td>
<td>0.6759</td>
<td>0.945180</td>
</tr>
<tr>
<td>Hamilton Semipar.</td>
<td>0.9230</td>
<td>0.7318</td>
<td>-</td>
</tr>
<tr>
<td>Kernel Semiparam.</td>
<td>0.8916</td>
<td>0.6634</td>
<td>0.844287</td>
</tr>
</tbody>
</table>

**Note.** Mean-Square Error and Mean-Square Forecast Error are defined as follows: $\text{MSE} = \mathbb{E} \{ (y_{t+1}^i - \hat{y}_{t+1}^i)^2 | I_T \}$ and $\text{MSFE} = \mathbb{E} \{ (y_T^i - \hat{y}_T^i)^2 | I_T \}$, respectively, where $\hat{y}_{t+1}^i$ is the in-sample estimation, $\hat{y}_{t+1}^i$ is the one-period ahead out-of-sample forecasting, and $I_T$ is the available information in $T$. 
Figure 13

p-values of non-linearity test for different oil price measures and for different subsamples
(Our data set)

Note: These figures represent the p-values of the non-linearity test for different oil price measures and for subsamples that run from the date indicated in the horizontal axis (ending in 1997:I) to 2001:III.
Note: These figures represent the p-values of the non-linearity test for different oil price measures and for subsamples that run from the date indicated in the horizontal axis (ending in 1997:I) to 2001:III, using Hamilton’s data set.
Figure 15
p-values of non-linearity test: Different window sizes
(Nominal oil price changes)
(Our data set)

Note: These figures represent the p-values of the non-linearity test for different window sizes \(T = 55, 60, 70, \ldots, 160\). We first establish a window with a fixed number of observations. We then displace this window over time and perform the non-linearity test. For instance, the first p-value represented in any charts plots the p-value of the non-linearity test for a sample that starts in 1947:II and ends \(T\) quarters later.
b) In the Mork case, for a window size less than 80 and any observations contained therein, we obtain the acceptance of the null hypothesis that the specification proposed is a correct representation of the non-linearity. Moreover, for window sizes of less than 130 and for observations that do not contain data beyond 1978, we obtain the rejection of the null hypothesis. We accept the null hypothesis, however, for all of the other window sizes considered.

c) In the SOPI case, we accept the null hypothesis that the specification proposed is a correct representation of the non-linearity for window sizes of less than 90 and for any observations contained therein, and for those that are longer than 120 (with some exceptions). We reject it for window sizes of 100 and 110 and for observations that do not contain data beyond 1976.

d) In the NOPI and NOPI3 cases, for window sizes of less than 110 and for observations that do not contain data beyond 1976, we obtain the rejection of the null hypothesis that the specification proposed is a correct representation of the non-linearity. Furthermore, for window sizes of less than 140 and for observations that do not contain data beyond 1984, the null hypothesis is rejected. It is also rejected when we consider observations that include both the 1970s and the 1980s or windows of any size.\(^{28}\)

We conclude, therefore, that “the non-linearity of the above-mentioned relationship is only due to the use of data from the mid-80s onwards” is not entirely clear. We observe the existence of a non-linear relationship in subsamples that do not contain such data. Moreover, although we reject linearity in the full sample, the non-linear transformations that ignore the oil-price declines do not solve the problem. Note that these specifications attribute the non-linearity to the oil-price declines, and that is why they choose to ignore them.

\(^{28}\)The results of Hamilton’s data set for nominal oil price changes are reported in Figure 16. They are similar to our results, except that we reject the null hypothesis of linearity if we consider observations that include both the 1970s and the 1980s, and windows of any size. When we consider the above mentioned non-linear specifications, the results are similar, except that we accept that SOPI is a correct specification of non-linearity for windows of any size and any observations contained therein.
Figure 16

p-values of non-linearity test: Different window sizes
(Nominal oil price changes)
(Hamilton data set)

Note: These figures represent the p-values of the non-linearity test for different window sizes ($T = 55, 60, 70, \ldots, 160$), using Hamilton’s data set. We first establish a window with a fixed number of observations. We then displace this window over time and perform the non-linearity test. For instance, the first p-value represented in any charts plots the p-value of the non-linearity test for a sample that starts in 1947:II and ends $T$ quarters later.
5 Approach to Nonlinear Inference

Having obtained that the relationship between GDP and oil prices is not linear, using the Hamilton’s non-linear test, and that the non-linear transformations proposed in the Literature are questionable, we shall now try to see what this relationship is like. To do so, we consider positive and negative changes in oil prices and use a non-linear function that relates GDP growth to oil prices. We estimate this function by means of kernel methods, avoiding the assumption of any particular distribution regarding the above function. We propose the use of a particular semiparametric model to reflect such a relationship. Next, we briefly point out Hamilton’s semiparametric specification (2001b), and finally, we highlight the advantage our model has over Hamilton’s.

5.1 Nonparametric approach

As a starting point, we consider a simple non-linear regression model of the form

\[ y_t = m(x_{t_k}) + \epsilon_t \]  

(4.1)

where \( y_t \) is the growth rate of chain-weighted real GDP; \( x_{t_k} \) is a uni-dimensional variable that refers to the growth rate of the nominal oil price\(^{29}\); \( k = 0; 1; \ldots; 4 \); \( \epsilon_t \) is the error term with \( E[\epsilon_t] = 0 \) and \( Var[\epsilon_t] = \sigma^2 \); \( m(\cdot) \) is an unknown function that relates the variables \( y_t \) to \( x_{t_k} \), and our goal is to estimate it. In fact, \( m(\cdot) \) is the conditional mean: \( m(x) = E(y_t|x_{t_k} = x) \), with \( x \) some fixed value of \( x_{t_k} \).

In order to estimate \( m(\cdot) \), we use the kernel methods. With such methods, one has to choose a density function \( k(x) \), the kernel function, and a smoothing parameter \( h \), the window width. In spite of the diversity of kernel estimates proposed in the Literature, to perform the above regression (4.1), we consider the one most used in Applied

\(^{29}\)We perform the regression for the current growth rate of the nominal oil price, \( \alpha_t \), and for each lagged growth rate: \( \alpha_{t_1}, \alpha_{t_2}, \alpha_{t_3}, \) and \( \alpha_{t_4} \).
Econometrics, Nadaraya-Watson\(^{30}\):

\[
m(x) = \frac{\sum_{i=1}^{n} \frac{k}{k \cdot x_{i}} \cdot \frac{y_i}{h}}{\sum_{i=1}^{n} \frac{k}{k \cdot x_{i}} \cdot \frac{y_i}{h}} \tag{4.2}
\]

There are different alternatives for choosing the kernel function, although the most efficient is the Epanechnikov kernel. Any other kernel (such as the squared, triangular, Gaussian and rectangular kernels) can be used with a minimal loss of efficiency. For this reason, we use the Gaussian kernel\(^{31}\),

\[
k(t) = \frac{1}{2\pi} \exp \left( -\frac{t^2}{2} \right).
\]

We must also choose the smoothing parameter, \(h\). To do so, we shall use the leave-one-out technique, in which one, say the \(j\)th, observation is left out

\[
m_j(x_j) = \frac{\sum_{i=1}^{n} \frac{k}{k \cdot x_{i}} \cdot \frac{y_i}{h}}{\sum_{i=1}^{n} \frac{k}{k \cdot x_{i}} \cdot \frac{y_i}{h}};
\]

with these modified smoothers, the \(h\) that minimizes the Cross Validation function is chosen, i.e.

\[
h_{\text{opt}} = \arg \min_h \frac{1}{n} \sum_{j=1}^{n} (y_j - m_j(x_j))^2 \tag{4.3}
\]

and we look for it in the interval\(^{32}\)

\[(h_{\text{opt1}}, h_{\text{opt2}}) = \left( \frac{1}{4}(1.06)^{3/4}n^{1/5}, \frac{3}{2}(1.06)^{3/4}n^{1/5} \right);
\]

with \(3/4\) being the standard deviation and \(n\) the sample size.

With this window width, we calculate \(m(x_{t_i}, k)\) with the Nadaraya-Watson estimator. We performed this process for different values of \(k\), \(k \in \{0, 1, \ldots, 4\}\). We observe that the nonparametric estimators that are based only on oil-price changes help us to account for

\(^{30}\)Nadaraya (1964) and Watson (1964).

\(^{31}\)We should emphasize that we are now considering the uni-dimensional case. When the multi-dimensional case is considered we shall then use the multi-dimensional Gaussian Kernel.

\(^{32}\)The estimation does not change if we consider a wider interval.
some of the negative movements of the GDP growth, but not for the positive ones. We can, therefore, observe the asymmetric response of GDP to oil-price changes. The above estimators also bring significant influence to bear on GDP growth, mainly in the oil crises considered in the sample. One of the most striking features of these estimations is the fact that the most significant negative influence of oil-prices on the GDP took place during the Arab-Israel War (1973), except when we use the first lag of oil price as explanatory variable (where the most one takes place during the Gulf War). It is noteworthy that the price-controls imposed by the U.S., after the 1973 oil crisis, led to a lesser dependence on imports during 1978-80, and the response to these crises was significantly less spectacular.

Although positive oil-price changes play an important role in the explanation some of the negative GDP movements, they are not enough to justify them plenty. This fact does not seem surprising, since the GDP growth depends on several different variables.

Now, we consider \( x_{t_i k} \) as a vector, i.e. \( x_{t_i k} = (o_t; o_{t-1}; o_{t-2}; o_{t-3}; o_{t-4})^0 \). The methodology is the same as the one for the uni-dimensional case. The results are shown in Figure 17. The most important aspect of the estimation is that it helps us to explain the GDP recessions that occurred during the oil crises considered. The tool’s accuracy is perfect for the oil crisis of 1973. Furthermore, unlike an OLS linear estimation of \( y_t \) on a constant, the current oil-price and four lags of oil price, \( x_{t_i k} \), which gives us a spurious positive movement of GDP in 1986 and mid-1987, the nonparametric estimation fully explains the true movements. Moreover, the linear estimation explains the recessions related to oil crises less accurately than the nonparametric estimation does. The main advantage that the multi-dimensional nonparametric specification has over the linear one, therefore, is its greater accuracy in reflecting the oil crises and the collapse of the petroleum market in the mid-80s.

The above remarks are maintained if we consider \( x_{t_i k} \) as a vector without considering \( k = 0 \), i.e. \( x_{t_i k} = (o_{t_i 1}; o_{t_i 2}; o_{t_i 3}; o_{t_i 4})^0 \).

Since these results are not entirely acceptable, as they only consider the growth rate of

---

33 The kernel function used is a multi-dimensional Gaussian kernel. The results should be considered with due caution, given the size of the sample.

34 It should be remembered that the average world oil prices fell by over 50 percent in 1986.
Figure 17

Nonparametric Estimation
(Multidimensional case)

Note: These figures represent the kernel nonparametric estimation in a multidimensional model, using the optimal bandwidth. The sample period runs from 1947:II to 2001:III.
the oil price as an explanatory variable, we shall now consider the lagged values of GDP growth as additional variables, which we shall include linearly.

5.2 Semiparametric Approach

5.2.1 Semiparametric Estimation of Partially Linear Model

Consider a non-linear model of the form

$$y_t = z_t^0 + g(v_t) + \varepsilon_t$$  \hspace{1cm} (4.4)

where $y_t$ is the growth rate of chain-weighted real GDP; $z_t$ is a $p$-dimensional vector with lags in GDP growth, for which linearity is assumed; $v_t$ is a $q$-dimensional vector which contains lags in oil price growth; $\bar{\theta}$ is a $p$-dimensional vector of unknown parameters; $g(\cdot)$ is an unknown one-dimensional regression function, $g : \mathbb{R}^q \mapsto \mathbb{R}$; and $\varepsilon_t$ is an error term such that $E[\varepsilon_t | z_t; v_t] = 0$.

Different ways to approximate nonparametric part may give the corresponding estimators of $\bar{\theta}$. We shall consider the estimation for $\bar{\theta}$ when kernel methods are considered$^{35}$.

In this paper, we compute a nonparametric estimate of $g(\cdot)$ and then construct a kernel-based estimator for the vector of unknown parameters $\bar{\theta}$.

Let $K(\cdot)$ be a kernel function satisfying certain conditions and $h$ be a window width parameter. Let $w_{T;i}(v) = w_i(v; V_1; \ldots; V_T)$ be a positive weight function depending on $v$ and the design points $V_1; \ldots; V_T$. The weight function is defined as

$$w_{T;i}(v) = \frac{K}{h} \prod_{j=1}^{p} \frac{K}{h} \frac{K}{h}$$

For every given $\bar{\theta}$, we define an estimator of $g(\cdot)$ by

$$g_T(v; \bar{\theta}) = \sum_{i=1}^{T} w_{T;i}(v)(y_i - z_t^0)$$

$^{35}$For identifiability, we assume that the pair $(\bar{\theta}; g)$ satisfies

$$E f(y_t; z_t^0; g(v_t))^2 = \min_{(\bar{\theta}; f)} E f(y_t; z_t^0; f(v_t))^2$$
Replacing \( g(v_t) \) by \( g_T(v_t; \hat{\cdot}) \) into (4.4) and using the LS criterion, then the least squares estimator of \( \hat{\cdot} \) is obtained as

\[
\hat{\cdot} = (Z^0\hat{Z})^{-1}Z^0\hat{y}:
\]

where \( Z^0 = (z_1; \ldots; z_T) \) with \( z_t = z_i \) i.e. \( \hat{E}[z_j \mid v_j] = z_j \) i.e. \( P_{i=1}^T w_{T,i}(v_j) z_j \), and \( \hat{y}^0 = (y_1; \ldots; y_T) \) with \( y_j = y_i \) i.e. \( \hat{E}[y_j \mid v_j] = y_j \) i.e. \( P_{i=1}^T w_{T,i}(v_j) y_j \). We estimate \( E[z_j \mid v_j] \) and \( E[y_j \mid v_j] \) by Nadaraya-Watson kernel estimators\(^{36}\). Furthermore, we use the product kernel \( K(v_t) = \prod_{l=1}^q k(v_t;l) \), where \( k(\cdot) \) is a univariate kernel and \( v_{t,i} \) is the \( i \)th component of \( v_t \):

As such, the nonparametric part \( g(v_t) \) is estimated by

\[
g_T(v) = \sum_{i=1}^q w_{T,i}(v)(y_i \mid z_i^{0\cdot}) = \hat{E}[y_j \mid v_j] \mid \hat{E}[z_j \mid v_j]^{0\cdot}
\]

We first consider \( p = 1 \) and \( q = 1 \) which gives us the following model

\[
y_t = \hat{\cdot} y_{t-1} + g(x_{t,i,k}) + e_t
\]

where \( \hat{\cdot} \) is a scalar parameter, \( x_{t,i,k} \) a uni-dimensional variable that represents the growth rate of the nominal price for oil with \( k = \{0; 1; \ldots; 4\} \).

As we pointed out at the beginning of this subsection, the choice of the bandwidth is of great importance, so we shall consider other window widths apart from the \( h \) which minimizes the Cross Validation function to see what happens. Although the results do not change essentially.

The results of the estimation with the \( h \) optimum are shown in Figure 18. The difference between the linear estimation\(^{37}\) and the semiparametric one is centered basically on three specific dates: those of the Yom Kippur War, the Persian Gulf War and the down movement of oil price in 1986. As such, the semiparametric estimation gives us a better fit for these three events.

We have, so far, considered uni-dimensional variables, but we would also like to know what would happen if vectors were allowed\(^{38}\).

\(^{36}\)For the nonparametric kernel estimator, we shall use the leave-one-out technique and we select the window width that minimizes Cross-Validation function.

\(^{37}\)It is an OLS linear estimation of \( y_t \) on a constant, its own immediate lag and the \( x_{t,i,k} \) corresponding to each \( k \).

\(^{38}\)It is necessary as an identification condition that \( x_{t,i,k} \) does not have unity as an element.
Note: These figures represent the kernel semiparametric estimation in a unidimensional model. The sample period runs from 1947:II to 2001:III.

Note: This figure plots the kernel semiparametric estimation in a multivariate model. The sample period runs from 1947:II to 2001:III.
5.2.2 Proposal of semiparametric specification

We shall now approach the relationship between changes in the price of crude oil and GDP growth through a semiparametric model of the following form

\[ y_t = -1y_{t-1} + -2y_{t-2} + -3y_{t-3} + -4y_{t-4} + g(o_{t-1}; \cdots; o_{t-4}) + \varepsilon_t; \tag{4.6} \]

and estimate it by means of kernel methods as we have been performing in the unidimensional case.

We look for the optimum in the interval \((h_{nopt1}; h_{nopt2})\), although the use of different \(h\) has been analyzed. The results of the estimation for \(h\) optimal are shown in Figure 19. We have also considered different values of \(h\) other than the optimum, but the results do not change significantly.

We shall now deal with the question of analyzing the semiparametric estimation using the \(h\) optimum:

\[ y_t = 0.26237310y_{t-1} + 0.091182855y_{t-2} + 0.058079640y_{t-3} \tag{4.7} \]
\[ + 0.16828811y_{t-4} + g(o_{t-1}; \cdots; o_{t-4}) \]

It is worth noting, at this point, that the periods in which the oil-price changes seem to have their greatest influence (i.e., the oil crises and the collapse of the petroleum market in 1986), the semiparametric estimation is a better approach than the linear one\(^{39}\), with a perfect fit in both the oil crisis of 1973 and in the collapse of the crude oil market. Furthermore, the linear estimation establishes a spurious increase in GDP growth during 1986 and 1987 the 1973 crisis much worse crisis. In fact, if we only consider the OLS linear estimation of \(y_t\) on a constant and its own four last lags, the GDP fluctuations on these dates can not be satisfactorily explained. We also compare the kernel semiparametric estimation with those of the non-linear specifications, and observe that the kernel

\(^{39}\) This conclusion is confirmed when we focus on the traditional forecasting measure, MSE/MSFE, observing that the kernel specification has a smaller MSE/MSFE (See Table 8). Moreover, the DM statistic, which is set up such that a positive value means that the kernel specification fits better than the other specifications considered, gives us the rejection, in-sample, of the null hypothesis of equal forecast accuracy (See Table 9).
### Table 7

<table>
<thead>
<tr>
<th>Specification</th>
<th>Our data set</th>
<th>Hamilton’s data set</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Linear</td>
<td>Non-linearity Test</td>
</tr>
<tr>
<td></td>
<td>(p-value)</td>
<td></td>
</tr>
<tr>
<td>Linear spec.</td>
<td>7.47\text{***}</td>
<td>39.99\text{***}</td>
</tr>
<tr>
<td>(0.00625)</td>
<td>(2.5AE { 010)</td>
<td></td>
</tr>
<tr>
<td>Mork spec.</td>
<td>0.05</td>
<td>2.98a</td>
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<tr>
<td>(0.62044)</td>
<td>(0.08434)</td>
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<td>0.51</td>
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<tr>
<td>(0.92681)</td>
<td>(0.47652)</td>
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<tr>
<td>NOPI spec.</td>
<td>0.21</td>
<td>3.84q</td>
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<tr>
<td>(0.64623)</td>
<td>(0.05008)</td>
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<tr>
<td>NOPI3 spec.</td>
<td>0.81</td>
<td>1.58</td>
</tr>
<tr>
<td>(0.36798)</td>
<td>(0.20826)</td>
<td></td>
</tr>
</tbody>
</table>

Note.- This Table reports the statistic value and the p-value of the non-linearity test performed in the full sample (1947:II-2001:III). p-values appear in parenthesis. One/two/three asterisks mean a p-value less than 10%/5%/1%.

### Table 8

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Linear</td>
<td>0.9288</td>
<td>0.7129</td>
<td>1.109119</td>
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<tr>
<td>Mork</td>
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<td>0.993695</td>
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<td>1.051724</td>
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<td>0.974486</td>
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<tr>
<td>NOPI3</td>
<td>0.8943</td>
<td>0.6759</td>
<td>0.945180</td>
</tr>
<tr>
<td>Hamilton Semipar.</td>
<td>0.9230</td>
<td>0.7318</td>
<td>-</td>
</tr>
<tr>
<td>Kernel Semiparam.</td>
<td>0.8916</td>
<td>0.6634</td>
<td>0.844287</td>
</tr>
</tbody>
</table>

Note.- Mean-Square Error and Mean-Square Forecast Error are defined as follows: $\text{MSFE} = E[\{(y_{T+1} - \hat{y}_{T+1})^2|I_T\}]$ and $\text{MSE} = E[\{(y_t - \hat{y}_t)^2|I_T\}]$, respectively, where $\hat{y}_t$ is the in-sample estimation, $\hat{y}_{T+1}$ is the one-period ahead out-of-sample forecasting, and $I_T$ is the available information in $T$. 

- \text{***} \quad \text{p}<0.01
- \text{**} \quad \text{0.01}<\text{p}<0.05
- \text{*} \quad \text{p}<0.1
<table>
<thead>
<tr>
<th>Diebold-Mariano Test</th>
<th>DM-$S_2$ test</th>
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<tbody>
<tr>
<td>$H_0$: equal forecast accuracy</td>
<td>(relative to Kernel Semi param: model)</td>
</tr>
<tr>
<td><strong>Statistics</strong></td>
<td><strong>($p$ value)</strong></td>
</tr>
<tr>
<td><strong>Linear</strong></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>+2.342$^{**}$</td>
</tr>
<tr>
<td>OUT</td>
<td>+1.081</td>
</tr>
<tr>
<td><strong>Mork</strong></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>+0.695</td>
</tr>
<tr>
<td>OUT</td>
<td>+0.705</td>
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<tr>
<td><strong>SOPI</strong></td>
<td></td>
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<tr>
<td>IN</td>
<td>$i$ 0.212</td>
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<tr>
<td>OUT</td>
<td>+0.871</td>
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<tr>
<td><strong>NOPI</strong></td>
<td></td>
</tr>
<tr>
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<tr>
<td>OUT</td>
<td>+0.634</td>
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<tr>
<td><strong>NOPI3</strong></td>
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<tr>
<td>IN</td>
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<tr>
<td>OUT</td>
<td>+0.517</td>
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<tr>
<td><strong>Hamilton Semip. spec.ation</strong></td>
<td></td>
</tr>
<tr>
<td>IN</td>
<td>+0.924</td>
</tr>
<tr>
<td>OUT</td>
<td>$i$</td>
</tr>
</tbody>
</table>

Note.—In-sample, the first column refers to the sample (1949:II-2001:III) where as the second one refers to the sample (1961:I-2001:III). Out-of-sample refers to the period that runs from 1972:II to 2001:III. The DM statistic tests the null hypothesis that there is any statistically significant difference between the in-column model and the linear or the kernel semi-parametric model. $p$-values based on two-sided tests appear in parenthesis (One/two/three asterisks mean a $p$-value less than 10%/5%/1%).
semiparametric estimation is better in-sample than the other specifications, considering
data from 1961:I onwards (See Table 8, second column). Furthermore, the one-period
ahead out-of-sample kernel forecasting improves those of the above-mentioned non-linear
transformations, in the sense that it has a smaller Mean-Square Forecast Error (See Table
8, third column).

We shall now compare in-sample Hamilton’s semiparametric specification to the kernel
specification.

5.2.3 Hamilton’s semiparametric specification

Hamilton (2000) proposed to consider the nonlinear model (3.1). He specifically consid-
ered

\[ r_t = \beta_0 + \beta_1 r_t + \varepsilon_t \]  

(4.8)

where \( \varepsilon_t \) is the realization of a scalar-valued Gaussian random variable with a mean of
zero, unit variance and covariance function, since \( k = 4 \), given by

\[ H_4(h_{st}) = 1 + (2-\alpha)^2 + 2h_{st}(1 + h_{st}^2)^{3/2} + h_{st}(1 + h_{st}^2)^{1/2} + 1/h_{st} \]

when \( h_{st} \), which is \((1=2)^{4/4} g_{st}^2 r_{is} r_{it}^2 \), does not exceed the unity, and zero
otherwise, with \( g_{st} \) governing the variability of the nonlinear part with respect to \( r_i = \beta_i \); i \( 2f1; \ldots; 4g \).

5.2.4 Hamilton’s approach versus our semiparametric kernel approach

Hamilton supposes that the function \( \varepsilon_t \) in (4.8) has a Gaussian distribution. However, the
semiparametric specification based on the kernel estimation method (4.6) does not
have such a distribution, but rather it is allowed more flexibility. We therefore obtain a

\( \beta_0 \) highlights the fact that the Lee-Ni-Ratti (1995) and Hamilton’s (1996) specifications
derive much of their apparent success from data in the 1950s. It is worth noting that the 1950s period is
one of relative stability in the oil price, with the only smooth movements in Suez Crisis (1956).
\( \beta_1 \) Despite the fact that the kernel specification has a smaller MSE/MSFE, and that we obtain positive
DM statistics, we cannot reject the null hypothesis of equal forecast accuracy (See Table 9).
\( \beta_2 \) It should be remembered that Hamilton (2001b) uses the generalization of the finite-difference
Brownian motion.
gain with the kernel model since \( g(.) \) is totally unrestricted with regard to any particular distribution. Although we use the Gaussian kernel function for our estimations in the semi-parametric kernel approach, any other kernel function may be considered (from the most efficient -Epanechnikov kernel function- to any other -squared, triangular or rectangular), and the process would still be valid. However, the normality assumption is necessary in any case with Hamilton’s approach. With the model proposed in the above subsection, we avoid assuming any particular distribution that might be inaccurate.

On observing the two estimations, we see that they are essentially different from 1980 onwards. Figures 20(a), 20(b) and 20(c), show that the kernel estimation is better in-sample than Hamilton’s is, as it has achieved greater accuracy (See Tables 8, second column, and 9)\(^{43}\).

6 Conclusions

In this paper, we present evidence of a non-linear relationship between GDP growth and changes in the price of crude oil. We argue that this non-linearity is not solely due to the use of data from the mid-1980s onwards, as many authors have been suggesting up to now. In particular, we find the existence of non-linearity with the use of data from before 1984, and indeed, even before 1977.

This paper also questions that the non-linear transformations of oil prices proposed in the Literature can reflect such non-linearity. We show that these transformations still do not solve the forecasting of a spurious increase in GDP growth in the mid-1980s. Furthermore, when data earlier than 1977 is considered, the non-linearity test shows that these specifications are not the most accurate in summarizing the non-linearity. It should also be pointed out that these transformations ignore oil-price declines, treating them as if nothing had happened, which is very questionable. There seems to be data-mining.

We also propose a semiparametric model in which GDP growth is explained by its own lags, linearly, and an unknown non-linear function that depends on the lags of the oil-price changes. We estimate it through kernel methods, avoiding any assumptions about its

\(^{43}\)Considering data from 1961:I onwards, we can in-sample reject the null hypothesis at a 10% critical level.
Note: These figures plot Hamilton’s (1980) and kernel semiparametric estimations from 1980 onwards, because these estimations are essentially different from this date onwards.
form. In-sample, the kernel estimation improves the linear estimation, and also improves both Hamilton’s (2001b) estimation and those of the above non-linear transformations, considering data from 1961:I onwards. Moreover, the one-period ahead out-of-sample kernel forecasting improves those of the above-mentioned non-linear transformations.
Appendix A

The distribution theory of the Hamilton’s test is asymptotic and has been derived under the assumption that the regressors are stationary. This excludes structural change in the marginal distribution of regressors. We observe that there is a structural change in the variance of oil price regressor variable.

We perform the Andrews (1993) and Andrews and Ploberger tests (1994), and we observe that the variance of oil price changes in 1973:I (we look for structural change from 1955:II to 1993:II).

<table>
<thead>
<tr>
<th>Estimated breakpoint: 1973:I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Structural Break Tests</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_0$: no structural change</td>
</tr>
<tr>
<td>Statistics</td>
</tr>
<tr>
<td>$(p_i \text{ value})$</td>
</tr>
</tbody>
</table>

We therefore realize that the results of the non-linearity test may change. When we perform this test in the full sample, we reject the null hypothesis of linearity with a p-value of 0.00625. Now, we consider a bootstrap by block with and without ..xed regressor bootstrap referred to oil price regressors.

We perform a bootstrap with 10,000 replications and blocks of six elements:

Step 1: Perform an OLS regression of $y_t$ on $x_t, z_t$ and a constant:

$y_t = X' + \hat{\theta}_t$.

Step 2: Calculate the OLS residuals, $\hat{\epsilon}_t$.

Step 3: Conduct a bootstrap block re-sampling residuals, $\epsilon^{\omega}_t$ (with and without seed).

Step 4: Generate 10,000 $y^{\omega}_t$:

- ..xed regressor bootstrap:

$y^{\omega}_t = \hat{\theta}_0 + \hat{\theta}_1 y^{\omega}_t 1 + \cdots + \hat{\theta}_4 y^{\omega}_t 4 + \hat{\omega}_1 \theta^{\omega}_t 1 + \cdots + \hat{\omega}_4 \theta^{\omega}_t 4 + \epsilon^{\omega}_t$

- non-..xed regressor bootstrap:

$y^{\omega}_t = \hat{\theta}_0 + \hat{\theta}_1 y^{\omega}_t 1 + \cdots + \hat{\theta}_4 y^{\omega}_t 4 + \hat{\omega}_1 \theta^{\omega}_t 1 + \cdots + \hat{\omega}_4 \theta^{\omega}_t 4 + \epsilon^{\omega}_t$

\[44\] We always consider the lags of GDP as ..xed regressors.

\[45\] Notice that $y_t$ is the real GDP growth, $x_t$ is a 4-dimensional vector which contains lags in oil price change, and $z_t$ is a 4-dimensional vector with lags in GDP growth.
Step 5: Calculate the Lagrange Multiplier statistic for each $y_t^*$:

We calculate the percentage of times we accept the null hypothesis at a 5% critical level.

<table>
<thead>
<tr>
<th>Number of replications</th>
<th>Fixed regressor bootstrap</th>
<th>Non-fixed regressor bootstrap</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>With Seed</td>
<td>Without Seed</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Times of acceptance</td>
<td>96.44 %</td>
<td>96.56 %</td>
</tr>
</tbody>
</table>

We observe that we accept the null hypothesis of linearity at a high percentage, indicating the fact that the asymptotic distribution of this test would be no unchanged to structural changes in the marginal distribution of regressors. For this reason, we should look at the results of this test with caution.
Data Appendix

The data used in this study, the sources, and all transformations are as follows. (Data are taken from the rst period of 1947, 1959, or 1960, up to 2000:III or 2001:III, depending on the case).

The United States:

GDP: Gross Domestic Product; Billions of chained 1996 dollars SAAR; NIPA; (Quarterly data); downloaded from the Bureau of Economic Analysis web page (http://www.bea.doc.gov/bea/dn/gdplev.htm); entered in log-differences.

ur: Standardised unemployment rate; Quarterly S.A., Percent; downloaded from the OECD Main Economic Indicators CD-ROM 2001.

poil: Price of West Texas Intermediate Crude, Monthly N.S.A., Dollars Per Barrel; from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter; entered in log-differences.

defator: Gross Domestic Product Implicit Price Deflator; (1996=100) S.A. (Quarterly data); from www.economagic.com; entered in log-differences.

CPI: All Urban Consumers-(CPI-U): U.S. city average: All items: 1982-84=100 (Monthly data); from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter; entered in log-differences.

Ir: Ten-year Treasury Constant Maturity (Monthly data); from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter.

fr: Federal Funds rate (Monthly data); downloaded from www.economagic.com; aggregated from monthly to quarterly using the monthly-average value of the quarter.

w: Average hourly earnings of production workers ;(Monthly data); downloaded from the Bureau of Labor Statistics (National Employment, Hours, and Earnings) web page (http://stats.bls.gov/datahome.htm); Seasonally Adjusted; aggregated from monthly to quarterly using the monthly-average value of the quarter; entered in log-differences.
References


