VOLATILITY TRANSMISSION PATTERNS AND TERRORIST ATTACKS

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ABSTRACT

The objective of this study is to analyze volatility transmission between the US and Eurozone stock markets considering the effects of the September 11, March 11 and July 7 financial crises. In order to do this, we use a multivariate GARCH model and take into account the asymmetric volatility phenomenon, the non-synchronous trading problem and the crises themselves. Moreover, a graphical analysis of the Asymmetric Volatility Impulse-Response Functions (AVIRF) is introduced, which takes into consideration the crisis effect. Results suggest that there is bidirectional and asymmetric volatility transmission and show the different impact that terrorist attacks had on both markets.

JEL Classification: C32; F30; G15

Keywords: International financial markets; Stock market crisis; Multivariate GARCH; Volatility spillovers.

RESUMEN

El objetivo de este estudio es analizar la transmisión de volatilidad entre los mercados de EEUU y la Eurozona, considerando el efecto de los ataques terroristas del 11 de septiembre, 11 de marzo y 7 de julio. Para ello, se utiliza un modelo GARCH multivariante, teniendo en cuenta el fenómeno de la volatilidad asimétrica y el problema de la negociación no simultánea. Asimismo, también se propone un análisis gráfico de la transmisión de volatilidad a través de funciones impulso-respuesta en volatilidad asimétricas (AVIRF) y que consideran la existencia o no de crisis financieras. Los resultados sugieren que la transmisión de volatilidad es asimétrica y bidireccional y muestran que los ataques terroristas tuvieron un impacto diferente en los mercados bursátiles considerados.

Palabras clave: Mercados financieros internacionales; Crisis financieras; GARCH multivariante; Transmisión de volatilidad.

1 Introduction

On September 11, 2001, March 11, 2004 and July 7, 2005, the cities of New York, Madrid and London experienced respectively devastating terrorist attacks. These attacks had an influence over several economic variables and they obviously affected financial markets. Taking into account the increasing global financial integration, an important question arises: How did these terrorist attacks affect interrelations between financial markets?

The main objective of this study is to analyze how volatility transmission patterns are affected by stock market crises. Moreover, we compare the different reactions of the markets to the particular terrorist attacks considered. In order to do this, we use a multivariate GARCH model and take into account both the asymmetric volatility phenomenon and the non-synchronous trading problem. In our empirical application, we focus on stock market crises as a result of terrorist attacks and analyze international volatility transmission between the US and Eurozone financial markets.

It must be highlighted that most existing studies on spillovers between developed countries focus on individual countries such as US, Canada, Japan, UK, France and Germany[1]. As far as we know, there are no many articles analyzing volatility transmission patterns between the US and the Eurozone as a global market. Moreover, this paper will be the first one to take into account the non-synchronous trading problem and to use a sample period that includes the September 11, March 11 and July 7 terrorist attacks.

As far as we know, no paper has analyzed until now the effects of the attacks of March 11 and July 7. Moreover, few studies have examined the effects of the attacks of September 11 on

[1] See Koutmous and Booth (1995), Karolyi (1995), Karolyi and Stulz (1996), Darbar and Deb (1997), Ramchand and Susmel (1998), Brooks and Henry (2000), Longin and Solnik (2001), Martens and Poon (2001) and Bera and Kim (2002), *inter alia*.

financial markets and they focus on the economy as a whole2 or in different concrete aspects of the economy. For instance, Poteshman (2006) analyzes whether there was unusual option market activity prior to the terrorist attacks. Ito and Lee (2005) and Blunk *et al*. (2006) assess the impact of the September 11 attack on US airline demand. Abadie and Dermisi (2006) analyze the effect of terrorist attacks on the real state. Glaser and Weber (2006) focus on how the terrorist attack influenced expected returns and volatility forecasts of individual investors. Chen and Siems (2004) investigate if terrorist and military attacks (including the September 11 attack) are associated with significant negative abnormal returns in global capital markets. Finally, Choudhry (2005) investigates the effects of the September 11 attack and the period after it on the time-varying beta of a few companies in the US. However, none of them analyzes volatility transmission patterns and how they have been affected by the event. As far as we know, the only papers that analyze changes in interrelations between stock markets are Hon *et al.* (2004) and Mun (2005), but they test whether the terrorist attack resulted in a change in correlation across global financial markets. We try to answer the following question: Were there differences in the reaction of the US and Eurozone stock markets to the different terrorist attacks considered? In order to do so, we propose a new version of the Asymmetric Volatility Impulse Response Functions (AVIRF) which takes into account stock market crises.

When studying asset price comovements and contagion between different financial markets, an important fact to take into account is the trading hours in each market. In the case of partially overlapping markets (like US and the Eurozone), a jump in prices can be observed in the first

[2] A special issue of the *Economic Policy Review* of the Federal Reserve Bank of New York (2002, Volume 8, Number 2) analyzes general economic consequences of September 11. A special issue of the *Journal of Risk and Uncertainty* (2003, Volume 26, Numbers 2/3) deals with the risks of terrorism with a special focus on September 11. A special issue of the *European Journal of Political Economy* (2004, Volume 20, Issue 2) deals with the economic consequences of terror.

market to open when the second one starts trading, reflecting information contained in the opening price. Therefore, this could make volatility increase in this first market. Moreover, as suggested by Hamao *et al.* (1990), a correlation analysis between partially overlapping markets using close to close (C-C) returns could produce false spillovers, both in mean and volatility. This is so because it is difficult to separate effects coming from the foreign market from those coming from the own market while it remains closed.

There are several solutions in order to artificially synchronize international markets. First of all, in the case of US, information transmission with other markets can be analyzed through American Depositary Receipts (ADRs), which will share trading hours with the US market. The problem is that there are no many ADRs, they are not actively traded and there are microstructure differences between the North American stock market and that from the original country [see Wongswan (2006)]. Some studies, such as Longin and Solnik (1995) and Ramchand and Susmel (1998), use weekly or monthly data in order to avoid the non-synchronous trading problem. However, the use of low frequency data leads to small samples, which is inefficient for multivariate modeling. On the other hand, some studies, such as Hamao *et al.* (1990) and Koutmos and Booth (1995), use daily non-synchronous open-to-close and close-to-open returns. Nevertheless, these studies cannot distinguish volatility spillovers from contemporaneous correlations. Finally, Martens and Poon (2001) use 16:00-to-16:00 synchronous stock market series in order to solve this problem. By doing this, they find a bidirectional spillover between US and France and between US and UK, contrary to previous studies that only found volatility spillovers from US to the other countries.

This study innovates with respect the existing literature in two ways. First, we study volatility transmission between US and the Eurozone using a sample period including the terrorist attacks occurred in New York, Madrid and London. As far as we know, these terrorist attacks have not yet been included in any paper analyzing volatility transmission in international markets. Second, we introduce a new version of Asymmetric Volatility Impulse Response Functions which takes into account stock market crises.

The rest of the paper is organized as follows. Section 2 presents the data and offers some preliminary analysis. Section 3 deals with the econometric approach and introduces the AVIRF with crises. Section 4 presents the empirical results and, finally, Section 5 summarizes the main results.

2 Data

The data consists of simultaneous daily stock market prices recorded at 15:00 GMT time for the US (S&P500 index) and the Eurozone (EuroStoxx50 index). At that time, the European markets are about to close and the US market has just started trading. We use stock market prices recorded at 15:00 GMT time, at the midpoint of the overlapping hours, in order to avoid the use of index prices recorded exactly at the open (US) and close (Eurozone) of trading.

The data is extracted from Visual Chart Group (www.visualchart.com) for the period January 18, 2000 to January 25, 2006. When there are no common trading days due to holidays in one of the markets, the index values recorded on the previous day are used.

Each terrorist attack considered had a different effect on financial markets. If we focus on the September 11 attack, both price indexes reached their minimum level on September 21. In the Eurozone, the EuroStoxx50 fell by 6.7% the day of the attack and between September 11 and September 21 was down 17.9%. The New York Stock Exchange did not open until September 17 and fell by 5.1%. Between that day and September 21, the S&P 500 decreased by 12.3%. In contrast with the effects of the September 11, the March 11 terrorist attack affected less both markets. The EuroStoxx50 decreased by 3.1% the day of the attack and, at the end of that month, it had returned to the pre-attack levels. In the same way, the S&P 500 suffered a small decline (1.5%) and recovered in less than a month. Finally, the July 7 attack had no effect on the S&P 500 and its impact on the EuroStoxx50 was small (1.7%). All in all, the three terrorist attacks affected much less the US market than the Eurozone market.

Figure 1 displays the daily evolution of the stock indexes S&P500 and EuroStoxx50 in the analyzed period.

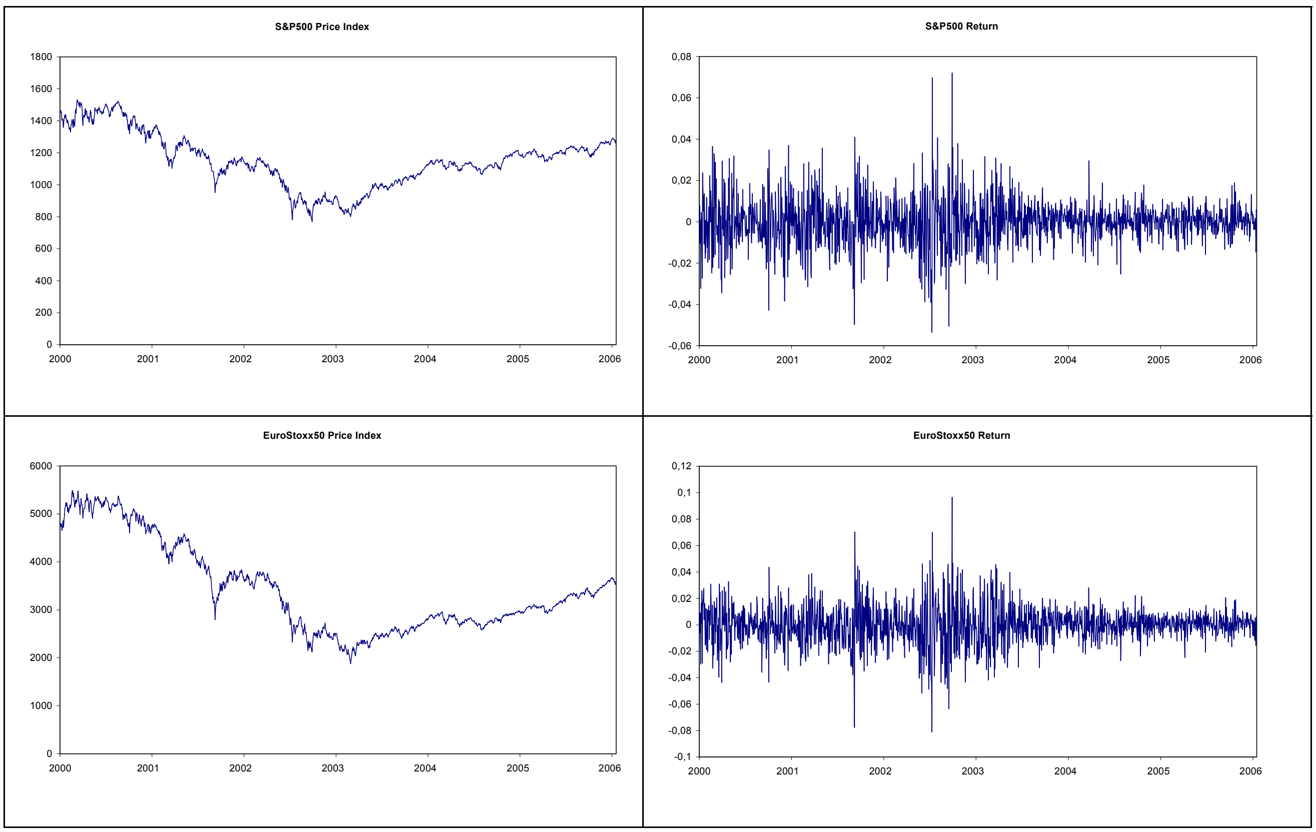

Figure 1. Price indexes and returns

Table 1 presents some summary statistics on the daily returns, which are defined as log differences of index values. The Jarque-Bera test rejects normality of the returns for both indexes. This is caused mainly by the excess kurtosis, suggesting that any model for equity returns should accommodate this characteristic of equity returns. The ARCH test reveals that returns exhibit conditional heteroskedasticity, while the Ljung-Box test (of twelfth order) indicates significant autocorrelation in both markets in squared returns but not in levels. Fat tails and non-normal distributions are common features of financial data. Finally, both the augmented Dickey Fuller (ADF) and Philips and Perron (PP) tests indicate that both series have a single unit root. Table 2 shows that both series are not cointegrated, being four the optimal lag length following the AIC criterion.

Table 1. Summary statistics.

	$R_{1,t}$	*p-value*	$R_{2,t}$	*p-value*
Mean	-0.00009		-0.00019	
Variance	0.00013		0.00021	
Skewness	0.11202	[0.0701]	0.00400	[0.9484]
Kurtosis	3.72923	[0.0000]	4.90041	[0.0000]
Bera-Jarque	782.423	[0.0000]	910.341	[0.0000]
Q(12)	23.2728	[0.0255]	28.8222	[0.0041]
Q²(12)	502.408	[0.0000]	842.236	[0.0000]
ARCH(12)	185.035	[0.0000]	255.721	[0.0000]
ADF(4)	-1.87522	[0.3443]	-1.52663	[0.5200]
PP(7)	-1.90664	[0.3295]	-1.53550	[0.5154]

Note: p-values displayed as [.]. $R_{1,t}$ and $R_{2,t}$ represent the log-returns of the S&P500 and the EuroStoxx50 indexes. The Bera-Jarque statistic tests for the normal distribution hypothesis and has an asymptotic distribution $X^2(2)$. Q(12) and Q²(12) are Ljung-Box tests for twelfth order serial correlation in the returns and squared returns. ARCH(12) is Engle's test for twelfth order ARCH, distributed as $X^2(12)$. The ADF (number of lags) and PP (truncation lag) refer to the Augmented Dickey and Fuller (1981) and Phillips and Perron (1988) unit root tests. Critical value at 5% significance level of Mackinnon (1991) for the ADF and PP tests (process with intercept but without trend) is -2.86.

Table 2. Johansen (1988) tests for cointegration.

Lags	Null	$\lambda_{trace}(r)$	Critical Value	$\lambda_{\max}(r)$	Critical Value
4	r = 0	11.81020	20.26184	7.685361	15.89
	r = 1	4.124843	9.16	4.124843	9.16

Note: The lag length is determined using the AIC criterion. λ_{trace} (r) tests the null hypothesis that there are at most r cointegration relationships against the alternative that the number of cointegration vectors is greater than r. λ_{max} (r) tests the null hypothesis that there are r cointegration relationships against the alternative that the number of cointegration vectors is greater than r + 1. Critical values are from Osterwald-Lenum (1992).

3 The Econometric Approach

3.1 The model

The econometric model is estimated in a three-step procedure. First, a VAR model is estimated to clean up any autocorrelation behavior. Then, the residuals of the model are orthogonalized. These orthogonalized innovations have the convenient property that they are uncorrelated both across time and across markets. Finally, the orthogonalized innovations will be used as an input to estimate a multivariate asymmetric GARCH model.

In order to take into account the September 11, March 11 and July 7 terrorist attacks, three dummy series are introduced in the conditional mean equations. These dummies equal one the days following the terrorist attacks in New York, Madrid and London respectively until the days where the indexes take their lowest values, and 0 otherwise.

Equation (1) models the mean equation as a VAR(5) process:

$$
\begin{aligned}
R_{1,t} &= \mu_1 + x_1 S11_t + y_1 M11_t + z_1 J7_t + \sum_{p=1}^{5} d_{11,p} R_{1,t-p} + \sum_{p=1}^{5} d_{12,p} R_{2,t-p} + u_{1,t} \\
R_{2,t} &= \mu_2 + x_2 S11_t + y_2 M11_t + z_2 J7_t + \sum_{p=1}^{5} d_{21,p} R_{1,t-p} + \sum_{p=1}^{5} d_{22,p} R_{2,t-p} + u_{2,t}
\end{aligned}
\qquad (1)
$$

where $R_{1,t}$ and $R_{2,t}$ are US and Eurozone returns, respectively, μ_i, x_i, y_i, z_i and $d_{ij,p}$ for $i,j=1,2$ and p=1,…,5 are the parameters to be estimated and $S11_t$, $M11_t$ and $J7_t$ are dummy series for the terrorist attacks. Finally, $u_{1,t}$ and $u_{2,t}$ are the non-orthogonal innovations. The VAR lag has been chosen following the AIC criterion.

The innovations $u_{1,t}$ and $u_{2,t}$ are non-orthogonal because, in general, the covariance matrix $\sum = E\left(u_t u_t^{'}\right)$ is not diagonal. In order to overcome this problem, in a second step, the non-orthogonal innovations ($u_{1,t}$ and $u_{2,t}$) are orthogonalized ($\varepsilon_{1,t}$ and $\varepsilon_{2,t}$). If we choose any matrix *M* so that $M^{-1} \sum M^{'-1} = I$, then the new innovations:

$$\varepsilon_t = u_t M^{-1} \tag{2}$$

satisfy $E\left(\varepsilon_t \varepsilon_t^{'}\right) = I$. These orthogonalized innovations have the convenient property that they are uncorrelated both across time and across equations. Such a matrix *M* can be any solution of $MM^{'} = \sum$.

To model the conditional variance-covariance matrix we use an asymmetric version of the BEKK model [Baba *et al.* (1989), Engle and Kroner (1995) and Kroner and Ng (1998)]. As done in the mean equations, we introduce dummy series in order to take into account the terrorist attacks.

The compacted form of this model is:

$$H_t = C'C + B'H_{t-1}B + A'\varepsilon_{t-1}\varepsilon_{t-1}^{'}A + G'\eta_{t-1}\eta_{t-1}^{'}G + S'\delta_{t-1}\delta_{t-1}^{'}S + M'\xi_{t-1}\xi_{t-1}^{'}M + L'\vartheta_{t-1}\vartheta_{t-1}^{'}L \tag{3}$$

where *C, B, A, G, S, M* and *L* are matrices of parameters to be estimated, being *C* upper-triangular and positive definite and H_t is the conditional variance-covariance matrix in *t*.

In the bivariate case, the BEKK model is written as follows:

$$\begin{bmatrix} h_{11t} & h_{12t} \\ \cdot & h_{22t} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix}^{'} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}^{'} \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ \cdot & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} + \tag{4}$$

$$+\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}'\begin{bmatrix} \varepsilon_{1,t-1}^2 & \varepsilon_{1,t-1}\varepsilon_{2,t-1} \\ \cdot & \varepsilon_{2,t-1}^2 \end{bmatrix}\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}+\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}'\begin{bmatrix} \eta_{1,t-1}^2 & \eta_{1,t-1}\eta_{2,t-1} \\ \cdot & \eta_{2,t-1}^2 \end{bmatrix}\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}+$$
$$+\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}'\begin{bmatrix} \delta_{1,t-1}^2 & \delta_{1,t-1}\delta_{2,t-1} \\ \cdot & \delta_{2,t-1}^2 \end{bmatrix}\begin{bmatrix} s_{11} & s_{12} \\ s_{21} & s_{22} \end{bmatrix}+\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}'\begin{bmatrix} \xi_{1,t-1}^2 & \xi_{1,t-1}\xi_{2,t-1} \\ \cdot & \xi_{2,t-1}^2 \end{bmatrix}\begin{bmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{bmatrix}+$$
$$+\begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix}'\begin{bmatrix} \vartheta_{1,t-1}^2 & \vartheta_{1,t-1}\vartheta_{2,t-1} \\ \cdot & \vartheta_{2,t-1}^2 \end{bmatrix}\begin{bmatrix} l_{11} & l_{12} \\ l_{21} & l_{22} \end{bmatrix}$$

where $c_{i,j}, b_{i,j}, a_{i,j}, g_{i,j}, s_{i,j}, m_{i,j}$ and $l_{i,j}$ for all *i,j=1,2* are parameters, $\varepsilon_{1,t}$ and $\varepsilon_{2.t}$ are the unexpected shock series coming from equation (2), $\eta_{1,t} = \max[0, -\varepsilon_{1,t}]$ and $\eta_{2,t} = \max[0, -\varepsilon_{2,t}]$ are the Glosten *et al.* (1993) dummy series collecting a negative asymmetry from the shocks and, finally, $h_{ij,t}$ for all *i,j=1*, are the conditional second moment series. Similarly to $\eta_{i,t}$, the variables $\delta_{i,t}$, $\xi_{i,t}$ and $\vartheta_{i,t}$ for all *i=1,2* are the dummy series for the terrorist attacks. They take the values of the shocks the days following the terrorist attacks in New York, Madrid and London respectively, until the days where the indexes take their lowest values and 0 otherwise.

Equation (4) allows for both own-market and cross-market influences in the conditional variance, therefore allowing the analysis of volatility spillovers between both markets. Moreover, the BEKK model guarantees by construction that the variance-covariance matrix will be positive definite.

In equation (4), parameters $c_{i,j}, b_{i,j}, a_{i,j}, g_{i,j}, s_{i,j}, m_{i,j}$ and $l_{i,j}$ for all *i,j=1,2* can not be interpreted individually. Instead, we have to interpret the non-linear functions of the parameters which form the intercept terms and the coefficients of the lagged variances, covariances and error terms. We follow Kearney and Patton (2000) and calculate the expected value and the standard error of those non-linear functions. The expected value of a non-linear function of random variables is calculated as the function of the expected value of the variables, if the estimated variables are unbiased. In order to calculate the standard errors of the function, a first-order Taylor approximation is used. This linearizes the function by using the variance-covariance matrix of the parameters as well as the mean and standard error vectors.

The parameters of the bivariate BEKK system are estimated by maximizing the conditional log-likelihood function:

$$L(\theta) = -\frac{TN}{2}\ln(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\left(\ln\left|H_t(\theta)\right| + \varepsilon_t^{'} H_t^{-1}(\theta)\varepsilon_t\right)$$

where T is the number of observations, N is the number of variables in the system and θ denotes the vector of all the parameters to be estimated. Numerical maximization techniques were used to maximize this non-linear log likelihood function based on the BFGS algorithm.

In order to estimate the model in equations (1) and (3), it is assumed that the vector of innovations is conditionally normal and a quasi-maximum likelihood method is applied. Bollerslev and Wooldridge (1992) show that the standard errors calculated using this method are robust even when the normality assumption is violated.

3.2 Asymmetric Volatility Impulse Response Functions (AVIRF) with crisis

The Volatility Impulse-Response Function (VIRF), proposed by Lin (1997), is a useful methodology for obtaining information on the second moment interaction between related markets. The VIRF, AVIRF and our proposed crisis version, measure the impact of an unexpected shock on the predicted volatility. This is:

$$R_{s,3} = \frac{\partial vechE\left[H_{t+s} \mid \psi_t\right]}{\partial dg(\varepsilon_t \varepsilon'_t)} \tag{5}$$

where $R_{s,3}$ is a 3x2 matrix, $s=1,2,...$ is the lead indicator for the conditioning expectation operator, H_t is the 2x2 conditional covariance matrix, $\partial dg(\varepsilon_t \varepsilon'_t) = (\varepsilon^2{}_{1,t}, \varepsilon^2{}_{2,t})'$, ψ_t is the set of conditioning information. The *vech* operator transforms a symmetric *NxN* matrix into a vector by stacking each column of the matrix underneath the other and eliminating all supradiagonal elements.

In volatility symmetric structures, it is not necessary to distinguish between positive and negative shocks, but with asymmetric structures the VIRF can change with the sign of the shock. The asymmetric VIRF (AVIRF) for the asymmetric BEKK model is introduced in Meneu and Torró (2003). Similarly, it would be interesting to distinguish between periods of relative stability and periods of financial distress. Therefore, in this article we introduce a version of the AVIRF which takes into account periods of stock market crisis. By applying (5) to (3), we obtain:

$$R_{s,3}^{+} = \begin{cases} a & s=1 \\ (a+b+1/2g+\alpha w)R_{s-1,3}^{+} & s>1 \end{cases} \tag{6}$$

$$R_{s,3}^{-} = \begin{cases} a+g & s=1 \\ (a+b+1/2g+\alpha w)R_{s-1,3}^{-} & s>1 \end{cases} \tag{7}$$

$$R_{s,3}^{+,c} = \begin{cases} a+w & s=1 \\ (a+b+1/2g+\alpha w)R_{s-1,3}^{+} & s>1 \end{cases} \tag{8}$$

$$R_{s,3}^{-,c} = \begin{cases} a+g+w & s=1 \\ (a+b+1/2g+\alpha w)R_{s-1,3}^{-} & s>1 \end{cases} \tag{9}$$

where $R_{s,3}^{+}(R_{s,3}^{-})$ represents the VIRF for positive (negative) initial shocks in periods of stability, $R_{s,3}^{+,c}(R_{s,3}^{-,c})$ represents the VIRF for positive (negative) initial shocks in periods of stock market crisis, *a, b* and *g* are 3x3 parameter matrices, α is the probability of occurrence of a crisis and *w* is a 3x3 parameter matrix that, in our case, equals *s*, *m* and *l* during the September 11, March 11 and July 7 terrorist attacks, respectively. Moreover, $a = D_N^+(A'\otimes A')D_N$, $b = D_N^+(B'\otimes B')D_N$, $g = D_N^+(G'\otimes G')D_N$ and $w = D_N^+(W'\otimes W')D_N$, where D_N is a duplication matrix, D_N^+ is its Moore-Penrose inverse and $\otimes$ denotes the Kronecker product between matrices, that is:

$$D_N = \begin{bmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,1\,0 \\ 0\,0\,1 \end{bmatrix} \qquad D_N^+ = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/2 & 0 & 0 \\ 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

It is important to note that this impulse response function examines how fast asset prices can incorporate new information. This fact lets us test for the speed of adjustment, analyze the dependence of volatilities across the returns of the S&P500 and the EusoStoxx50, distinguish between negative and positive shocks and distinguish between crisis periods and non-crisis periods.

4 Empirical Results

4.1 Model estimation

Table 3 displays the estimated BEKK model of equation (3). In order to keep an appropriate length of the paper the results of the estimated VAR(5) are not included, although they are available upon request. The low p-values obtained for most of the parameters show that the model fits well the data. Table 4 shows the standardized residuals analysis. It can be observed that the standardized residuals appear free from serial correlation and heteroskedasticity.

Table 3. Estimation results

Multivariate GARCH model estimation

$$C = \begin{bmatrix} \underset{(0.00)}{-0.001006} & \underset{(0.70)}{0.000017} \\ & \underset{(0.00)}{-0.000511} \end{bmatrix} \qquad B = \begin{bmatrix} \underset{(0.00)}{0.950495} & \underset{(0.07)}{0.001525} \\ \underset{(0.00)}{-0.008417} & \underset{(0.00)}{0.967856} \end{bmatrix}$$

$$A = \begin{bmatrix} \underset{(0.00)}{-0.046510} & \underset{(0.00)}{0.118528} \\ \underset{(0.00)}{0.202174} & \underset{(0.00)}{-0.098272} \end{bmatrix} \qquad G = \begin{bmatrix} \underset{(0.00)}{0.295050} & \underset{(0.00)}{0.034440} \\ \underset{(0.00)}{0.105456} & \underset{(0.00)}{0.202836} \end{bmatrix}$$

$$S = \begin{bmatrix} \underset{(0.00)}{0.114195} & \underset{(0.87)}{-0.018969} \\ \underset{(0.00)}{-0.192744} & \underset{(0.00)}{-0.152630} \end{bmatrix} \qquad M = \begin{bmatrix} \underset{(0.43)}{0.042863} & \underset{(0.00)}{0.320945} \\ \underset{(0.23)}{-0.105324} & \underset{(0.69)}{0.043739} \end{bmatrix}$$

$$L = \begin{bmatrix} \underset{(0.29)}{-0.199620} & \underset{(0.00)}{1.307097} \\ \underset{(0.92)}{0.091490} & \underset{(0.00)}{-6.347604} \end{bmatrix}$$

Note: This table shows the estimation of the model defined in equation (3). P-values appear in brackets. The necessary conditions for the stationarity of the process are satisfied.

Table 4. Summary statistics for the standardized residuals of the model.

	$\varepsilon_{1,t} / \sqrt{h_{11,t}}$		$\varepsilon_{2,t} / \sqrt{h_{22,t}}$	
Q(12)	12.41548	[0.41291]	4.36900	[0.97582]
Q²(12)	11.23055	[0.50927]	13.43020	[0.33856]
ARCH(12)	5.903165	[0.92088]	7.484829	[0.82398]

Note: Q(12) and Q²(12) are Ljung-Box tests for twelfth order serial correlation in the standardized residuals and squared residuals. ARCH(12) is Engle's test for twelfth order ARCH, distributed as $\chi^2(12)$. The p-value of these tests are displayed as [.].

As it has been mentioned above, the parameters of Table 3 can not be interpreted individually. Instead, we have to focus on the non-linear functions that form the intercept terms and the coefficients of the lagged variance, covariance and error terms. Table 5 displays the expected value and the standard errors of these non-linear functions.

Table 5. Results of the linearized multivariate BEKK model.

S&P500 conditional variance equation

$h_{11,t} =$									
$1{,}01\text{x}10^{-6}$	$+0{,}9034\, h_{11,t-1}$	$-0{,}0160\, h_{12,t-1}$	$+7{,}0845\text{x}10^{-5}\, h_{22,t-1}$	$+0{,}0021\varepsilon_{1,t-1}^2$	$-0{,}0188\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	$+0{,}0408\varepsilon_{2,t-1}^2$	$+0{,}0870\eta_{1,t-1}^2$	$+0{,}0617\eta_{1,t-1}\eta_{2,t-1}$	$+0{,}0109\eta_{2,t-1}^2 +$
$1{,}10\text{x}10^{-7}$	0,0036	0,0019	1,7103	0,0005	0,0024	0,0031	0,0043	0,0069	0,0021
(9,1454)	(244,72)	(−8,3242)	(4,1422)	(3,9943)	(−7,7722)	(12,864)	(19,968)	(8,8912)	(5,1711)

$+0{,}01419\,\delta_{1,t-1}^2$	$-0{,}04592\,\delta_{1,t-1}\delta_{2,t-1}$	$+0{,}0371\,\delta_{2,t-1}^2$	$+0{,}0018\xi_{1,t-1}^2$	$-0{,}0090\xi_{1,t-1}\xi_{2,t-1}$	$+0{,}0112\xi_{2,t-1}^2$	$+0{,}0398\vartheta_{1,t-1}^2$	$-0{,}365\vartheta_{1,t-1}\vartheta_{2,t-1}$	$+0{,}0083\vartheta_{2,t-1}^2$
0,0030	0,0028	0,0096	0,0047	0,0070	0,0188	0,0767	0,3712	0,1708
(4,6098)	(−16,105)	(3,8458)	(0,3902)	(−1,2803)	(0,5957)	(0,5190)	(−0,0983)	(0,0489)

EuroStoxx50 conditional variance equation

$h_{22,t} =$									
$2{,}61x10^{-7}$	$+2{,}32x10^{-6}h_{11,t-1}$	$+0{,}0029h_{12,t-1}$	$+0{,}9387h_{22,t-1}$	$+0{,}0140\varepsilon_{1,t-1}^2$	$-0{,}0232\varepsilon_{1,t-1}\varepsilon_{2,t-1}$	$+0{,}0096\varepsilon_{2,t-1}^2$	$+0{,}0011\eta_{1,t-1}^2$	$+0{,}0139\eta_{1,t-1}\eta_{2,t-1}$	$+0{,}0411\eta_{2,t-1}^2 +$
$5{,}22x10^{-7}$	$2{,}61x10^{-6}$	0,0016	0,0040	0,0006	0,0013	0,0010	0,0003	0,00245	0,0027
(0,5005)	(0,8897)	(1,7836)	(232,43)	(21,3179)	(−17,7817)	(9,3201)	(3,3495)	(5,7023)	(14,7990)

$+0{,}0003\,\delta_{1,t-1}^2$	$-0{,}0057\,\delta_{1,t-1}\delta_{2,t-1}$	$+0{,}0232\,\delta_{2,t-1}^2$	$+0{,}1030\xi_{1,t-1}^2$	$+0{,}0280\xi_{1,t-1}\xi_{2,t-1}$	$+0{,}0019\xi_{2,t-1}^2$	$+1{,}7085\vartheta_{1,t-1}^2$	$-16{,}5938\vartheta_{1,t-1}\vartheta_{2,t-1}$	$+40{,}2920\vartheta_{2,t-1}^2$
0,0044	0,0362	0,0098	0,0209	0,0726	0,0097	0,1203	1,5037	4,8040
(0,0813)	(−0,1599)	(2,3750)	(4,9072)	(0,3865)	(0,1960)	(14,1995)	(−11,0350)	(8,3871)

Note: h_{11} and h_{22} denote the conditional variance for the S&P500 and EuroStoxx50 return series, respectively. Below the estimated coefficients are the standard errors, with the corresponding t-values given in parentheses.

The expected value is obtained taking expectations to the non-linear functions, therefore involving the estimated variance-covariance matrix of the parameters. In order to calculate the standard errors, the function must be linearised using first order Taylor series expansion. This is sometimes called the "delta method". When a variable Y is a function of a variable X, i.e., Y =F(X), the delta method allows us to obtain approximate formulation of the variance of Y if: (1) Y is differentiable with respect to X and (2) the variance of X is known. Therefore:

$$V(Y) \approx (\Delta Y)^2 \approx \left(\frac{\partial Y}{\partial X}\right)^2 (\Delta X)^2 \approx \left(\frac{\partial Y}{\partial X}\right)^2 V(X)$$

When a variable Y is a function of variables X and Z in the form of Y = F(X, Z), we can obtain approximate formulation of the variance of Y if: (1) Y is differentiable with respect to X and Z and (2) the variance of X and Z and the covariance between X and Z are known. This is:

$$V(Y) \approx \left(\frac{\partial Y}{\partial X}\right)^2 V(X) + \left(\frac{\partial Y}{\partial Z}\right)^2 V(Z) + 2\left(\frac{\partial Y}{\partial X}\right)\left(\frac{\partial Y}{\partial Z}\right) Cov(X, Z)$$

Once the variances are calculated it is straightforward to calculate the standard errors.

The S&P500 volatility is directly affected by its own volatility $(h_{1,1})$ and by the EuroStoxx50 volatility $(h_{2,2})$. Our findings suggest that the S&P500 volatility is affected by its own shocks (ε_1^2) and the EuroStoxx50 shocks (ε_2^2). Finally, the coefficient for its own asymmetric term (η_1^2) and the EuroStoxx50 asymmetric term are significant (η_2^2), indicating that negative shocks on any market affect more volatility than positive shocks.

The behavior of the EuroStoxx50 volatility does not differ much from that of the S&P500. The EuroStoxx50 volatility is affected by its own volatility $(h_{2,2})$, but not by the S&P500 volatility[4]. Interestingly, the EuroStoxx50 volatility is affected by the S&P500 shocks (ε_1^2) and its own shocks (ε_2^2). Finally, the coefficient for its own asymmetric term (η_1^2) and the EuroStoxx50 asymmetric term are significant (η_2^2), indicating that negative shocks on any market affect more volatility than positive shocks.

Regarding dummies, from the analysis of the coefficients significance, the most appealing results are: (1) the September 11 terrorist attack had an influence over volatility of both the US and Eurozone markets, although in the case of the Eurozone, the effect was indirectly transmitted through its own shocks. (2) Both the March 11 and July 7 terrorist attacks did not affect the S&P500 volatility. (3) The July 7 terrorist attack in London had an effect over volatility in the Eurozone. However, the March 11 terrorist attack only affected volatility in the Eurozone indirectly through shocks coming from the S&P500.

In general, there is bidirectional volatility transmission between the US and the Eurozone stock markets. However, the terrorist attack occurred in New York in September 11 affected

[4] This could be due to the fact that we use prices recorded at 15:00 GMT, when European markets are about to close and the US market has just started trading.

volatility in the Eurozone stock markets but the terrorist attacks occurred in Madrid and London in March 11 and July 7 respectively did not affect volatility in the US market.

4.2 Asymmetric Volatility Impulse Response Functions (AVIRF) with crisis

Figures 2 to 6 present the AVIRFs with crisis, computed following Lin (1997) and Meneu and Torró (2003), as explained in section 3.2. Results add evidence in favor of the bidirectional volatility transmission between the US and the Eurozone stock markets and the different impact that the terrorist attacks had on both markets. These graphical representations also allow us to test for the speed of adjustment, analyze the dependence of volatilities across the returns of the S&P500 and the EusoStoxx50, distinguish between negative and positive shocks and distinguish between crisis periods and non-crisis periods.

Figure 2 represents the AVIRF when unexpected shocks are positive and there is a period of financial stability as opposed to stock market crisis periods caused by terrorist attacks. The graphical analysis shows that there exist bidirectional volatility spillovers between the S&P500 and the EuroStoxx50 (about 4% and 1.5% of the shock, respectively, Figures 2B and 2C). Positive shocks in the EuroStoxx50 have a relatively small effect on its own volatility (Figure 2D), whereas past positive shocks in the S&P500 have no effect on current volatility (Figure 2A).

If unexpected shocks are negative and there is a period of financial stability, Figure 3 shows that there are also bidirectional volatility spillovers between the S&P500 and the EuroStoxx50 (Figures 3B and 3C). Negative shocks in the S&P500 have an important effect on its own volatility (Figure 3A). Negative shocks in the EuroStoxx50 also have an important effect on its own volatility (Figure 3D), though they are less important than in the case of the S&P500. It is interesting to note that own positive shocks do not have any effect on S&P500 volatility, whereas own negative shocks have a very significant effect. In all cases, there is evidence of asymmetry: negative socks have a higher effect on volatility than positive shocks. The only exception is the effect of shocks from the S&P500 on the EuroStoxx50, where both kinds of shock have a similar and relatively small impact on volatility.

One of the most appealing contributions of the new version of the AVIRF introduced in this paper is that it allows to differentiate between periods of relative financial stability and periods of stock market crisis caused, in this case, by terrorist attacks. Figure 4 represents the AVIRF to negative unexpected shocks during the crisis period produced by the September 11 terrorist attack. Similarly, Figures 5 and 6 represent the AVIRF to negative unexpected shocks during the March 11 and July 7 crisis periods, respectively. In order to interpret these graphs, it is important to compare the figures with those obtained in Figure 3, AVIRF to negative unexpected shocks in a no-crisis period.

In general, the most appealing results are: (1) Conditional variances are more sensitive to negative than to positive shocks; (2) The September 11 terrorist attack (Figure 4) had an influence over volatility of both the US and Eurozone markets, because all figures have increased their initial response to a shock when compared to Figure 3. In the case of the Eurozone, the effect was indirectly transmitted through its own shocks (Figure 4D). (3) Both the March 11 and July 7 terrorist attacks did not affect the S&P500 volatility (Figures 5A, 5B, 6A and 6B are either non-significative or they do not change when compared to Figure 3). (4) The March 11 and July 7 terrorist attacks had an effect over volatility in the Eurozone (Figures 5C, 5D, 6C and 6D). However, the March 11 terrorist attack (Figure 5) only affected volatility in the Eurozone indirectly through shocks coming from the S&P500 (Figure 5C), as Figure 5D does not change when compared to Figure 3D.

Figure 2A. A positive shock in the S&P500

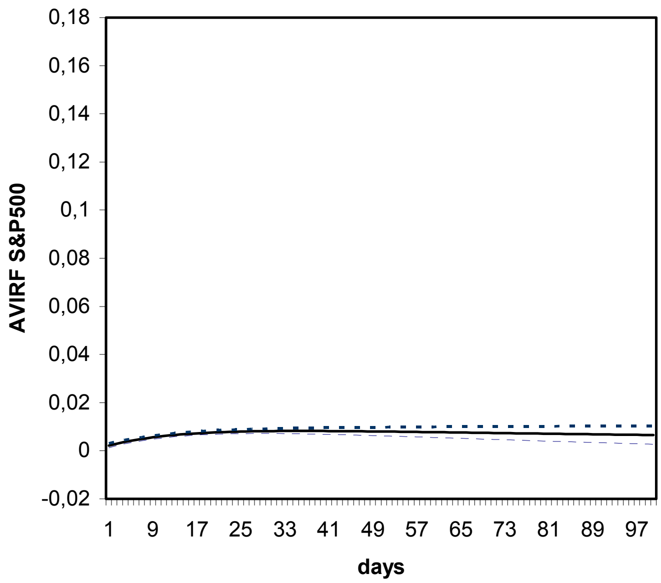

Figure 2B. A positive shock in the EuroStoxx50

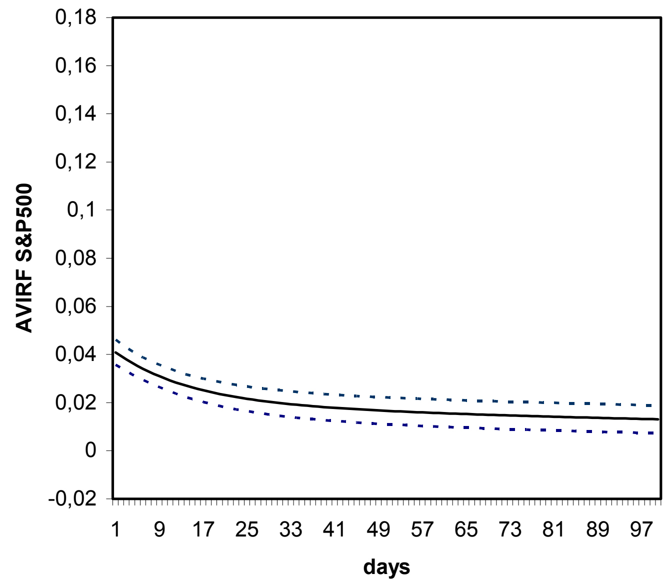

Figure 2C. A positive shock in the S&P500

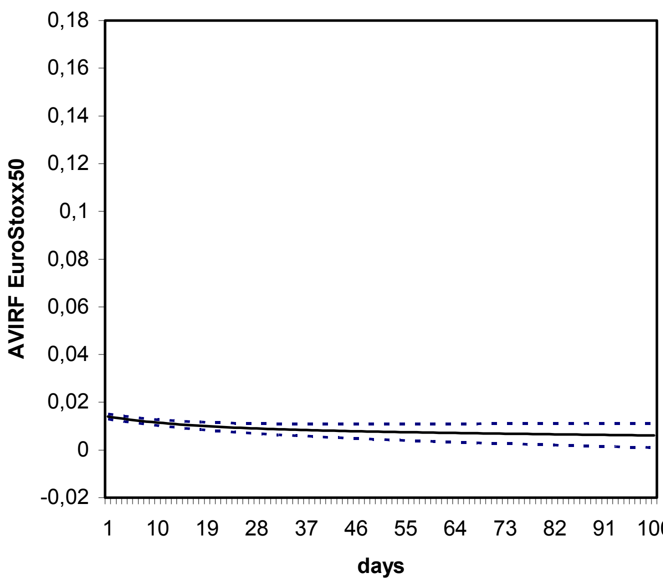

Figure 2D. A positive shock in the EuroStoxx50

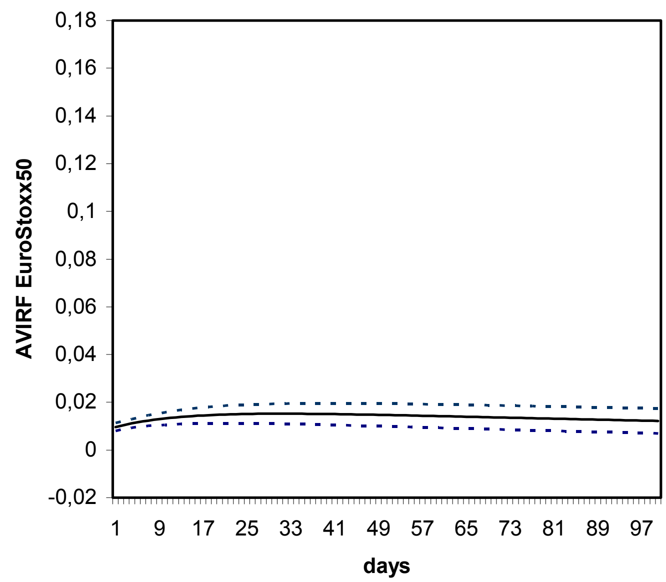

Figure 2. AVIRF to positive unexpected shocks from the VAR-Asymmetric BEKK
No Crisis Period
(Dashed lines display the 90% confidence interval)

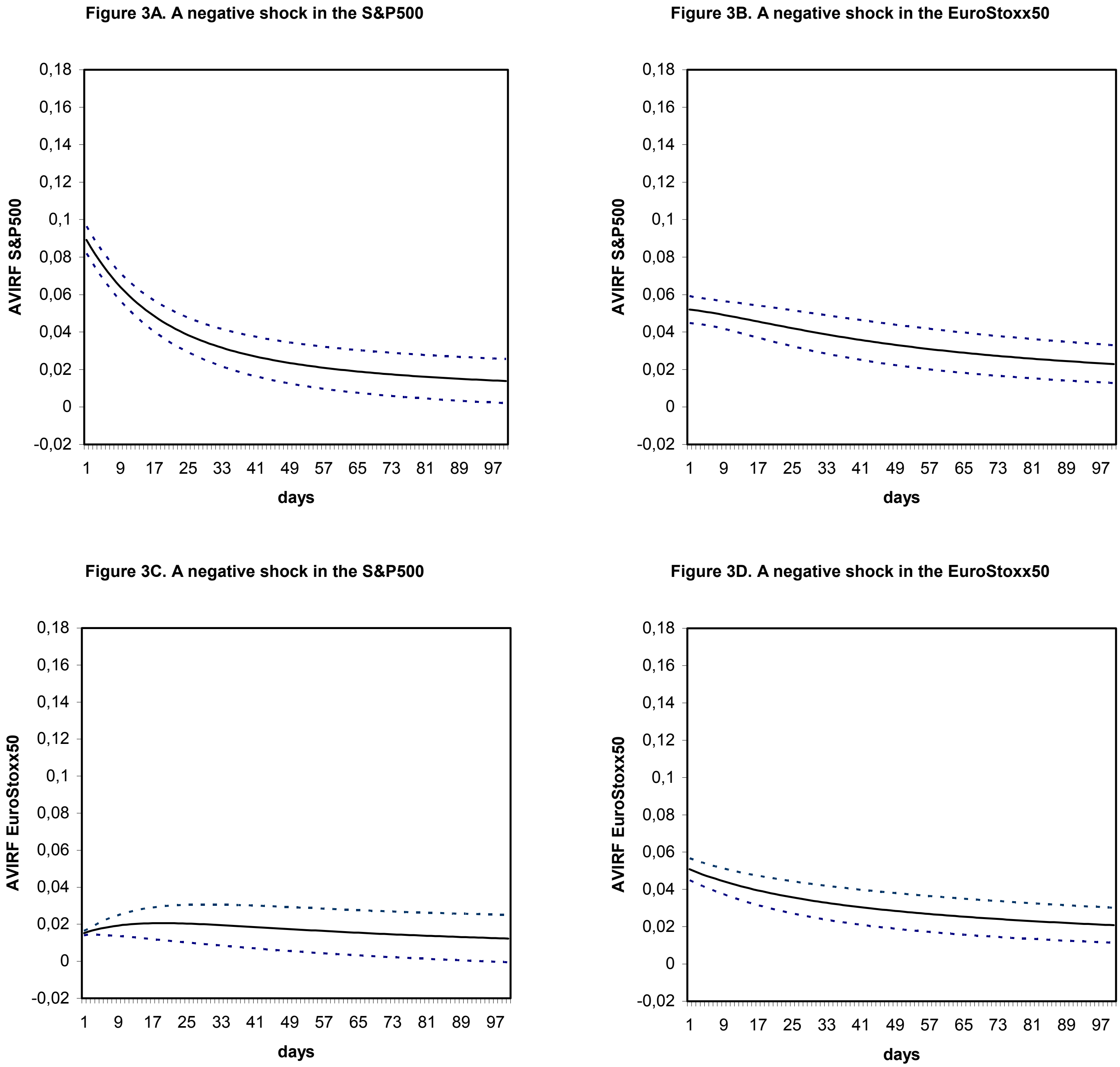

Figure 3. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK
No Crisis Period
(Dashed lines display the 90% confidence interval)

Figure 4A. A negative shock in the S&P500

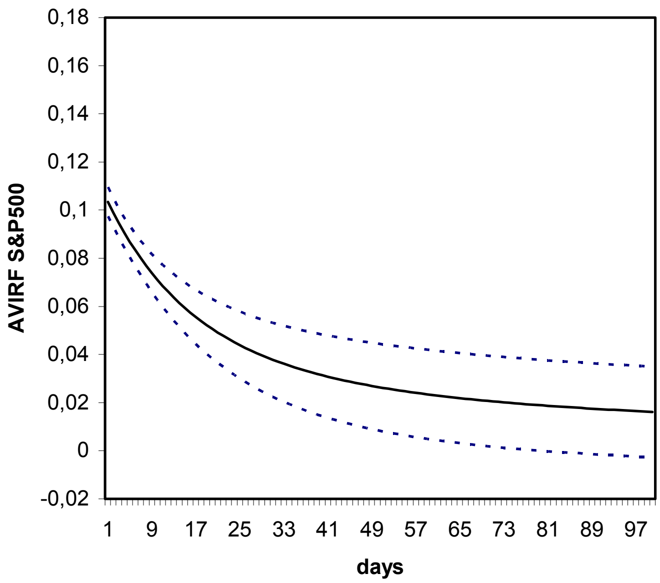

Figure 4B. A negative shock in the EuroStoxx50

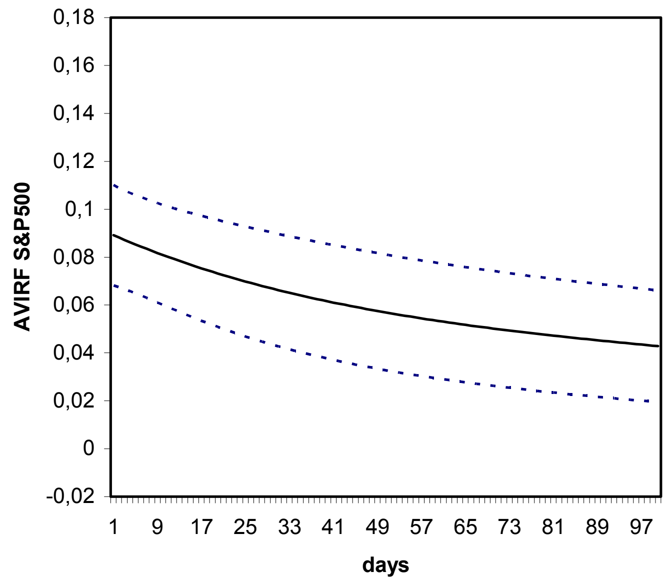

Figure 4C. A negative shock in the S&P500

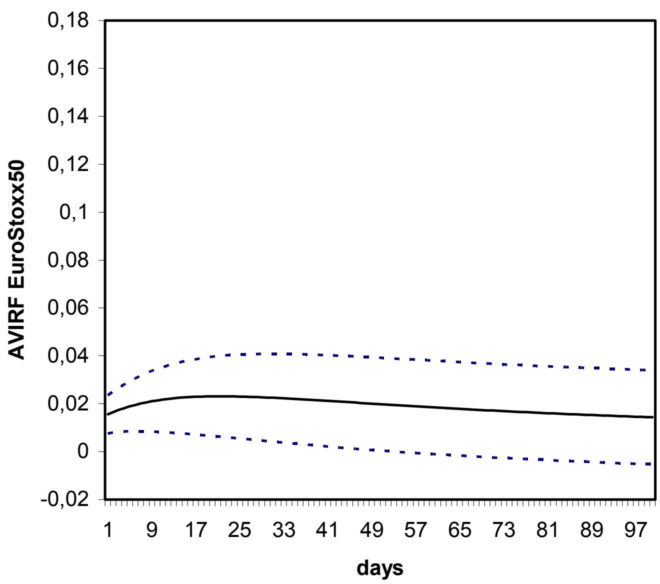

Figure 4D. A negative shock in the EuroStoxx50

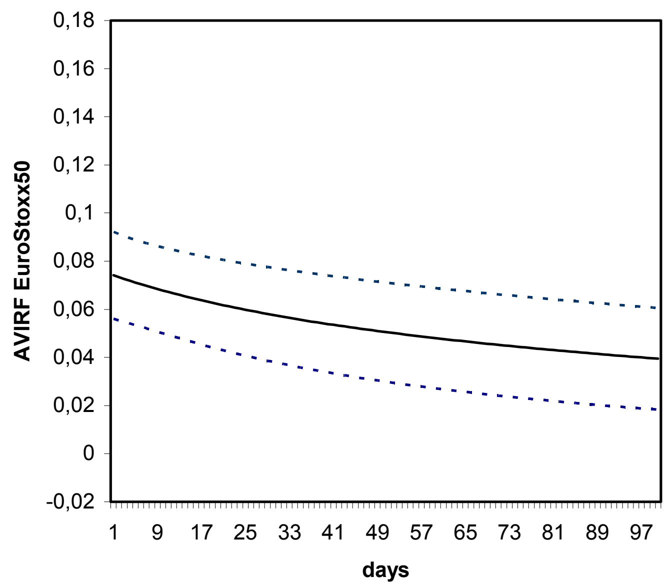

Figure 4. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK
Crisis Period (September 11)
(Dashed lines display the 90% confidence interval)

Figure 5A. A negative shock in the S&P500

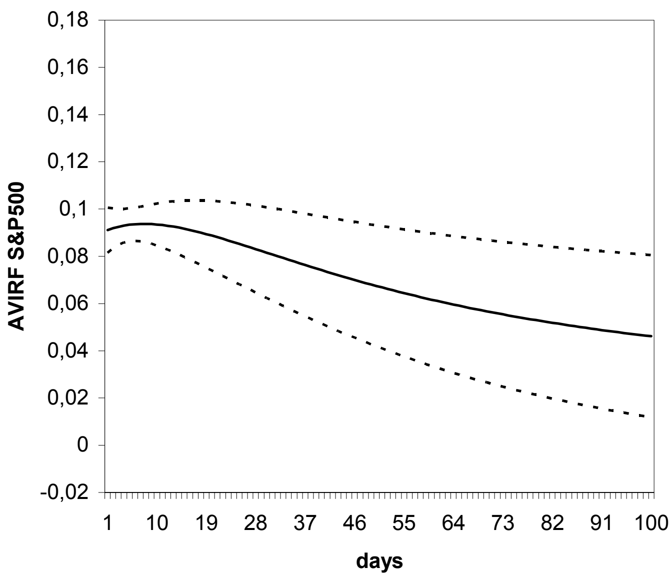

Figure 5B. A negative shock in the EuroStoxx50

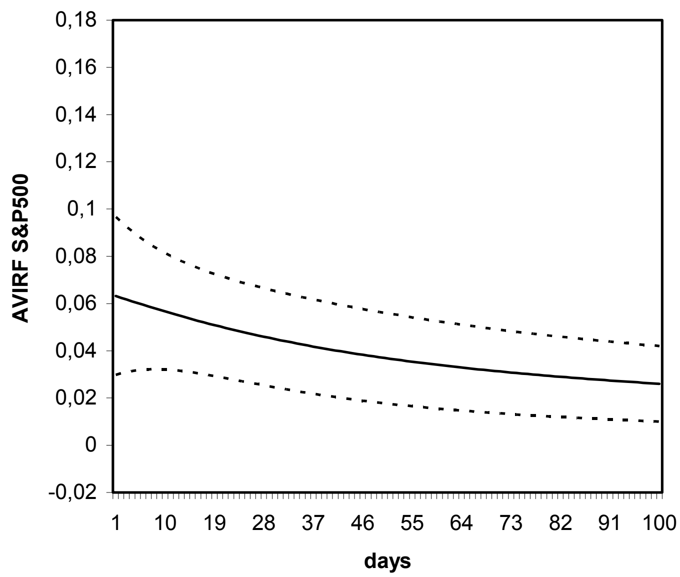

Figure 5C. A negative shock in the S&P500

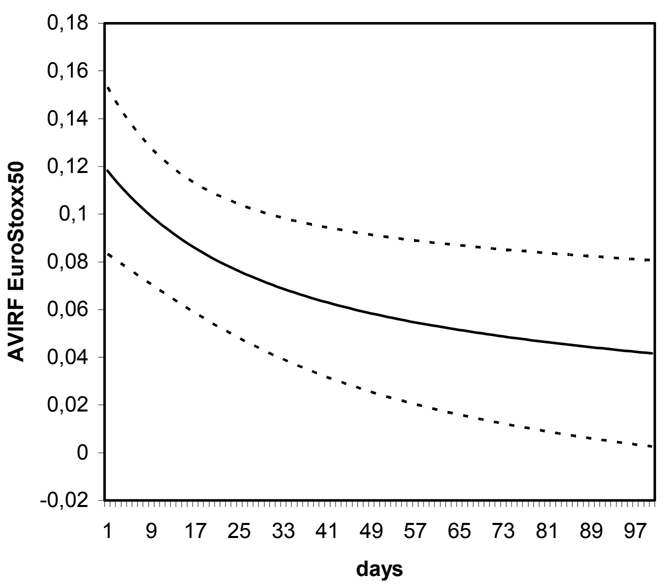

Figure 5D. A negative shock in the EuroStoxx50

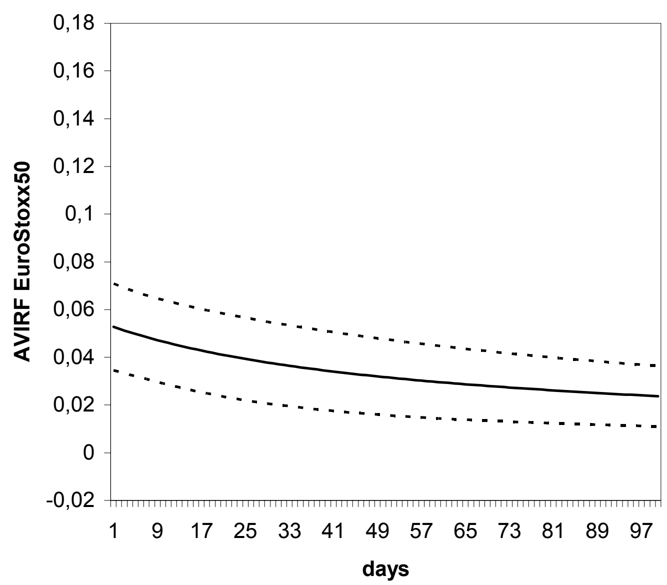

Figure 5. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK
Crisis Period (March 11)
(Dashed lines display the 90% confidence interval)

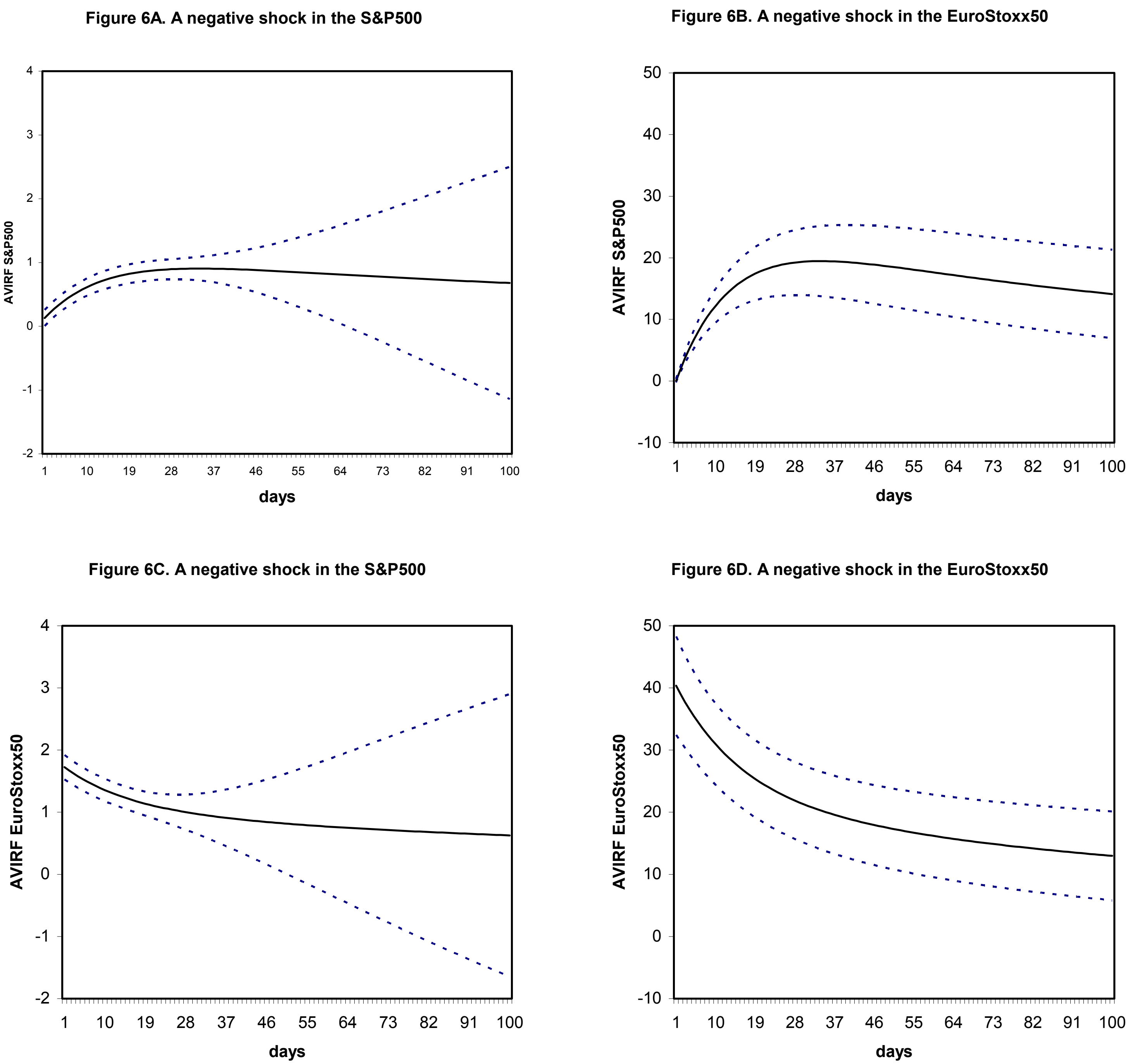

Figure 6. AVIRF to negative unexpected shocks from the VAR-Asymmetric BEKK Crisis Period (July 7)
(Dashed lines display the 90% confidence interval)

Therefore, these results add evidence in favor of the hypothesis of bidirectional variance causality between the S&P500 and the EuroStoxx50, but also in favor of the hypothesis of different reactions to each particular stock market crisis due to a terrorist attack.

5 Conclusion

The main objective of this study is to analyze how volatility transmission patterns are affected by stock market crises. In order to do this, we use a multivariate GARCH model and take into account both the asymmetric volatility phenomenon and the non-synchronous trading problem. In our empirical application, we focus on stock market crises as a result of terrorist attacks and analyze international volatility transmission between the US and Eurozone financial markets.

In particular, an asymmetric VAR-BEKK model is estimated with daily stock market prices recorded at 15:00 GMT time for the US (S&P500 index) and Eurozone (EuroStoxx50 index).

We also introduce a complementary analysis, the Asymmetric Volatility Impulse Response Functions (AVIRF) with crisis, which distinguishes both a) effects coming from a positive shock from those coming from a negative shock, and b) effects coming from periods of stability from those coming from periods of crisis.

The results confirm that there exist asymmetric volatility effects in both markets and that volatility transmission between the US and the Eurozone is bidirectional. The terrorist attack occurred in New York in September 11 affected volatility in the Eurozone stock markets but the terrorist attacks occurred in Madrid and London in March 11 and July 7, respectively, did not affect volatility in the US market.

Based on Johnston and Nedelescu (2006), there are several possible explanations for the differences in stock market reactions to the three terrorist attacks considered. Firstly, the September 11 terrorist attack had a direct impact on several financial markets, such as the aeronautical, tourism, banking or insurance industries. These industries were not so badly affected in the case of the other terrorist attacks considered. Secondly, while the attacks in New York were perceived as a global shock, the attacks on Madrid and London were perceived as mostly having a local and regional effect, respectively. Finally, while the events of September 11 occurred in the midst of a global economic downturn, the terrorist attacks in Madrid and London occurred at a time when the world economy was growing strongly.

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